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Research Article

Multiobjective Programming Approaches to Obtain the Priority Vectors under Uncertain Probabilistic Dual Hesitant Fuzzy Preference Environment

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ABSTRACT

This paper develops uncertain probabilistic dual hesitant fuzzy numbers (UPDHFN), which includes six types of dual hesitant fuzzy sets (DHFNs). Next, the UPDHFN is applied to the uncertain probabilistic dual hesitant fuzzy preference relation (UPDHFPR). Furthermore, the (acceptable) expected consistency, method of obtaining uncertain probabilistic information, and consistency-increasing iterative algorithm for flexible application of UPDHFPRs are explained respectively. Then, the UPDHFPRs and these approaches are applied to group decision-making procedure. Two operators are established to aggregate the UPDHFPRs and the integrated preference relations are also UPDHFPRs. In this model, due to the aggregated UPDHFPRs may be inconsistent. Thus an acceptable group consistency algorithm is designed. The group decision-making process is summarized under the UPDHFPR situation. Eventually, an illustrate example that selects the optimal alternative from three listed candidates is provided to verify our methods.

1. INTRODUCTION

Through specific mathematical notation, fuzzy idea [1] is an effective tool to express the epistemic uncertainty and vagueness of data. Fuzzy set has played an increasingly important role in many practical engineering fields, which include decision-making problems [2], algebraic structure [3], fuzzy preference relations (PR) [4], fuzzy information clustering [5], and fuzzy granular computing [6]. According to practical situations, different forms of fuzzy set have been proposed, which involve interval fuzzy set [7,8], intuitionistic fuzzy (IF) set [9], (probabilistic) hesitant fuzzy set [10], fuzzy rough set [11], shadowed set [12], proportional hesitant fuzzy linguistic term set [13,14], dual hesitant fuzzy set (DHFS)[15], neutrosophic set [16].

The DHFS as a new theory that that specifies membership within a set by giving multiple hesitant values and nonmembership within a set by assigning multiple hesitant values to each membership. Therefore, the DHFS can depict uncertain memberships and nonmemberships more flexibility than other fuzzy theory, and has received extensive attention from scholars. Besides, some extended DHFSs have been established, like interval-valued DHFS [17], dual interval-valued hesitant fuzzy set [18], dual hesitant fuzzy soft set [19], dual hesitant fuzzy rough set [17]. The probabilities of the factors in the above DHFSs are same, which is obviously unrealistic.

Then, the occurrence probabilistic dual hesitant fuzzy set (PDHFS) was established and applied in risk evaluation [20]. Since then, more and more scholars have devoted themselves to the research in this field, Ren *et al.* studied the strategy selection problem on artificial intelligence with analytic hierarchy process (AHP) method under probabilistic dual hesitant fuzzy environment [21]. Garg and Kaur presented an algorithm based on aggregation operators with new distance measures, and applied to probabilistic hesitant fuzzy environment [22]. Garg and Kaur established a robust correlation coefficient for PDHFSs and applied in a case study based on personnel selection [23]. Meanwhile Garg and Kaur used PDHFS to solve the problem of quantification of gesture information of patients with cerebral hemorrhage, and achieved excellent results [24].

But in the actual operation of PDHFS, we found that it is difficult to accurately and fully give the objective information of the membership function under the probabilistic dual hesitant fuzzy environment. For instance, a decision maker (DM) depicts the satisfaction of an alternative with a DHFN $\langle\{0.4, 0.5, 0.6\}, \{0.1, 0.2, 0.3\}\rangle$. DM holds that the satisfaction level associated with 0.4 and dissatisfaction level associated with 0.3 are certain, the probabilities can be determined as 0.2 and 0.3, respectively. The satisfaction levels associated with 0.5, 0.6

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and dissatisfaction levels associated with 0.1, 0.2 are hesitant, their probability are uncertain, In this situation, the DHFN and PDHFS are invalid. After research, the uncertain probabilistic dual hesitant fuzzy set (UPDHFS) is more flexible to express the decision information in practical circumstance. Thus, in this paper, the UPDHFS and its PRs are focused.

Recently, PRs based on different conditions have been considered, including the fuzzy PRs [25,26], the additive (multiplicative) PRs [27,28], the interval-value intuitionistic fuzzy preference relations (IVIPRs) [29], intuitionistic fuzzy preference relation (IVFPR) [30], incomplete HFPRs [31], which extended methods the DMs' epistemic information by assigning to each factor in a universe some different membership degrees. Jin and Garg presented multiplicative consistency adjustment model and applied to data envelopment analysis driven decision-making problems under the probabilistic hesitant fuzzy PRs [32]. As a new idea for expressing the epistemic information, the DHFS depicts membership and nonmembership by a set of multi values, respectively. Therefore, the dual hesitant fuzzy preference relations (DHFPRs) could be more practical to express epistemic uncertainty preference information than other types of fuzzy PR. Because DM provides the epistemic uncertain information over candidates, he (or she) may provides the degrees that one situation is preferred and nonpreferred to another, in which are described by a set of multi values, respectively. The key advantage of the DHFPRs is that it can describes more epistemic information by expressing the hesitant degrees than one situation is preferred to another, and the hesitant degrees that the situation is nonpreferred to another.

When faced with group decision-making problems, it is difficult for DMs to cognitively background the involved situation in the same method since they mostly come from different domains and have different conceptions on the situation [33]. Then, some dissimilar might happen. Usually, it is promised to adjust these epistemic information through communication and argument until a satisfactory consistency is concluded. In the consistent analysis, the DMs often get into the tricky situations surrounding the measurement of cognitive consistency of DMs. One method to calculate the cognitive consistency is to require DMs to individually complete a questionary measuring that has been used to evaluate content domains [34]. The lack of acceptable consistency, which means that there are major differences among the selections, can make for the dissatisfied or unreasonable decision conclusions. The compatibility to estimate the difference among multiplicative PRs was presented [35]. Xu [36] came up with a compatibility index among PRs under the interval fuzzy situations. Next, in Xu [37] established different compatibility measures for IF preference relations (IFPRs) and interval-valued IFPR (IVIFPRs), and proposed some consistent improving procedures based on above measures. Jiang *et al.* [38] came up with the compatibility index among intuitionistic multiplicative preference relations (IMPRs), and presented some consistency models.

According to the above analysis, three significant issues should be focused in exploiting new PRs: (1) developed DHFPRs from different viewing angle (in this paper, the DHFPRs are investigated based on the presented UPDHFS); (2) consistency analysis and development of new DHFS; (3) group decision-making models based on new DHFPRs. Therefore, an effective extension of the UPDHFS is to establish a corresponding preference mode which can be utilized to the preference (group) decision-making courses.

The following remainder of this paper is introduced as follows: the development of UPDHFS is presented in Section 2. the definition of UPDHFPRs are proposed and its expected consistency is defined. The approach that calculate its uncertain probability information is constructed. An iterative algorithm that develops consistency is designed in Section 3. In Section 4, the UPDHFPRs are applied to group decision making (GDM) by considering the group consistency. In Section 5, an elucidative example which about selecting the optimal candidate from three newly listed candidates to explain the practicability and flexibility of established method. Eventually, the conclusion are summarized.

The following table is used to explain the important abbreviations in this paper.

Abbreviation	Explanation
DHFS	Dual hesitant fuzzy set
DHFN	Dual hesitant fuzzy number
IF	Intuitionistic fuzzy set
PDHFS	Probabilistic hesitant fuzzy set
PR	Preference relations
DHFPR	Dual hesitant fuzzy PR
DM	Decision maker

2. PRELIMINARY KNOWLEDGE

In this part, we will introduce some basic notions and properties of PDHFS and distance measure.

Definition 1. [20] Suppose that X is a finite reference set, a PDHFS is described by the following formula,

$$\tilde{P} = \left\{ \left\langle x, \tilde{h}(x)|\tilde{p}(x), \tilde{g}(x)|\tilde{q}(x) \right\rangle \mid x \in X \right\} \tag{1}$$

The $\tilde{h}(x)|\tilde{p}(x), \tilde{g}(x)|\tilde{q}(x)$ are two factors of some hesitant information where $h(x), g(x)$ depict the corresponding hesitant fuzzy membership information and hesitant fuzzy nonmembership information of x. $\tilde{p}(x), \tilde{q}(x)$ are the probabilistic values for these two types of hesitant fuzzy data. $\tilde{P}$ holds the following conditions, $0 \leqslant \tilde{\gamma}_i, \tilde{\eta}_j \leqslant 1, 0 \leqslant \tilde{\gamma}^+ + \tilde{\eta}^+ \leqslant 1, \tilde{p}_i \in [0,1], \tilde{q}_j \in [0,1]$

$$\sum_{i=1}^{\#\tilde{h}} \tilde{p}_i = \sum_{i=1}^{\#\tilde{g}} \tilde{q}_j = 1 \tag{2}$$

$\#\tilde{h}, \#\tilde{g}$ are the cardinal number factors in $\tilde{h}(x)|\tilde{p}(x), \tilde{g}(x)|\tilde{q}(x)$, respectively.

Generally, we define the formula $\tilde{P} = \langle \tilde{h}(x)|\tilde{p}(x), \tilde{g}(x)|\tilde{q}(x)\rangle$ as a probabilistic dual hesitant fuzzy number (PDHFN), described by $\tilde{P} = \langle \tilde{h}|\tilde{p}, \tilde{g}|\tilde{q}\rangle$ for convenient.

