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Optimal Control for Smoking Epidemic Model

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ABSTRACT
In this study, we used a mathematical model of the smoking epidemic which was divided into three sub-classes. In this model we provide four optimal controls, namely education campaign about cigarettes, vaccines, treatment and rehabilitation. To solve the problem of optimal control, we use the principle of maximum pontryagin, where the objective function is set to increase the population of individuals who quit smoking and reduce the number of smokers. In the solution, we use the numerical method namely Runge Kutta forward-backward sweep order4 and simulated using MATLAB with the values of initial parameters and conditions are given based on the results of a survey that conducted on high school students in several schools in Lamongan. The control provided in this study shows a decrease in the number of individual population of potential smokers and active smokers, as well as an increase in the number of individual ex-smokers. These results also indicate that the system gets the optimal solution according to certain values of the control level.

Keywords: *Mathematical Model, Optimal Control, Runge Kutta Orde-4.*

1. INTRODUCTION

Smoking it is the largest preventable cause of death worldwide and therefore special attention is needed to control it. Illnesses caused by smoking can cause death, the life expectancy for individuals who smoke is cut by 10-12 years [1]. There are about 1 billion people worldwide who are smokers. Smoking is still a habit in some individuals because of the low levels of treatment available and long-term smoking cessation [2]. When compared, between non-smokers and smokers, smokers had a 70% higher risk of having a heart attack than non-smokers. Likewise for lung cancer cases, smokers have a ten times greater risk and smoking will be the cause of death of one in ten people. Smoking causes 80% of several diseases such as cardiovascular, emphysema or bronchitis., 29% comes from cancer lung, and heart disease accounted for 24% [3]. Without this disease, smoking can also cause cancer. All this is not surprising because cigarette smoke contains more than 4000 chemical compounds and toxins, all of which have very dangerous components for human health. In certain cases, smoking can be considered as a contagious pain caused by addiction.

The negative effects caused by smoking can attack almost every organ and can reduce health every people. Causes of death are so many, but smoking can be one of the preventable causes of death. Human health of all ages in Indonesia and around the world can be affected by the negative effects of smoking. In this study, we conducted a survey of high school students in several schools in Lamongan. We consider the data from 200 male students, among the students who participated, 52% had never smoked, 24,5% is a active smoker and 23,5% were ex-smokers that stopped to smoke permanently. The results of these studies indicate that adolescents in Indonesia, especially in Lamongan district, need a cigarette prevention and control program. Many studies about mathematical model of smoking and optimal control that have been done are published, where the compartment model was formulated to investigate their addiction to smoking and infection. Castillo-Garsow et al. [4] Three variables are assumed in that study, namely: potential smoker (P), smoker (S), and stop smoking (Q). Sharomi and Gumel [5] they introducing the lighter class of the previous model by presenting their progress and the public health impacts of smoking.

In this research, we used a mathematical model of the smoking epidemic that was done by Gul Zaman et al [6], from this mathematical model we provide four optimal controls, namely education campaign about cigarettes, vaccines, treatment and rehabilitation. From giving the control, it will be seen how the control behavior in the model that used. Where for the formulation of models with optimal control and the settlement of these controls we use the principle of maximum pontryagin [7][8]. The data that will be used as the initial conditions of the three sub-class are taken from survey thar we have conducted. Therefore in this study we use optimal control technique that describe in the next chapter, to minimize of potential smoker and active smoker also increase the population of ex-smoker.

2. LITERATURE REVIEW

2.1. Mathematical Model

In this part, we discuss the mathematical model of smoking epidemics that will be used in this study. In real epidemics as smallpox, leptospirosis, smoking cessation also chickenpox can use optimal control for the desired purpose [9]. There are many mathematical theories about the concepts of disease and epidemic. Basic ideas in theory this is that everyone in the community is getting healthy. Smoking can infect every individual, and individuals who are infected can become healthy and not infected. For example, total population $N(t)$ shows (shown constant) at time t; $P(t)$ is the number at time t of potential smokers; $T(t)$ is the number of people who smoking every day in the population at time t; $Q(t)$ is the number of ex-smokers who stopped to smoke permanently in population at time t. Three state variables of differential equations non-linier [6].

$$\begin{cases} \dfrac{dP(t)}{dt} = \mu N(t) - \mu P(t) - \dfrac{\alpha P(t)T(t)}{N(t)}, P(0) = P_0 \geq 0, \\ \dfrac{dT(t)}{dt} = \dfrac{\alpha P(t)T(t)}{N(t)} - (\omega + \mu)T(t), T(0) = T_0 \geq 0, \\ \dfrac{dQ(t)}{dt} = \omega T(t) - \mu Q(t), Q(0) = Q_0 \geq 0, \end{cases} \quad (1)$$

where the system rate $N = P(t) + T(t) + Q(t), \mu > 0, \omega > 0$ rate per smoker stop per unit time and effectiveness level α at time t. Rate of the standard incidence in this model of $\alpha P(t)T\,(t)/N$ is considered. Let α the level of effectiveness of the influence is the average of the product of total contacts c and the probability of q being a smoker, namely $\alpha = cq$.

2.2 Optimal Control

Optimal control is a way to determine which control variables will be causing the process to meet multiple physical constraints and to minimize the objective function that has been determined in this study. The formulation requires the process of the control for a description mathematical (or model), the specification of the index performance, and state boundary and physical constraints statement and/or control [10].

The control from the initial state $x(t_0)$ at time t_0 to $x(t_f)$ at the end t_f, in such a way as to provide the maximum or minimum value for a particular objective functional. System dynamics can be expressed mathematically by:

$$\dot{x} = f(x(t), u(t), t) \quad (2)$$

With f is a continuous function that depends on the variable state x and the control variable u for each t. In order for the solution obtained to be the desired solution, criteria are needed, which means that each control $u(t)$ and state variable $x(t)$ is associated with the following function:

$$J = \int_0^{t_f} f_0(x(t), u(t), t)dt \quad (3)$$

with f_0 is a given function, t_f does not have to be determined and $x(t_f)$ has certain conditions.

Among all the functions or control variables obtained, one is determined so that J reaches the maximum value or the minimum value. such control is called optimum control. The optimum control problem can be expressed as a problem of maximizing or minimizing a functional (3) with constraints (2) [11].

2.3 Pontryagin Maximum Principle

The optimum control problem is optimizing (maximizing or minimizing) the functional objective (3). Theorem 1 give the Pontryagin's maximum principle below.

Theorem 1 [12]
Let $u^*(t)$ be an admissible control vector which transfers (x_0, t_0) to a target $(x(T), T)$ where $x(T)$ and T are not specified in general. Let $x^*(t)$ be the trajectory corresponding to $u^*(t)$. In order for that $u^*(t)$ be optimal, it is necessary that there exist a non-zero, continuous vector function $p^*(t) = (p_1^*(t), p_2^*(t), \ldots, p_n^*(t))$ and a constant scalar p_0 such that

(a) $p^*(t)$ and $x^*(t)$ are the solution of the canonical system

$$\dot{x}^*(t) = \frac{\partial H}{\partial p}(x^*, p^*, u^*, t) \quad (4)$$

$$\dot{p}^*(t) = -\frac{\partial H}{\partial x}(x^*, p^*, u^*, t) \quad (5)$$

where

$$H \equiv \sum_0^n p_i f_i(x, p, u, t)$$
$$\equiv f_0(x, u, t) + \sum_1^n p_i f_i(x, u, t) \quad (6)$$

the usual Hamiltonian, with $p_0 \equiv 1$.

(b) $H(x^*, u^*, P^*, t) \geq H(x^*, u, p, t)$.

(c) All boundary conditions be satisfied.

3. DISCUSSION

The strategy for optimal control is investigated in the mathematical model in equation (1). Of the many branches of mathematics, optimal control method that is in great demand and use by other fields of science such as engineering. The theory of optimal control aims to explore the optimal law of control in a system for a certain criteria so that these criteria can be achieved [5]. From these findings, it will be done optimizing the control of smokers in the community with an effective strategy. We will investigate model (1) to control smoking from cigarettes [13]. The control variable that used in the present model is u_1, u_2, u_3, u_4 are represents the education campaigns, vaccination, reducing the pharmacological effects of nicotine and rehabilitation.

First of all, assume that all smokers have the opportunity to be aware of the dangers of smoking (education campaigns) $u_1(t)$, vaccination is $u_2(t)$, so that the potential fraction of smokers $u_1(t)$ and $u_2(t)$ is transferred from class $P(t)$ to class $Q(t)$. Active treatment of $u_3(t)$ against nicotine can reduce smoking relapse rates in heavy smokers by reducing the pharmacological effects of nicotine (anti-nicotine drug / medicine), rehabilitation is also given to smokers $u_4(t)$, resulting in a potential fraction smokers $u_3(t)$ and $u_4(t)$ are transferred from class $T(t)$ to class $Q(t)$. Now, the four optimal controls will be used in model (1). So, we get the following equations [11]:

$$\begin{cases} \dfrac{dP(t)}{dt} = \mu N(t) - (\mu + u_1 + u_2)P(t) - \dfrac{\alpha P(t)T(t)}{N(t)} \\ \dfrac{dT(t)}{dt} = \dfrac{\alpha P(t)T(t)}{N(t)} - (\omega + \mu + u_3 + u_4)T(t) \\ \dfrac{dQ(t)}{dt} = \omega T(t) - \mu Q(t) + (u_1 + u_2)P(t) + (u_3 + u_4)T(t) \end{cases}$$