Definition 2. [20] The complement of a PDHFN $\tilde{P} = \langle \tilde{h}|\tilde{p}, \tilde{g}|\tilde{q}\rangle$ is defined as

$$\tilde{P}^c = \begin{cases} \cup_{\tilde{\gamma}\in\tilde{h},\tilde{\eta}\in\tilde{g}} \langle \{\tilde{\eta}|q_{\tilde{\eta}}\}, \{\tilde{\gamma}|p_{\tilde{\gamma}}\}\rangle, & if\ \tilde{h} \neq \phi, \tilde{g} \neq \phi \\ \cup_{\tilde{\gamma}\in\tilde{h}} \langle \{1-\tilde{\gamma}|\tilde{p}_{\tilde{\gamma}}\}, \{\phi\}\rangle, & if\ \tilde{h} \neq \phi, \tilde{g} = \phi \\ \cup_{\tilde{\eta}\in\tilde{g}} \langle \{\phi\}, \{1-\tilde{\eta}|\tilde{q}_{\tilde{\eta}}\}\rangle, & if\ \tilde{h} = \phi, \tilde{g} \neq \phi \end{cases} \tag{3}$$

In this paper, we only consider the situation of $\tilde{h} \neq \phi, \tilde{g} \neq \phi$. The other situations are omitted.

Generally, the mathematical notation $\tilde{P} = \langle \tilde{h}|\tilde{p}, \tilde{g}|\tilde{q}\rangle$ depicts a PDHFN, the functions

$$S(\tilde{P}) = \sum_{i=1}^{\#\tilde{h}} \tilde{\gamma}_i \cdot \tilde{p}_i - \sum_{j=1}^{\#\tilde{g}} \tilde{\eta}_j \cdot \tilde{q}_j \tag{4}$$

$$D(\tilde{P}) = \left(\sum_{i=1}^{\#\tilde{h}} (\tilde{\gamma}_i - S(\tilde{P})^2) \cdot \tilde{p}_i + \sum_{j=1}^{\#\tilde{g}} (\tilde{\eta}_j - S(\tilde{P})^2 \cdot \tilde{q}_j \right)^{\frac{1}{2}} \tag{5}$$

expresses the score function and deviation function of PDHFN, respectively. Hao *et al.* further establish the comparison approach and some basic operation laws,

1. The comparison method of PDHFNs $\tilde{P}_1, \tilde{P}_2$, If $S(\tilde{P}_1) \geq S(\tilde{P}_2)$, then $\tilde{P}_1 \geq \tilde{P}_2$. If $S(\tilde{P}_1) = S(\tilde{P}_2)$, then (i) if $D(\tilde{P}_1) \geq D(\tilde{P}_2)$, then $\tilde{P}_1 \geq \tilde{P}_2$; (ii) if $S(\tilde{P}_1) \leq S(\tilde{P}_2)$, then $\tilde{P}_1 \leq \tilde{P}_2$; (iii) if $S(\tilde{P}_1) = S(\tilde{P}_2)$, the $\tilde{P}_1 = \tilde{P}_2$.
2. The basic operation laws of PDHFNs $P = (\tilde{h}|\tilde{p}, \tilde{g}|\tilde{q})$, $\tilde{P}_1 = (\tilde{h}_1|\tilde{P}_1, \tilde{g}_1|\tilde{q}_1)$, $\tilde{P}_2 = (\tilde{h}_2|\tilde{P}_2, \tilde{g}_2|\tilde{q}_2)$, $\lambda \geq 0$,

$$\tilde{P}_1 \oplus \tilde{P}_2 = \bigcup_{\substack{\tilde{\gamma}_1 \in \tilde{h}_1, \tilde{\eta}_1 \in \tilde{g}_1; \\ \tilde{\gamma}_2 \in \tilde{h}_2, \tilde{\eta}_2 \in \tilde{g}_2}} \left\langle \{(\tilde{\gamma}_1 + \tilde{\gamma}_2 - \tilde{\gamma}_1\tilde{\gamma}_2)|\tilde{p}_{\tilde{\gamma}_1}\tilde{p}_{\tilde{\gamma}_2}\}, \{(\tilde{\eta}_1\tilde{\eta}_2)|\tilde{q}_{\tilde{\eta}_1}\tilde{q}_{\tilde{\eta}_2}\} \right\rangle \tag{6}$$

$$\tilde{P}_1 \otimes \tilde{P}_2 = \bigcup_{\substack{\tilde{\gamma}_1 \in \tilde{h}_1, \tilde{\eta}_1 \in \tilde{g}_1; \\ \tilde{\gamma}_2 \in \tilde{h}_2, \tilde{\eta}_2 \in \tilde{g}_2}} \left\langle \{(\tilde{\gamma}_1\tilde{\gamma}_2)|\tilde{p}_{\tilde{\gamma}_1}\tilde{p}_{\tilde{\gamma}_2}\}, \{(\tilde{\eta}_1 + \tilde{\eta}_2 - \tilde{\eta}_1\tilde{\eta}_2)|\tilde{q}_{\tilde{\eta}_1}\tilde{q}_{\tilde{\eta}_2}\} \right\rangle \tag{7}$$

$$\lambda\tilde{P} = \bigcup_{\tilde{\gamma}\in\tilde{h},\tilde{\eta}\in\tilde{g}} \left\langle \{(1-(1-\tilde{\gamma})^{\lambda})|\tilde{p}_{\lambda}\}, \{\tilde{\eta}^{\lambda}|\tilde{q}_{\tilde{\eta}}\} \right\rangle \tag{8}$$

$$\tilde{p}^{\lambda} = \bigcup_{\tilde{\gamma}\in\tilde{h},\tilde{\eta}\in\tilde{g}} \left\langle \{\tilde{\gamma}^{\lambda}|\tilde{p}_{\lambda}\}, \{(1-(1-\tilde{\eta})^{\lambda})|\tilde{q}_{\tilde{\eta}}\} \right\rangle \tag{9}$$

It is noteworthy that the corresponding probabilities of hesitant fuzzy membership (nonmembership) degrees are different to get or estimate. Thus, some probability information are lose in PDHFNs, then the notion of UPDUFS is introduced as follows,

Definition 3. Let X be a finite reference set. A UPDHFS is depicted as

$$P = \{\langle x, h(x)|p(x), g(x)|q(x)\rangle | x \in X\} \tag{10}$$

where $h(x)|p(x), g(x)|q(x)$ expresses two sets of some factors, $p_i \in p(x)$ describes a given probability or an lose probability of $\tilde{\gamma} \in h(x)$, $q_j \in q(x)$ describes a given probability or an lose probability of $\tilde{\eta} \in g(x)$. P_U satisfies the following requests, $0 \le \gamma_i, \eta_j \le 1, 0 \le \gamma^+ + \eta^+ \le 1, p_i \in [0,1], q_j \in [0,1]$,

$$\sum_{i=1}^{\#h} p_i = \sum_{j=1}^{\#g} q_j = 1 \tag{11}$$

The mathematical notation $\#h, \#g$ are the corresponding cardinal number factors in $h(x)|p(x), g(x)|q(x)$.

Generally, we rule the mathematical symbol $P = \langle h(x)|p(x), g(x)|q(x)\rangle$ is an uncertain probabilistic dual hesitant fuzzy number (UPDHFN), denoted by $P = (h|p, g|q)$.

Next, some special cases of UPDHFN are listed as follows:

1. If probabilities of all types of membership elements is unknown, then UPDHFN is called full uncertain probabilistic dual hesitant fuzzy number (FUPDHFN), denoted by

$$\left\langle \begin{array}{l} \{\gamma_1|x_1, \gamma_2|x_2, \cdots, \gamma_{\#h}|x_{\#h}\}, \\ \{\eta_1|x_1', \eta_2|x_2', \cdots, \eta_{\#g}|x_{\#g}'\} \end{array} \right\rangle \tag{12}$$

   E.g., $\langle\{0.2|x_1, 0.3|x_2, 0.4|x_3\}, \{0.1|x_1', 0.4|x_2', 0.5|x_3'\}\rangle$;

2. If probabilities of some of the elements are unknown, then UPDHFN is called partially uncertain probabilistic dual hesitant fuzzy number (PUPDHFN), denoted by

$$\left\langle \begin{array}{l} \{\gamma_1|p_1, \gamma_2|x_2, \cdots, \gamma_{\#h}|p_{\#h}\}, \\ \{\eta_1|x_1', \eta_2|q_2, \cdots, \eta_{\#g}|q_{\#g}\} \end{array} \right\rangle \tag{13}$$

   E.g., $\langle\{0.2|0.4, 0.3|x_2, 0.4|0.5\}, \{0.1|x_1', 0.4|0.2, 0.5|x_3'\}\rangle$;

3. If probabilities of all types of membership elements is known, then UPDHFN reduces to a PDHFN, denoted by

$$\left\langle \begin{array}{l} \{\gamma_1|p_1, \gamma_2|p_2, \cdots, \gamma_{\#h}|p_{\#h}\}, \\ \{\eta_1|q_1, \eta_2|q_2, \cdots, \eta_{\#g}|q_{\#g}\} \end{array} \right\rangle \tag{14}$$

   E.g., $\langle\{0.2|0.4, 0.3|0.1, 0.4|0.5\}, \{0.1|0.2, 0.4|0.2, 0.5|0.6\}\rangle$;

4. If probabilities of all types of membership elements are equal to 1, then UPDHFN reduces to a DHFN, denoted by

$$\langle\{\gamma_1, \gamma_2, \cdots, \gamma_{\#h}\}, \{\eta_1, \eta_2, \cdots, \eta_{\#g}\}\rangle \tag{15}$$

   E.g., $\langle\{0.2, 0.3, 0.4\}, \{0.1, 0.4, 0.5\}\rangle$;

5. If probabilities of all types of membership elements are equal to 1 and $\{g(x)|q(x)\} = \phi$, then UPDHFN reduces to a HFN, denoted by $\langle\{\gamma_1, \gamma_2, \cdots, \gamma_{\#h}\}, \rangle$, E.g., $\langle\{0.2, 0.3, 0.4\}\rangle$;

6. If probabilities of membership degrees are known and $\{g(x)|q(x)\} = \phi$, then UPDHFN reduces to a PHFN, denoted by

$$\langle\{\gamma_1, \gamma_2, \cdots, \gamma_{\#h}\}\rangle \tag{16}$$

   E.g., $\langle\{0.2|0.4, 0.3|0.1, 0.4|0.5\}\rangle$.

According to above special cases, we can get the following consequences:

1. The UPDHFN is an extended DHFN, includes six types of HFNs, the FUPDHFN, PUPDHFN, PDHFN, DHFN, PHFN, HFN.

2. The UPDHFN is a general situation of DHFN, where all probability values are equal to 1.
3. Since probabilities can be regarded as unknown parameters, thus the FUPDHFN and PUPDHFN are more comment to describe the uncertain situations than PDHFN and DHFN. Absolutely, the unknown probability parameters create more difficult contexts for final evaluation conclusion. Thus, establish a method to obtain the unknown parameters is important.
4. The FUPDHFN, PUPDHFN and PDHFN integrate more uncertain data than DHFN.
5. The PDHFN is a special FUPDHFN (PUPDHFN), since all subjective information is known in a PDHFN.

By the above introduction, it is a key step to calculate optimal probability values. Therefore, in this paper, we pay attention to the FUPDHFN and PUPDHFN.

3. UNCERTAIN PROBABILISTIC DUAL HESITANT FUZZY PREFERENCE RELATIONS AND PROBABILITY COMPUTE

Based on the notion of UPDHFS, a UPDHFN is more comprehensive and flexible to explain objective and subjective cognitive information from the evaluators than other types of DHFNs. Thus, a flexible generalization of the UPDHFN is to establish the corresponding PRs which is conducive to solve group decision-making problems under the preference consistency environment. First of all, we establish a uncertain probabilistic dual hesitant fuzzy preference relations (UPDHFPRs). Secondly, we construct a consistency test and a consistent enrichment approach. Meanwhile, we project a method to calculate the unknown probabilistic information of the UPDHFPR.

3.1. UPDHFPR and Expected Consistency

Similarly to IVFPRs and hesitant fuzzy PRs, the UPDHFPR is established as follows:

Definition 4. Let $X = \{x_1, x_2, \cdots, x_m\}$ be a finite reference set, then a matrix $U = (d_ij)_{m\times m} \in X \times X$ is defined as a UPHFPR on X, where $i, j = 1, 2, \cdots, m$, $d_{ij} = (h_{ij}|p_{ij}, g_{ij}|q_{ij}) = (\{\gamma_{ij,1}|p_{ij,1}, \gamma_{ij,2}|p_{ij,2}, cdots, \gamma_{ij,\#h}|p_{ij,\#h},\}, \{\eta_{ij,1}|q_{ij,1}, \eta_{ij,2}|q_{ij,2}, cdots, \eta_{ij,\#g}|q_{ij,\#h},\})$ is a UPDHFN, h_{ij} describes all possible degrees that x_i is preferred to x_j, p_{ij} is the corresponding probability of h_{ij}, which can be obtain probabilities or unknown probabilities. g_{ij} describes all possible degrees that x_i is nonpreferred to x_j, q_{ij} is the corresponding probability of h_{ij}, which can be obtain probabilities or unknown probabilities. On the side, d_{ij} must hold the following demands,

1. (i) $h_{ij} = h_{ji}, p_{ij} = p_{ji}; g_{ij} = g_{ji}, q_{ij} = q_{ji}$ when $h_{ij} \neq \phi, g_{ij} \neq \phi$. (ii) $\gamma_{ij,l} = 1 - \gamma_{ji,\#h-l+1}, p_{ij,l} = p_{ji,\#h-l+1}$ when $h_{ij} \neq \phi, g_{ij} = \phi$. (iii) $\eta_{ij,l} = 1 - \eta_{ji,\#g-l+1}, q_{ij,l} = q_{ji,\#h-l+1}$ when $h_{ij} = \phi, g_{ij} \neq \phi$. $i, j = 1, 2, \cdots, m, i \neq j$.
2. $d_{ii} = \langle\{0.5|1\}, \{0.5|1\}\rangle$.
3. $\gamma_{ij,l} < \gamma_{ij,l+1}, \gamma_{ji,l+1} < \gamma_{ji,l}, \eta_{ij,k} < \eta_{ij,k+1}, \eta_{ji,k+1} < \eta_{ji,k}$, where $i, j = 1, 2, \cdots, m, i < j$, $\sum_{l=1}^{\#h_{ij}} p_{ij,l} = \sum_{l=1}^{\#h_{ji}} p_{ji,l} = 1, \sum_{k=1}^{\#g_{ij}} q_{ij,k} = \sum_{k=1}^{\#g_{ij}} q_{ji,k} = 1$.

Based on Definition 4, we can acquire the following consequences:

1. The basic factors in UPDHFPRs are UPDHFNs.
2. About the upper triangular matrix of UPDHFPRs, the factors in the UPDHFNs are nondecreasing. Meanwhile, about the lower triangular matrix of UPDHFPRs, the factors in the UPDHFNs are nonincreasing.
3. The diagonal line of the UPDHFPRs are $d_{ii} = \langle\{0.5|1\}, \{0.5|1\}\rangle$, where is coincident with IVFPR and HFPRs.

It is found that (2) is considered to a tool for simplified compute under a UPDHFPR situation.

Generally, the consistency is identified as a key element in uncertain PRs. "$\prec$" denotes a PR, if $A \prec B, B \prec C$, then $A \prec C$. Due to the complexity of the reality situations and epistemic uncertainty of estimator, the estimation information can not hold consistency PRs. Thus, researchers established a consistency test to demonstrate this "$\prec$" relation. However, the UPDHFPR is considered to a general fuzzy PR, it should also hold this condition. Until now, multiplicative consistency and additive consistency are identified as two different forms of expression of fuzzy PRs. In this paper, the multiplicative consistency of UPDHFPRs are discussed, the multiplicative expected consistency test approach is established.

Definition 5. [39] If $X = \{x_1, x_2, \cdots, x_m\}$ be a finite reference set, $\tilde{R} = (c_{ij})_{m\times m} = \{\gamma_{ij}, \eta_{ij}\}_{m\times m}$ be a IVFPR, it holds the following conditions:

$$\gamma_{ij} = \eta_{ji}, \eta_{ij} = \gamma_{ji}, r_{ij} = \{0.5, 0.5\} \tag{17}$$

Let $w = (w_1, w_2, \cdots, w_m) = ([w_{1l}, w_{1u}], [w_{2l}, w_{2u}], \cdots, [w_{ml}, w_{mu}])$ be an interval priority vector of the multiplicative consistent IFIPR R, then

$$\gamma_{ij} = T\frac{w_{il}}{w_{il} + w_{ju}}, 1 - \eta_{ij} = T\frac{w_{iu}}{w_{jl} + w_{iu}} \tag{18}$$

where T is a parameter.

Definition 6. [40] The interval priority vector $w = (w_1, w_2, \cdots, w_m) = ([w_{1l}, w_{1u}], [w_{2l}, w_{2u}], \cdots, [w_{ml}, w_{mu}])$ is called to normalized if and only if $(i = 1, 2, \cdots, m)$

$$w_{il} + \sum_{j=1, j\neq i}^{m} w_{ju} \geq 1; w_{iu} + \sum_{j=1, j\neq i}^{m} w_{jl} \leq 1 \tag{19}$$

Definition 7. Let $P = \langle h|p, g|q \rangle$ be a UPDHFN, then its expected value can be described by

$$\begin{aligned} E(d_{ij}) &= e = \frac{e_1 + e_2}{2} \\ &= \frac{\sum_{a=1}^{\#h} \gamma_a p_a + (1 - \sum_{b=1}^{\#g} \eta_b q_b)}{2} \end{aligned} \tag{20}$$

where $\#h, \#g$ are the total numbers of factors in $h|p, g|q$, respectively.

According to Definition 6, an (multiplicative) expected consistency for a UPDHFPR is defined.

Definition 8. Let $X = \{x_1, x_2, \cdots, x_m\}$ be a finite reference set, $R = (d_{ij})_{m\times m} = (h_{ij}|p_{ij}, g_{ij}|q_{ij})_{m\times m}$ be a UPDHFPR matrix, in which d_{ij} is a UPDHFN, then R holds the expected consistency, if

$$e_{ij,1} = \sum_{l=1}^{\#h} \gamma_{ij,l} p_{ij,l} = T\frac{w_{il}}{w_{il} + w_{ju}} \tag{21}$$

$$e_{ij,2} = 1 - \sum_{k=1}^{\#g} \eta_{ij,k} q_{ij,k} = T\frac{w_{iu}}{w_{jl} + w_{iu}} \tag{22}$$

$$\begin{aligned} e_{ij,3} &= e_{ij,2} - e_{ij,1} \\ &= T\frac{w_{iu}}{w_{jl}+w_{iu}} - T\frac{w_{il}}{w_{il}+w_{ju}} \end{aligned} \tag{23}$$

where $i, j = 1, 2, \cdots, m$ and $i < j$, $\#h, \#g$ are the corresponding total numbers of factors in $h_{ij}|p_{ij}, g_{ij}|q_{ij}$. p_{ij}, q_{ij} represses the corresponding probability of h_{ij}, g_{ij}, $w = \{w_1, w_2, \cdots, w_m\}$ is a normalized interval priority vector of R, $w_i \in [0, 1]$ and

$$w_{il} + \sum_{j=1, j\neq i}^{m} w_{iu} \geq 1, i = 1, 2, \cdots, m \tag{24}$$

$$w_{iu} + \sum_{j=1, j\neq i}^{m} w_{il} \leq 1, i = 1, 2, \cdots, m \tag{25}$$

Definition 9. [41] For a priority vector $\Omega = (w_1, w_2, \cdots, w_n)$ of the consistent UPDHFPRs A, the interval vector Ω to the form of the intuitionistic fuzzy elements

$$\begin{aligned} \psi = &((w_{1l}, 1 - w_{1u}, w_{1u} - w_{1l}), (w_{2l}, 1 - w_{2u}, w_{2u} - w_{2l}), \\ &\cdots, (w_{nl}, 1 - w_{nu}, w_{nu} - w_{nl})) \end{aligned} \tag{26}$$

where w_{il} is explained as the hesitant membership degrees of the importance of x_i, $1 - w_{iu}$ is explained as the hesitant nonmembership degrees of the importance of x_i, $w_{iu} - w_{il}$ is explained as the hesitation degree of the importance of x_i, $i \in m$. ψ is defined as the priority vector of the multiplicative consistent UPDHFPRs R, if Eqs. (21–23) are held.