The optimal control problem in this study has the objective function given by:

$$J(u_1, u_2, u_3, u_4) = \int_0^{t_{end}} \Big(P(t) + T(t) - Q(t) + \frac{1}{2}(k_1 u_1^2(t) + k_2 u_2^2(t) + k_3 u_3^2(t) + k_4 u_4^2(t)) \Big) dt$$

Here positive constans k_1, k_2, k_3 and k_4 are to keep the control size is balance u_1, u_2, u_3, u_4, respectively. The aim of this research is using the possible minimal control variables u_1, u_2, u_3, u_4 to minimize the population of potential smokers and active smoker also to maximize the number of ex-smokers who stopped smoking permanently. Let the assumption $P_0(t), T_0(t)$ and $Q_0(t)$ is the initial values that have chosen steady state with expected value, and μ and ω are the chosen values as follows:

$$P_E(t) = \frac{(\omega + \mu)}{\alpha}, T_E(t) = \left(\frac{1}{\omega + \mu} - \frac{1}{\alpha}\right), Q_E(t) = \left(\frac{1}{\omega + \mu} - \frac{1}{\alpha}\right)\omega N$$

From that equation we may obtain a value for $\alpha = 0.0022$ [6].

Table 1. Smoking statistical data

Variable	P_i	T_i	Q_i
Grade ($10^{th} - 12^{th}$)	104	49	47
Percentage	52%	24,5%	23,5%

3.1 Optimal Control Solution

In this study, we form a lagrangian function as follows:

$$L(P, T, Q, u_1, u_2, u_3, u_4) = P(t) + T(t) - Q(t) + \frac{1}{2}k_1 u_1^2(t) + \frac{1}{2}k_2 u_2^2(t) + \frac{1}{2}k_3 u_3^2(t) + \frac{1}{2}k_4 u_4^2(t)$$

and the Hamiltonian $\mathcal{H}$ is:

$$\begin{aligned}\mathcal{H}(P, Q, T, u_i, \lambda_i, t) &= P(t) + T(t) - Q(t) + \frac{1}{2}(k_1 u_1^2(t) + k_2 u_2^2(t) + k_3 u_3^2(t) + k_4 u_4^2(t)) \\ &+ \lambda_1 \left(\mu N(t) - (\mu + u_1(t) + u_2(t))P(t) - \frac{\alpha P(t)T(t)}{N(t)} \right) \\ &+ \lambda_2 \left(\frac{\alpha P(t)T(t)}{N(t)} - (\omega + \mu + u_3(t) + u_4(t))T(t) \right) \\ &+ \lambda_3 \big(\omega T(t) - \mu Q(t) + (u_1(t) + u_2(t))P(t) + (u_3(t) + u_4(t))T(t) \big)\end{aligned}$$

Where $u_i, i = 1,2,3,4$ and the adjoint functions is $\lambda_i, i = 1,2,3$.

Stationary condition is a condition in which the derivative $\mathcal{H}$ against u_i equals 0, as follows:

$$\frac{\partial \mathcal{H}}{\partial u_1} = k_1 u_1{}^*(t) - \lambda_1 P(t) + \lambda_3 P(t) = 0$$

$$u_1{}^*(t) = \frac{\lambda_1 P(t) - \lambda_3 P(t)}{k_1}$$

$$\frac{\partial \mathcal{H}}{\partial u_2} = k_2 u_2{}^*(t) - \lambda_1 P(t) + \lambda_3 P(t) = 0$$

$$u_2{}^*(t) = \frac{\lambda_1 P(t) - \lambda_3 P(t)}{k_2}$$

$$\frac{\partial \mathcal{H}}{\partial u_3} = k_3 u_3{}^*(t) - \lambda_2 T(t) + \lambda_3 T(t) = 0$$

$$u_3{}^*(t) = \frac{\lambda_2 T(t) - \lambda_3 T(t)}{k_3}$$

$$\frac{\partial \mathcal{H}}{\partial u_4} = k_4 u_4{}^*(t) - \lambda_2 T(t) + \lambda_3 T(t) = 0$$