Definition 10. [42] Let $\psi_1 = ((w_{1l}, 1-w_{1u}, w_{1u}-w_{1l}))$ and $\psi_2 = ((w_{2l}, 1-w_{2u}, w_{2u}-w_{2l}))$ be two priority vectors, $\Delta(S_1) = w_{1l}-(1-w_{1u})$ and $\Delta(S_2) = w_{2l}-(1-w_{2u})$ be the corresponding scores of ψ_1 and ψ_2. Let $\Delta(H_1) = w_{1u}+w_{1l}$ and $\Delta(H_2) = w_{2u}+w_{2l}$ be the corresponding accuracy degrees of ψ_1 and ψ_2. Then

- When $\Delta(S_1) \le \Delta(S_2)$, then $\psi_1 \le \psi_2$.
- When $\Delta(S_1) = \Delta(S_2)$, $\Delta(H_1) \le \Delta(H_2)$, then $\psi_1 \le \psi_2$.
- When $\Delta(S_1) = \Delta(S_2)$, $\Delta(H_1) = \Delta(H_2)$, then $\psi_1 = \psi_2$.

Lemma 1. If $e_{ij} = T\dfrac{w_{il}}{w_{il}+w_{ju}} + T\dfrac{w_{iu}}{w_{jl}+w_{iu}}$ is right, then

$$e_{ji} = T\frac{w_{ju}}{w_{il}+w_{ju}} + T\frac{w_{jl}}{w_{jl}+w_{iu}} \tag{27}$$

Proof. For $d_{ji} = (h_{ji}|p_{ji}, g_{ji}|q_{ji})$, based on Definition 4, then

$$\begin{aligned}
e_{ji} &= \sum_{l=1}^{\#h\prime} \gamma_{ji,l}p_{ji,l} + \left(1 - \sum_{k=1}^{\#g\prime} \eta_{ji,k}q_{ji,k}\right)\\
&= \sum_{k=1}^{\#g} \eta_{ij,k}q_{ij,k} + \left(1 - \sum_{l=1}^{\#h} \gamma_{ij,l}p_{ji,l}\right)\\
&= -\left(\sum_{l=1}^{\#h} \gamma_{ij,l}p_{ji,l}\right) - \left(1 - \sum_{k=1}^{\#g} \eta_{ij,k}q_{ij,k}\right) + 2\\
&= \left[1 - \left(\sum_{l=1}^{\#h} \gamma_{ij,l}p_{ji,l}\right)\right] + \left[1 - \left(1 - \sum_{\#g}^{k=1} \eta_{ij,k}q_{ij,k}\right)\right]\\
&= \left[1 - T\frac{w_{il}}{w_{il}+w_{ju}}\right] + \left[1 - T\frac{w_{iu}}{w_{jl}+w_{iu}}\right]\\
&= T\frac{w_{ju}}{w_{il}+w_{ju}} + T\frac{w_{ji}}{w_{jl}+w_{iu}}
\end{aligned} \tag{28}$$

which completes the proof of Lemma 1. ☒

On the other hand, it is easy to hold UPDHFPRs with perfectly expected consistency. Hence, an acceptable expected consistency is defined as follows.

Definition 11. Let $U = (d_{ij})_{m\times m}$ be a UPDHFPR matrix, $d_{ji} = (h_{ji}|p_{ji}, g_{ji}|q_{ji})$, if

$$\begin{aligned}
CI &= \tfrac{1}{2n(n-1)}\left(\left|e_{ij,1} - \tfrac{w_{il}}{w_{il}+w_{ju}}\right| + \left|e_{ij,2} - \tfrac{w_{iu}}{w_{jl}+w_{iu}}\right|\right)\\
&\le \varepsilon,
\end{aligned} \tag{29}$$

then we call U holds the acceptable expected consistency, where ξ is a threshold value.

Then a rule needs to be defined: When $CI = 0$, then the UPDHFPRs hold expected consistency, the confidence value is 99%. When $CI \le 0.01$, then the confidence value of the expected consistency is 99%. The confidence value of expected consistency is 97%, when $CI \le 0.03$. Generally, when $CI \le 0.05$, we consider the expected consistency of UPDHFPRs is acceptable. Under most circumstances, the expected consistency may be unacceptable. Next, we investigate a iterative algorithm to increase the confidence value of consistency.

3.2. Probability Assessments for UPDHFPRs

Under the practical circumstances, the UPDHFPRs often are inconsistent. Furthermore, the probability information of every hesitant fuzzy membership value is significative. Therefore, by increasing the consistency and assessing probability information, the UPDHFPRs can be more effectively applied to decision problems.

In this section, we establish an approach to assess probability values for UPDHFPRs and increase the confidence value of consistency. First of all, we need to a model to calculate probability information.

$$\min \xi_{ij} = \frac{1}{2}\left(\left|e_{ij,1} - T\frac{w_{il}}{w_{il}+w_{ju}}\right| + \left|e_{ij,2} - T\frac{w_{iu}}{w_{jl}+w_{iu}}\right|\right)$$
$$= \frac{1}{2}\left(\left|\sum_{a=1}^{\#h}\gamma_{ij,a}p_{ij,a} - T\frac{w_{il}}{w_{il}+w_{ju}}\right| + \left|\left(1-\sum_{\#g}^{b=1}\eta_{ij,b}q_{ij,b}\right) - T\frac{w_{iu}}{w_{jl}+w_{iu}}\right|\right) \tag{30}$$

$$s.t.\begin{cases}\sum_{a=1}^{\#h} p_{ij,a} = 1, p_{ij,a}\in[0,1]\\ \sum_{k=b}^{\#g} q_{ij,b} = 1, q_{ij,b}\in[0,1]\\ w_{iu}-w_{il}\geqslant 0, w_{iu}, w_{il}\geqslant 0\\ w_{il}+\sum_{j=1,}^{m} w_{il}\leqslant 1, iw_{iu}\geqslant 1, i=1,2,\cdots,m\\ w_{iu}+\sum_{j=1,j\neq i}^{m} w_{il}\leqslant 1, i=1,2,\cdots,m\\ i,j=1,2,\cdots,m\end{cases}$$

According to Eq. (30) and Lemma 1, we can get Lemma 2 and.

Lemma 2. If $i=j$, then $\frac{1}{2}\left(\left|\sum_{a=1}^{\#h}\gamma_{ij,a}p_{ij,a} - T\frac{w_{il}}{w_{il}+w_{ju}}\right| + \left|\left(1-\sum_{b=1}^{\#g}\eta_{ij,b}q_{ij,b}\right) - T\frac{w_{iu}}{w_{jl}+w_{iu}}\right|\right) = \frac{1-T}{2}$.

Proof. By Lemma 1, the conclusion is obviously. If $T=1$, then

$$\frac{1}{2}\left(\left|\sum_{a=1}^{\#h}\gamma_{ij,a}p_{ij,a} - T\frac{w_{il}}{w_{il}+w_{ju}}\right| + \left|\left(1-\sum_{b=1}^{\#g}\eta_{ij,b}q_{ij,b}\right) - T\frac{w_{iu}}{w_{jl}+w_{iu}}\right|\right) = 0.$$

□

Lemma 1. From Eq. (30), we have

$$\frac{1}{2}\left(\left|e_{ij,1} - T\frac{w_{il}}{w_{il}+w_{ju}}\right| + \left|e_{ij,2} - T\frac{w_{iu}}{w_{jl}+w_{iu}}\right|\right) = \frac{1}{2}\left(\left|e_{ji,1} - T\frac{w_{jl}}{w_{jl}+w_{iu}}\right| + \left|e_{ji,2} - T\frac{w_{ju}}{w_{il}+w_{ju}}\right|\right) \tag{31}$$

Proof. According to Eq. (30), we know

$$\left|\sum_{a=1}^{\#h}\gamma_{ij,a}p_{ij,a} - T\frac{w_{il}}{w_{il}+w_{ju}}\right| = \left|\sum_{b'=1}^{\#g'}\eta_{ji,b'}q_{ji,b'} - \left(1 - T\frac{w_{ju}}{w_{il}+w_{ju}}\right)\right|$$
$$= \left|\left(1-\sum_{b'=1}^{\#g'}\eta_{ji,b'}q_{ji,b'}\right) - T\frac{w_{ju}}{w_{il}+w_{ju}}\right)\right|$$

Similarity, the following equation is obtained

$$\left|\left(1-\sum_{b=1}^{\#g}\eta_{ij,b}q_{ij,b}\right) - T\frac{w_{iu}}{w_{jl}+w_{iu}}\right| = \left|\sum_{a'=1}^{\#h'}\gamma_{ji,a'}p_{ji,a'} - T\frac{w_{jl}}{w_{jl}+w_{iu}}\right|.$$

Then,

$$\left|\sum_{a=1}^{\#h}\gamma_{ij,a}p_{ij,a} - T\frac{w_{il}}{w_{il}+w_{ju}}\right| + \left|\left(1-\sum_{b=1}^{\#g}\eta_{ij,b}q_{ij,b}\right) - T\frac{w_{iu}}{w_{jl}+w_{iu}}\right|$$
$$= \left|\sum_{a'=1}^{\#h'}\gamma_{ji,a'}p_{ji,a'} - T\frac{w_{jl}}{w_{jl}+w_{iu}}\right| + \left|\left(1-\sum_{b'=1}^{\#g'}\eta_{ji,b'}q_{ji,b'}\right) - T\frac{w_{ju}}{w_{il}+w_{ju}}\right)\right|.$$

which completes the proof of Lemma 1□

Based on Lemma 2 and 1, the Eq. (30) that assesses the probability information can be transformed into Eq. (32).

$$\begin{aligned}\min \xi_{ij} &= \frac{1}{2}\left(\left|e_{ij,1} - T\frac{w_{il}}{w_{il}+w_{ju}}\right| + \left|e_{ij,2} - T\frac{w_{iu}}{w_{jl}+w_{iu}}\right|\right) \\ &= \frac{1}{2}\left(\left|\sum_{a=1}^{\#h} \gamma_{ij,a}p_{ij,a} - T\frac{w_{il}}{w_{il}+w_{ju}}\right| + \left|\left(1-\sum_{b=1}^{\#g}\eta_{ij,b}q_{ij,b}\right) - T\frac{w_{iu}}{w_{jl}+w_{iu}}\right|\right)\end{aligned} \tag{32}$$

$$s.t.\begin{cases}\sum_{a=1}^{\#h}\gamma_{ij,a}p_{ij,a} = 1, p_{ij,l}\in[0,1] \\ \sum_{b=1}^{\#g}\eta_{ij,b}q_{ij,b} = 1, q_{ij,k}\in[0,1] \\ w_{iu}-w_{il}\geqslant 0, w_{iu}, w_{il}\geqslant 0 \\ w_{il}+\sum_{j=1,j\neq i}^{m} w_{iu}\geqslant 1, i=1,2,\cdots,m \\ w_{iu}+\sum_{j=1,j\neq i}^{m} w_{il}\leqslant 1, i=1,2,\cdots,m \\ i,j=1,2,\cdots,m \\ i<j\end{cases}$$

Obviously, Eq. (32) is a multi-objective programming model, it is difficulty to be straightly settled. In order to solve this problem, an optimization model to help us obtain an optimal result.

$$\min \xi_{ij} = \sum_{i=1}^{m-1}\sum_{j=i+1}^{m}(A_{ij}x_{ij}^{+} + B_{ij}x_{ij}^{-} + C_{ij}y_{ij}^{+} + D_{ij}y_{ij}^{-}) \tag{33}$$

$$s.t.\begin{cases}\sum_{l=1}^{\#h}\gamma_{ij,l}p_{ij,l}\left(w_{il}+w_{ju}\right) - Tw_{il} - A_{ij}x_{ij}^{+} + B_{ij}x_{ij} = 0 \\ \left(1-\sum_{k=1}^{\#g}\eta_{ij,k}q_{ij,k}\right)\left(w_{iu}+w_{jl}\right) - Tw_{iu} - C_{ij}y_{ij}^{+} + D_{ij}y_{ij} = 0 \\ \sum_{l=1}^{\#h}p_{ij,l} = 1, p_{ij,l}\in[0,1] \\ \sum_{k=1}^{\#g}q_{ij,k} = 1, q_{ij,k}\in[0,1] \\ w_{il}+\sum_{j=1,j\neq i}^{m} w_{ju}\geqslant 1 \\ w_{iu}+\sum_{j=1,j\neq i}^{m} w_{jl}\leqslant 1 \\ w_{iu}-w_{il}\geqslant 0 \\ x_{ij}^{+}\leqslant 0, x_{ij}^{-}\leqslant 0, y_{ij}^{+}\leqslant 0, y_{ij}^{-}\leqslant 0 \\ i,j=1,2,\cdots,m.\end{cases}$$

where x_{ij}^{+} and y_{ij}^{+} describe the positive deviation, x_{ij}^{-} and y_{ij}^{-} describe the negative deviation. Meanwhile, A_{ij}, B_{ij}, C_{ij} and D_{ij} are the weights to $x_{ij}^{+}, x_{ij}^{-}, y_{ij}^{+}$ and y_{ij}^{-}, respectively.

Obviously, Eq. (33) includes five unknown parameters, $A_{ij}, B_{ij}, C_{ij}, D_{ij}$ and T. Generally, we sign that all targets ξ_{ij} are equitable, namely, $A_{ij} = B_{ij} = C_{ij} = D_{ij} = T = 1$. Therefore, Eq. (1) can be reduced to Eq. (34).

$$\min \xi = \sum_{i=1}^{m-1}\sum_{j=i+1}^{m}(x_{ij}^{+} + x_{ij}^{-} + y_{ij}^{+} + y_{ij}^{-}) \tag{34}$$

$$s.t.\begin{cases}\sum_{l=1}^{\#h}\gamma_{ij,l}p_{ij,l}\left(w_{il}+w_{ju}\right)-w_{il}-x_{ij}^{+}+x_{ij}^{-}=0\\ \left(1-\sum_{k=1}^{\#g}\eta_{ij,k}q_{ij,k}\right)\left(w_{iu}+w_{jl}\right)-w_{iu}-y_{ij}^{+}+y_{ij}^{-}=0\\ \sum_{l=1}^{\#h}p_{ij,l}=1,p_{ij,l}\in[0,1]\\ \sum_{k=1}^{\#g}q_{ij,k}=1,q_{ij,k}\in[0,1]\\ w_{il}+\sum_{j=1,j\neq i}^{m}w_{ju}\geqslant 1\\ w_{iu}+\sum_{j=1,j\neq i}^{m}w_{jl}\leqslant 1\\ w_{iu}-w_{il}\geqslant 0\\ x_{ij}^{+}\leqslant 0,x_{ij}^{-}\leqslant 0,y_{ij}^{+}\leqslant 0,y_{ij}^{-}\leqslant 0\\ i,j=1,2,\cdots,m\end{cases}$$

Eqs. (30) and (32–34) are four models to assess probability information for UPDHFPRs. But, we can find only Eq. (34) can be calculated. Next, we utilize Eq. (34) to compute the probability information of the UPDHFPRs.

Thus, our proposed model not only makes the modified fuzzy preference information have consistency, but also gives the optimal result for the unknown probability information while retaining the original data of the decision maker as much as possible.

4. GROUP DECISION-MAKING UNDER THE GROUP CONSISTENCY

In decision support system, the completely multiplicative consistent UPDHFPRs are not commonly provided by experts. Because experts have their own inherent values, it is difficult to avoid the disagreement among experts. Therefore, group consistency is very important in group decision-making. Group consistency can ensure that the decision-making results can be accepted by all members in the case of dissenting minority members.

Established the probability-assessing method, we can obtain unknown probabilities and get the ranking weights of the UPDHFPR matrix. There is one UPDHFPR matrix when a DM is contained in the decision-making situation. In order to add the application situation of the built UPDHFPRs. Next, we further research the group decision-making situations under the UPDHFPR environment. To accomplish this course, firstly, we investigate the group consistency under the UPDHFPR situation, then introduce the consistent group decision-making steps.

4.1. Group Consistency

To integrate all UPDHFPR information and test the whole consistency, we establish two aggregate operators for UPDHFPRs

Definition 12. Let $R_1=(d_{ij}^1)_{m\times m}=(\{h_{ij}^1|p_{ij}^1,g_{ij}^1|q_{ij}^1\})_{m\times m}$, $R_2=(d_{ij}^2)_{m\times m}=(\{h_{ij}^2|p_{ij}^2,g_{ij}^2|q_{ij}^2\})_{m\times m}$ be two UPDHFPRs, r_1,r_2 be corresponding weight information of R^1,R^2, $r_1+r_2=1$, then their weighted multiplicative operator $R=(d_{ij})_{m\times m}$ is defined as

$$d=d_1^{r_1}\otimes d_2^{r_2} \tag{35}$$

1. When $i\le j$,

$$d=d_1^{r_1}\otimes d_2^{r_2}=\bigcup_{\substack{\gamma_1\in h_{ij}^1,\eta_1\in g_{ij}^1;\\ \gamma_2\in h_{ij}^2,\eta_2\in g_{ij}^2}}\left\langle\{\gamma_1^{r_1}\cdot\gamma_2^{r_2}|p_{\gamma_1}p_{\gamma_2}\},\{1-(1-\eta_1)^{r_1}\cdot(1-\eta_2)^{r_2}|q_{\eta_1}q_{\eta_2}\}\right\rangle,$$

2. When $i>j$,

$$d=d_1^{r_1}\otimes d_2^{r_2}=\bigcup_{\substack{\gamma_1\in h_{ij}^1,\eta_1\in g_{ij}^1;\\ \gamma_2\in h_{ij}^2,\eta_2\in g_{ij}^2}}\left\langle\{1-(1-\gamma_1)^{r_1}\cdot(1-\gamma_2)^{r_2}|p_{\gamma_1}p_{\gamma_2}\},\{\eta_1^{r_1}\cdot\eta_2^{r_2}|q_{\eta_1}q_{\eta_2}\},\right\rangle$$

Theorem 1. *Let $R_1 = (d^1_{ij})_{m\times m} = (\{h^1_{ij}|p^1_{ij}, g^1_{ij}|q^1_{ij}\})_{m\times m}$, $R_2 = (d^2_{ij})_{m\times m} = (\{h^2_{ij}|p^2_{ij}, g^2_{ij}|q^2_{ij}\})_{m\times m}$ be two UPDHFPRs, r_1, r_2 be two corresponding weights, then the integrated PRs $R = (d_{ij})_{m\times m} = (\{h_{ij}|p_{ij}, g_{ij}|q_{ij}\})_{m\times m}$ are UPDHFPRs.*

Proof.