$$u_4{}^*(t) = \frac{\lambda_2 T(t) - \lambda_3 T(t)}{k_4}$$

because $u_1{}^*(t) = u_2{}^*(t) = u_3{}^*(t) = u_4{}^*(t)$ closed set [0,1], then

$$u_1{}^*(t) = min\left\{max\left\{0, \frac{\lambda_1 P(t) - \lambda_3 P(t)}{k_1}\right\}, 1\right\}$$

$$u_2{}^*(t) = min\left\{max\left\{0, \frac{\lambda_1 P(t) - \lambda_3 P(t)}{k_2}\right\}, 1\right\}$$

$$u_3{}^*(t) = min\left\{max\left\{0, \frac{\lambda_2 T(t) - \lambda_3 T(t)}{k_3}\right\}, 1\right\}$$

$$u_4{}^*(t) = min\left\{max\left\{0, \frac{\lambda_2 T(t) - \lambda_3 T(t)}{k_4}\right\}, 1\right\}$$

State Equation $\dot{P}^*(t), \dot{T}^*(t)$ and $\dot{Q}^*(t)$ is obtained by reducing $\mathcal{H}^*$ to λ_i where $i = 1,2,3$ as follows:

$$\dot{P}^*(t) = \frac{\partial \mathcal{H}^*}{\partial \lambda_1}$$

$$= \mu N^*(t) - (\mu + u_1{}^*(t) + u_2{}^*(t))P^*(t) - \frac{\alpha P^*(t) T^*(t)}{N^*(t)}$$

$$\dot{T}^*(t) = \frac{\partial \mathcal{H}^*}{\partial \lambda_2}$$

$$= \frac{\alpha P^*(t) T^*(t)}{N^*(t)} - (\omega + \mu + u_3{}^*(t) + u_4{}^*(t))T^*(t)$$

$$\dot{Q}^*(t) = \frac{\partial \mathcal{H}^*}{\partial \lambda_3}$$

$$= \omega T^*(t) - \mu Q^*(t) + (u_1{}^*(t) + u_2{}^*(t))P^*(t) + (u_3{}^*(t) + u_4{}^*(t))T^*(t)$$

We form a Hamiltonian function with each state variable so that the adjoint equations is obtained $\lambda_i{}^*(t)$ where $i = 1,2,3$ as follows:

$$\lambda_1{}^*(t) = -\frac{\partial \mathcal{H}^*}{\partial P^*} = -\Bigg(1 - \left(\mu + u_1{}^*(t) + u_2{}^*(t) - \frac{\alpha T^*(t)}{N^*(t)}\right)\lambda_1 + \lambda_2 \frac{\alpha T^*(t)}{N^*(t)} + (u_1{}^*(t) + u_2{}^*(t))\lambda_3\Bigg)$$

$$\lambda_2{}^*(t) = -\frac{\partial \mathcal{H}^*}{\partial T^*} = -\Bigg(1 - \lambda_3 \frac{\alpha P^*(t)}{N^*(t)} - \left(\omega + \mu + u_3{}^*(t) + u_4{}^*(t) - \frac{\alpha P^*(t) T^*(t)}{N^*(t)}\right)\lambda_2 + (\omega + u_3{}^*(t) + u_4{}^*(t))\lambda_3\Bigg)$$

$$\lambda_3{}^*(t) = -\frac{\partial \mathcal{H}^*}{\partial Q^*} = -(-1 - \mu\lambda_3).$$

4. NUMERICAL SIMULATIONS

In this section, to solve the optimally system we used a numerical method. The several variables we used forward-backward sweep method is essentially identical to the scheme used with one state and one control. First, for each control we made an initial condition, this initial condition is over the interval ($u \equiv 0$ is almost always sufficient), then the all states will be solved forward in time and for the adjoints function will be solved backward in time and this process is repeated until the system convergence. Any differential systems solver can be used to solve the states and adjoints. As Runge-Kutta 4 has been used to this point, we will now make use of Runge-Kutta 4 for systems.

Settlement the control in this study using forward-backward sweep Runge-Kutta order4. Numerical settlement to optimize the system in this problem was solved using MATLAB with a hypothetical parameter value realistic and given initial conditions $\mu = 0.019$, $\omega = 0.00015$, $\alpha = 0.0022$, $k_1 = k_2 = k_3 = k_4 = 100$, $P(0) = 104, T(0) = 49, Q(0) = 47$.