1. When $h_{ij}, g_{ij} \neq \phi$, $i \leq j$, (i)Since

$$d_{ij} = \bigcup_{\substack{\gamma_1 \in h^1_{ij}, \eta_1 \in g^1_{ij}; \\ \gamma_2 \in h^2_{ij}, \eta_2 \in g^2_{ij}}} \left\langle \{\gamma_1^{r_1} \cdot \gamma_2^{r_2} | p_{\gamma_1} p_{\gamma_2}\}, \{1-(1-\eta_1)^{r_1}(1-\eta_2)^{r_2} | q_{\eta_1} q_{\eta_2}\} \right\rangle$$

thus we have $\gamma_1^{r_1} \cdot \gamma_2^{r_2}, 1-(1-\eta_1)^{r_1}(1-\eta_2)^{r_2} \in [0,1]$, $p_{\gamma_1}p_{\gamma_2}, q_{\eta_1}q_{\eta_2} \in [0,1]$, $\sum p_{\gamma_1}p_{\gamma_2} = \sum q_{\eta_1}q_{\eta_2} = 1$. (ii) And

$$\begin{aligned} d_{ii} &= \left\langle \{\gamma_1^{r_1} \cdot \gamma_2^{r_2} | p_{\gamma_1} p_{\gamma_2}\}, \{1-(1-\eta_1)^{r_1}(1-\eta_2)^{r_2} | q_{\eta_1} q_{\eta_2}\} \right\rangle. \\ &= \langle \{0.5|1\}, \{0.5|1\} \rangle. \end{aligned}$$

(iii) Meanwhile,

$$\begin{aligned} d_{ji} &= \left\langle \{1-(1-\gamma_1')^{r_1} \cdot (1-\gamma_2')^{r_2} | p_{\gamma_1'} p_{\gamma_2'}\}, \{\eta_1'^{r_1} \cdot \eta_2'^{r_2} | q_{\eta_1'} q_{\eta_2'}\} \right\rangle \\ d_{ij} &= \left\langle \{\gamma_1^{r_1} \cdot \gamma_2^{r_2} | p_{\gamma_1} p_{\gamma_2}\}, \{1-(1-\eta_1)^{r_1} \cdot (1-\eta_2)^{r_2} | q_{\eta_1} q_{\eta_2}\} \right\rangle \end{aligned}$$

by Definitions 4 and 12, obviously, $d_{ij} = d_{ji}$.

2. When $h_{ij} \neq \phi, g_{ij} = \phi$.
(i) On account of $d_{ij} = \cup_{\gamma_1 \in h^1_{ij}, \gamma_2 \in h^2_{ij}} \langle \{\gamma_1^{r_1} \cdot \gamma_2^{r_2} | p_{\gamma_1} p_{\gamma_2}\}, \{\phi\} \rangle$. Hence $\gamma_1^{r_1} \cdot \gamma_2^{r_2} \in [0,1]$, $p_{\gamma_1}p_{\gamma_2} \in [0,1]$, $\sum p_{\gamma_1}p_{\gamma_2} = 1$.
(ii) Furthermore, we know

$$\begin{aligned} d_{ii} &= \bigcup_{0.5 \in h^1_{ii}, 0.5 \in h^2_{ii}} \langle \{0.5^{r_1} \cdot 0.5^{r_2} | p_{\gamma_1} p_{\gamma_2}\}, \{\phi\} \rangle \rangle \\ &= \langle \{0.5|1\}, \{\phi\} \rangle. \end{aligned}$$

(iii) Simultaneously, Definition 12,

$$\begin{aligned} d_{ij} &= \bigcup_{\gamma_1 \in h^1_{ij}, \gamma_2 \in h^2_{ij}} \langle \{\gamma_1^{r_1} \cdot \gamma_2^{r_2} | p_{\gamma_1} p_{\gamma_2}\}, \{\phi\} \rangle \\ d_{ji} &= \bigcup_{\gamma_1' \in h^1_{ij}, \gamma_2' \in h^2_{ij}} \left\langle \{1-(1-\gamma_1')^{r_1}(1-\gamma_2')^{r_2} | p_{\gamma_1'} p_{\gamma_2'}\}, \{\phi\} \right\rangle \end{aligned}$$

Next, by Definition 4, $\gamma_1' = 1-\gamma_1, \gamma_2' = 1-\gamma_2, p_{\gamma_1} = p_{\gamma_1'}, p_{\gamma_2} = p_{\gamma_2'}$, then $1-(1-\gamma_1')^{r_1} \cdot (1-\gamma_2')^{r_2} + \gamma_1^{r_1} \cdot \gamma_2^{r_2} = 1$, $p_{\gamma_1}p_{\gamma_2} = p_{\gamma_1'}p_{\gamma_2'}$.

3. When $h_{ij} = \phi, g_{ij} \neq \phi$, the process of proof is similarity to (2), thus it is limited.

Summarized results, the integrated PR $R = (d_{ij})_{m\times m} = (\{h_{ij}|p_{ij}, g_{ij}|q_{ij}\})_{m\times m}$ is a UPDHFPR. ☒

Definition 13. Let $R_v = (d^v_{ij})_{m\times m} = (\{h^v_{ij}|p^1_{ij}, g^v_{ij}|q^v_{ij}\})_{m\times m}$ be V UPDHFPRs ($v = 1, 2, \cdots, V$), $r = (r_1, r_2, \cdots, r_V)$ be a corresponding weight information, $\sum_{v=1}^{V} r_v = 1$, the wight UPDHFPR aggregation operator $R = (d_{ij})_{m\times m}$ is described as follows

$$d = \otimes_{v=1}^{v} d(v_{ij})^{u_v} \tag{36}$$

1. When $i \leq j$,

$$d = \otimes_{v=1}^{v} d(v_{ij})^{u_v} = \bigcup_{\substack{\gamma_1 \in h^1_{ij}, \gamma_2 \in h^2_{ij}, \cdots, \gamma_V \in h^V_{ij}; \\ \eta_1 \in h^1_{ij}, \eta_2 \in h^1_{ij}, \cdots, \eta_V \in h^V_{ij}}} \left\langle \left\{ \prod_{v=1}^{V} \gamma_v^{r_v} \middle| \prod_{v=1}^{V} p_{\gamma_v} \right\}, \left\{ 1-\prod_{v=1}^{V} (1-\eta_v)^{r_v} \middle| \prod_{v=1}^{V} q_{\eta_v} \right\}, \right\rangle$$

2. When $i > j$,

$$d = \otimes_{v=1}^{v} d(v_{ij})^{u_v} = \bigcup_{\substack{\gamma_1 \in h_{ij}^1, \gamma_2 \in h_{ij}^2, \cdots, \gamma_V \in h_{ij}^V; \\ \eta_1 \in h_{ij}^1, \eta_2 \in h_{ij}^1, \cdots, \eta_V \in h_{ij}^V}} \left\langle \left\{ 1 - \prod_{v=1}^{V} (1-\gamma_v)^{r_v} \middle| \prod_{v=1}^{V} p_{\gamma_v} \right\}, \left\{ \prod_{v=1}^{V} \eta_v^{v_1} \middle| \prod_{v=1}^{V} q_{\eta_v} \right\} \right\rangle$$

Theorem 3. *Let $R_v = (d_{ij}^v)_{m\times m}$ be V UPDHFPRs ($v = 1, 2, \cdots, V$), $r = (r_1, r_2, \cdots, r_V)$ be a corresponding weight information, $\sum_{v=1}^{V} r_v = 1$, then the integrated PRs $R = (d_{ij})_{m\times m} = (\{h_{ij}|p_{ij}, g_{ij}|q_{ij}\})_{m\times m}$ are UPDHFPRs.*

Proof. The progress of proof is similarity to Theorem 1.□

Definition 14. Let $R_v = (d_{ij}^v)_{m\times m}$ $v = 1, 2, \cdots, V$ be V UPDHFPRs ($v = 1, 2, \cdots, V$), $r = (r_1, r_2, \cdots, r_V)$ be a corresponding weight information, $\sum_{v=1}^{V} r_v = 1$, then the integrated UPDHFPRs operator is described as $R = (d_{ij})_{m\times m} = (\{h_{ij}|p_{ij}, g_{ij}|q_{ij}\})_{m\times m}$, then

1. When

$$\sum_{l=1}^{\#h} \gamma_{ij,l} p_{ij,l} = T\frac{w_{il}}{w_{il} + w_{ju}}, 1 - \sum_{k=1}^{\#g} \eta_{ij,k} q_{ij,k} = T\frac{w_{iu}}{w_{jl} + w_{iu}}.$$

then we call R holds group consistency.

2. When $\frac{1}{2m(m-1)}\left(\left|e_{ij,1} - \frac{w_{il}}{w_{il}+w_{ju}}\right| + \left|e_{ij,2} - \frac{w_{iu}}{w_{jl}+w_{iu}}\right|\right) \le \xi$, then, we call R holds acceptable expected group consistency.

4.2. Iterative Algorithm for Increase Consistency Level

Step 1. Utilize Eq. (34) to compute the deviations $x_{ij}^+, x_{ij}^-, y_{ij}^+, y_{ij}^-$, unknown probabilities $p_{ij,l}, q_{ij,k}$, weight values w_i of the integrated UPDHFPRs $R = \{d_{ij}\}_{m\times m}$, $i, j = 1, 2, \cdots, m$.

Step 2. Utilize the established consistency test method, namely, Eq. (11), to calculate CI. If $CI \le \xi$, then skip to **Step 5**. If $CI \ge \xi$, then go to **Step 3**.

Step 3. Based on Eq. (34), calculate the maximum deviations x_{max}, y_{max}.

$$x_{max} = max\{x_{ij}^+, x_{ij}^- | i, j = 1, 2, \cdots, m; i < j\}$$
$$y_{max} = min\{y_{ij}^+, y_{ij}^- | i, j = 1, 2, \cdots, m; i < j\}$$

Step 4. Then we face to four situations as follows: $i < j$

1. When $x_{max} = x_{ij}^+$ ($i = 1, 2, \cdots, m-1; j = 2, 3, \cdots, m; i \le j$). Next, the corrected hesitant fuzzy membership degree $\gamma_{ij,l}^* = \gamma_{ij,l} - x_{ij}^+$, where $l = 1, 2, \cdots, \#h$;
2. When $x_{max} = a_{ij}^-$ ($i = 1, 2, \cdots, m-1; j = 2, 3, \cdots, m; i \le j$). Next, the corrected hesitant fuzzy membership degree $\gamma_{ij,l}^* = \gamma_{ij,l} + x_{ij}^-$;
3. When $y_{max} = y_{ij}^+$ ($i = 1, 2, \cdots, m-1; j = 2, 3, \cdots, m; i \le j$). Next, the corrected hesitant fuzzy nonmembership degree $1 - \eta_{ij,k}^* = (1 - \eta_{ij,l}) - y_{ij}^+$;
4. When $y_{max} = y_{ij}^-$ ($i = 1, 2, \cdots, m-1; j = 2, 3, \cdots, m; i \le j$). Next, the corrected hesitant fuzzy nonmembership degree $1 - \eta_{ij,k}^* = (1 - \eta_{ij,l}) + y_{ij}^-$.

Step 5. When $\gamma_{ji}^* = \eta_{ij}^*, p_{ji} = q_{ij}, \eta_{ji}^* = \gamma_{ij}^*, q_{ji} = p_{ij}$.

Step 6. The new UPDHFPRs are constructed. Then iteration returns to **Step 1**.

Step 7. Output $p_{ij,l}, q_{ij,k}$ and w_i ($i, j = 1, 2, \cdots, m; l = 1, 2, \cdots, \#h; k = 1, 2, \cdots, \#g$).

4.3. Group Decision-Making

Based on the Definition 14 and the consistency-improving iterative algorithm, the group consistency can be investigated and acceptable group consistencies for the integrated UPDHFPRs. According to the consistency-improving iterative algorithm, the (acceptable) group consistencies for the (aggregated) UPDHFPRs can be improved. Eventually, the priority vectors be calculated; next, the group decision process is terminated. Then the group decision-making process is showed based on the UPDHFPRs.

Step 1 Suppose Z evaluation experts give their PRs under the UPDHFPRs environment; therefore, Z UPDHFPR matrixes $R_z = (d_{ij}^z)_{m\times m}$ can be described $(i, j = 1, 2, \cdots, m; z = 1, 2, \cdots, Z)$.

Step 2 Utilize Eq. (34) to obtain the uncertain probability information. Then, the corrected probabilities and priority vector w can be calculated.

Step 3 Use Eq. (11) to calculate the CI level to evaluate the acceptable consistency of R^z. When $CI \le \xi$, continue the **Step 4**. If not, utilize the iterative algorithm to correct UPDHFPR matrices, returning to **Step 2**.

Step 4 Z completed UPDHFPR matrices are shown. Next, according to known weight information $\tilde{W}$ and Definition 14 to calculate the integrated the UPDHFPRs $R = (\tilde{d}_{ij})_{m\times m}$.

Step 5 Utilize Eq. (34) to analyze the priority vector w in R.

Step 6 Compute the CI level with Eq. (11). Then go to **Step 6** when $CI \le \xi$. If not, based on the iterative algorithm, the corrected aggregated UPDHFPR matrix $R = (\tilde{d}_{ij})_{m\times m}$ is shown. Next, back to **Step 5**.

Step 7 Utilize Eq. (34) to obtain the ranking results w_i, $(i = 1, 2, \cdots, m)$, and select the optimal selection, the bigger of w_i, the better the A_i.

5. ELUCIDATIVE EXAMPLE

In this section, the above approaches and notions are applied to practical group decision-making situations.

5.1. Example and Analyze

A company's interview rules stipulate that when interviewing candidates, three interviewers $(z = 1, 2, 3)$ are required to participate in the selection of talents, representing the needs of company with the purpose of finding the excellent talents. The selection refers to the process of recruiting those who have the ability and interest to the company in order to meet the needs of development, according to the requirements of human resources planning and job analysis, and select the appropriate personnel to hire them to ensure the enterprise. The activities were carried out normally. The three interviewers consider four newly listed candidates $(v = 1, 2, 3)$ that represent four emerging candidates that show promise.

Note that only the resumes of the four listed candidates for the resent years were shown. The resume information show that the candidates' education background and communication ability are unreliable and do not show the development potential of the candidates. Thus, the three interviewers consider to select the optimal candidate based on their knowledge and experience, under the DHFPRs. Further, the interviewers prefer to utilize the UPDHFPNs to explain their subjective dual hesitant fuzzy preference information, which could more exactly depict their uncertainty and hesitancy than other fuzzy methods. Therefore, the following three UPDHFPR matrices $R_v = (d_{ij}^z)_{m\times m}$ $z = 1, 2, 3$. Simultaneously, the weight information of the three interviewers is given according to their fairness, namely, $r = (0.5, 0.3, 0.2)$.

$$R_1 = \begin{pmatrix} \left\langle \begin{matrix}\{0.5 \mid 1\}, \\ \{0.5 \mid 1\}\end{matrix} \right\rangle, & \left\langle \begin{matrix}\left\{0.5|p_{12,1}^1, 0.8|0.1, 0.9 \mid p_{12,3}^1\right\}, \\ \{0.2 \mid 1\}\end{matrix} \right\rangle, & \left\langle \begin{matrix}\{0.6|1\}, \\ \left\{0.1|q_{13.1}^1, 0.01|q_{13,2}^1\right\}\end{matrix} \right\rangle \\ \left\langle \begin{matrix}\{0.2 \mid 1\} \\ \left\{0.5|p_{12,1}^1, 0.8|0.5, 0.9 \mid p_{12,3}^1\right\}\end{matrix} \right\rangle, & \left\langle \begin{matrix}\{0.5 \mid 1\}, \\ \{0.5 \mid 1\}\end{matrix} \right\rangle, & \left\langle \begin{matrix}\{0.4|1\}, \\ \left\{0.2|0.4, 0.1, q_{23,2}^1, 0.3|q_{23,3}^1\right.\end{matrix} \right\rangle \\ \left\langle \begin{matrix}\left\{0.1|q_{13,1}^2, 0.01|q_{13,2}^1\right\}, \\ \{0.6 \mid 1\}\end{matrix} \right\rangle, & \left\langle \begin{matrix}\left\{0.2|0.4, 0.1|q_{23,2}^1, 0.3 \mid q_{23,3}^1\right\}, \\ \{0.4 \mid 1\}\end{matrix} \right\rangle, & \left\langle \begin{matrix}\{0.5 \mid 1\}, \\ \{0.5 \mid 1\}\end{matrix} \right\rangle \end{pmatrix}$$

$$R_2 = \begin{pmatrix} \left\langle \begin{matrix}\{0.5 \mid 1\}, \\ \{0.5 \mid 1\}\end{matrix} \right\rangle, & \left\langle \begin{matrix}\{0.3 \mid 1\}, \\ \left\{0.4|q_{12,1}^2, 0.5|q_{13,2}^2\right\}\end{matrix} \right\rangle, & \left\langle \begin{matrix}\left\{0.1|p_{13,1}^2, 0.4|p_{13,2}^2, 0.8 \mid p_{13,3}^2\right\}, \\ \{0.1 \mid 1\}\end{matrix} \right\rangle \\ \left\langle \begin{matrix}\left\{0.4 \mid q_{12,1}^2, \left\{0.5 \mid q_{13,2}^2\right\}, \right. \\ \{0.3 \mid 1\}\end{matrix} \right\rangle, & \left\langle \begin{matrix}\{0.5 \mid 1\}, \\ \{0.5 \mid 1\}\end{matrix} \right\rangle, & \left\langle \begin{matrix}\left\{0.2|p_{23,1}^2, 0.8|p_{23,2}^2\right\} \\ \left\{0.1|q_{23,1}^2, 0.2|q_{23,2}^2\right\}\end{matrix} \right\rangle \\ \left\langle \begin{matrix}\{0.1 \mid 1\}, \\ \left\{0.1|p_{13,1}^2, 0.4|p_{13,2}^2, 0.8 \mid p_{13,3}^2\right\}\end{matrix} \right\rangle, & \left\langle \begin{matrix}\left\{0.1|q_{23,1}^2, 0.2|q_{23,2}^2\right\}, \\ \left\{0.2|p_{23,1}^2, 0.8|p_{23,2}^2\right\}\end{matrix} \right\rangle, & \left\langle \begin{matrix}\{0.5 \mid 1\}, \\ \{0.5 \mid 1\}\end{matrix} \right\rangle \end{pmatrix}$$