4.1 Potential smoker population $P(t)$

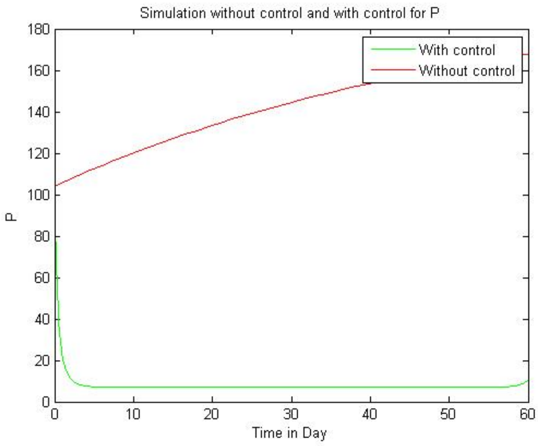

Figure 1 Potential smoker population $P(t)$ with control and without control.

The red line shows that potential smoker without control and the green line shows that potential smoker after giving control optimal. At t_f shows that the potential smokers has decreased and at the end of control shows that stop increasing. In the first week shows that the population of potential smokers has decreased sharply. Potential smoker systems with controls stabilized around days 4-58 and after day 58 increased slightly.

4.2 Smoker population $T(t)$

It shows the population system of active smokers before being given a control and after being given a control. Population of Individual smokers with controls decrease dramatically and go to 0.

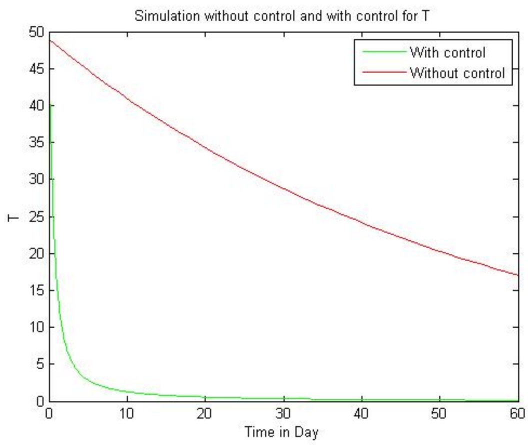

Figure 2 Smoker population $T(t)$ with control and without control

4.3 Population of ex-smokers $Q(t)$

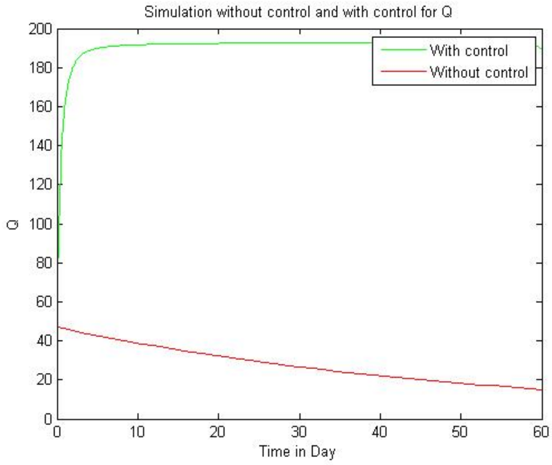

Figure 3 Population of ex-smokers $Q(t)$ with control and without control

Before control was carried out the number of individuals who stopped smoking decreased. after control was assigned to a population of ex-smokers, indicating that the population of ex-smokers increased. This suggests that giving control to a population of ex-smokers works well.

It can be seen in the figure that has been presented, where in the simulation we used the initial conditions taken from a survey that we conducted on high school students in several schools in Lamongan. That for the first subclass, namely the population of potentially smoking individuals $P(t)$ without control showed an increase at time t, on the other hand after being given a control population showed a significant decrease. For the second subclass, namely the individual smoker population $T(t)$, compared to without control and after being given control, it decreased significantly and was closer to 0 at the end of the time t_f. In the third subclass, before being given a control, the population of individuals who quit smoking $Q(t)$ had decreased, whereas after being given control showed an increase. It shows that the control provided can work well and fulfill the objective function that was given, that is to increase the population of potential smoking individuals and the smokers population and increase the population of individuals who quit smoking.

5. CONCLUSION

In this study, the problem of optimal control of the epidemic model is carried out and numerical simulation on the model with four control variables. In order for the pain to go away the stable model must be downgraded to baseline in case of the initial infectious invasion rate. In this study, there are four optimal control strategies, namely education campaigns, vaccines, rehabilitation and treatment to minimize population of potential smokers and active smokers also maximizing individuals who stop smoking. From the numerical calculations and simulations carried out in this study, it shows of susceptible individuals or potential smokers and active smokers decreases, and ex-smokers increase optimally. These results also indicate that the system gets the optimal solution according to certain values of the control level.

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