$$R_3 = \begin{pmatrix} \left\langle \begin{matrix} \{0.5 \mid 1\}, \\ \{0.5 \mid 1\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \{0.6 \mid 1\}, \\ \{0.4 \mid 1\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \{0.2 \mid 1\}, \\ \{0.8 \mid 1\} \end{matrix} \right\rangle \\ \left\langle \begin{matrix} \{0.4 \mid 1\}, \\ \{0.6 \mid 1\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \{0.5 \mid 1\} \\ \{0.5 \mid 1\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \left\{ 0.05|p^3_{23,1}, 0.35|p^3_{23,2} \right\} \\ \left\{ 0.01|q^3_{23,1}, 0.65|q^3_{23,2} \right\} \end{matrix}, \right\rangle \\ \left\langle \begin{matrix} \{0.8 \mid 1\}, \\ \{0.2 \mid 1\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \left\{ 0.01|q^3_{23,1}, 0.65|q^3_{23,2} \right\} \\ \left\{ 0.05|p^3_{23,1}, 0.35|p^3_{23,2} \right\} \end{matrix}, \right\rangle, \left\langle \begin{matrix} \{0.5 \mid 1\}, \\ \{0.5 \mid 1\} \end{matrix} \right\rangle \end{pmatrix}$$

$$\begin{aligned} \min \xi = {} & x^+_{12} + x^-_{12} + y^+_{12} + y^-_{12} + x^+_{13} + x^-_{13} + y^+_{13} + y^-_{13} + x^+_{14} + x^-_{14} + y^+_{14} + y^-_{14} \\ & + x^+_{23} + x^-_{23} + y^+_{23} + y^-_{23} x^+_{24} + x^-_{24} + y^+_{24} + y^-_{24} + x^+_{34} + x^-_{34} + y^+_{34} + y^-_{34} \end{aligned}$$

$$s.t. \begin{cases} (0.5p^1_{12,1} + 0.8 + 0.9p^1_{12,3})(w_{1l} + w_{2u}) - w_{1l} - x^+_{12} + x^-_{12} = 0 \\ (1 - 0.2) * (w_{1u} + w_{2l}) - w_{1u} - y^+_{12} + y^-_{12} = 0 \\ 0.6(w_{1l} + w_{3u}) - w_{1l} - x^+_{13} + x^-_{13} = 0 \\ (1 - 0.1q^1_{13,1} - 0.01q^1_{13,2})(w_{1u} + w_{3l}) - w_{1u} - y^+_{13} + y^-_{13} = 0 \\ (0.4) * (w_{2l} + w_{3u}) - w_{2l} - x^+_{23} + x^-_{23} = 0 \\ (1 - 0.2 * 0.4 - 0.1q^1_{23,2} - 0.3q^1_{23,3})(w_{2u} - w_{3l}) - w_{2u} - y^+_{23} - y^-_{23} = 0 \\ 0.2(W_{2l} + w_{1u}) - w_{2l} - x^+_{21} + x^-_{21} = 0 \\ (1 - 0.5p^1_{12,1} - 0.8 * 0.1 - 0.9p^1_{12,3})(w_{2u} + w_{1l}) - w_{2u} - y^+_{21} + y^-_{21} = 0 \\ (0.1q^1_{13,1} + 0.01q^1_{13,2})(w_{3l} + w_{1u}) - w_{3l} - x^+_{31} + x^-_{31} = 0 \\ (1 - 0.6)(w_{3u} + w_{1l}) - w_{3u} - y^+_{31} + y^-_{31} = 0 \\ (0.2 * 0.4 + 0.1q^1_{23,2} + 0.3q^1_{23,3})(w_{3l} + w_{2u}) - w_{3l} - x^+_{32} + x^-_{32} = 0 \\ (1 - 0.4)(w_{3u} + w_{2l}) - w_{3u} - y^+_{32} + y^-_{32} = 0 \\ p^1_{12,1} + P^1_{12,3} = 0.9, q^1_{13,1} + q^1_{13,2} = 1, p^1_{14,1} + p^1_{14,2} = 1, q^1_{23,2} + q^1_{23,3} = 0.6 \\ w_{1l} + w_{2u} + w_{3u} \geqslant 1, w_{2l} + w_{1u} + w_{3u} \geqslant 1, w_{3l} + w_{1u} + w_{2u} \geqslant 1, \\ w_{1u} + w_{2l} + w_{3l} \leqslant 1, w_{2u} + w_{1l} + w_{3l} \leqslant 1, w_{3u} + w_{1l} + w_{1l} \leqslant 1, \\ w_{1u} - w_{1l} \geqslant 0, w_{2u} - w_{2l} \geqslant 0, w_{3u} - w_{3l} \geqslant 0, w_{4u} - w_{4l} \geqslant 0 \\ w_{il}, w_{iu} \geqslant 0 \\ x^+_{ij}, x^-_{ij}, y^+_{ij}, y^-_{ij} \geqslant 0 \\ i, j = 1, 2, 3, 4 \end{cases} \tag{37}$$

Based on the group decision-making approach under the UPDHFPR situation, the specific calculation steps are explained as follows,

Step 1. Three UPDHFPR matrices are established based on the fuzzy PRs given by three interviewers.

Step 2. Utilize Eq. (34) to compute the unknown probability information of R_1, R_2, R_3. Take R_1 as an example. By Eq. (34), the basic programming model is constructed to calculate the probabilities of R_1. Based on this model, the results are listed as follows,

$$\begin{aligned} p^1_{12,1} &= 0.8308, p^1_{12,3} = 0.0682, q^1_{13,1} = 0.7840 \\ q^1_{13,2} &= 0.2160, q^1_{23,2} = 0.4792, q^1_{23,3} = 0.1208. \end{aligned}$$

Then, the interval priority vector of R_1 as $w^1_1 = (0.4193, 0.7357)$, $w^1_2 = (0.1839, 0.3282)$, $w^1_3 = (0.0645, 0.2759)$. The priority vector of R_1 are $\psi^1_1 = (0.4193, 0.2643, 1.1550)$, $\psi^1_2 = (0.1839, 0.6718, 0.5121)$, $\psi^1_3 = (0.06450.72410.3404)$. Meanwhile, the positive (negative) deviations of R_1 are listed,

$$\begin{aligned} x^+_{12} = x^-_{12} = y^+_{12} = y^-_{12} = x^+_{13} = x^-_{13} = y^+_{13} = y^-_{13} = 0 \\ x^+_{23} = x^-_{23} = y^+_{23} = y^-_{23} = x^+_{21} = x^-_{21} = y^+_{21} = y^-_{21} = 0 \\ x^+_{31} = x^-_{31} = y^+_{31} = y^-_{31} = x^+_{32} = x^-_{32} = y^+_{32} = y^-_{32} = 0. \end{aligned}$$

Similarity, for R_2, R_3, the results are calculated,

$$
\begin{aligned}
& x_{12}^{+} = x_{12}^{-} = y_{12}^{+} = y_{12}^{-} = x_{13}^{+} = x_{13}^{-} = y_{13}^{+} = y_{13}^{-} = 0 \\
& x_{23}^{+} = x_{23}^{-} = y_{23}^{+} = y_{23}^{-} = x_{21}^{+} = x_{21}^{-} = y_{21}^{+} = y_{21}^{-} = 0 \\
& x_{31}^{+} = x_{31}^{-} = y_{31}^{+} = y_{31}^{-} = x_{32}^{+} = x_{32}^{-} = y_{32}^{+} = y_{32}^{-} = 0. \\
& q_{12,1}^{2} = 0.2122, q_{12,2}^{2} = 0.7878, p_{13,1}^{2} = 0.4396, \\
& p_{13,2}^{2} = 0.2954, p_{13,3}^{2} = 0.2650, p_{23,1}^{2} = 0.3839, \\
& p_{23,2}^{2} = 0.6162, q_{23,1}^{2} = 0.9701, q_{23,2}^{2} = 0.0299. \\
& w_{1}^{2} = (0.2021, 0.4873), w_{2}^{2} = (0.4476, 0.4716), \\
& w_{3}^{2} = (0.0541, 0.3381) \\
& \psi_{1}^{2} = (0.2021, 0.5127, 0.6894), \\
& \psi_{2}^{2} = (0.4476, 0.5284, 0.9192), \\
& \psi_{3}^{2} = (0.05410.66190.3922). \\
& x_{12}^{+} = x_{12}^{-} = y_{12}^{+} = y_{12}^{-} = x_{13}^{+} = x_{13}^{-} = y_{13}^{+} = y_{13}^{-} = 0 \\
& x_{23}^{+} = x_{23}^{-} = y_{23}^{+} = y_{23}^{-} = x_{21}^{+} = x_{21}^{-} = y_{21}^{+} = y_{21}^{-} = 0 \\
& x_{31}^{+} = x_{31}^{-} = y_{31}^{+} = y_{31}^{-} = x_{32}^{+} = x_{32}^{-} = y_{32}^{+} = y_{32}^{-} = 0. \\
& p_{23,1}^{3} = 0.7686, p_{23,2}^{3} = 0.2314, q_{23,1}^{3} = 0, p_{23,2}^{3} = 1, \\
& w_{1}^{3} = (0.2817, 0.3554), \\
& w_{2}^{2} = (0.0862, 0.2146), \\
& w_{3}^{2} = (0.4261, 0.6349) \\
& \psi_{1}^{3} = (0.2817, 0.6446, 0.6371), \\
& \psi_{2}^{3} = (0.44760.52840.9192), \\
& \psi_{3}^{3} = (0.42610.36511.0610).
\end{aligned}
$$

Step 3. Based on Eq. (11), we have $CI_1 = CI_2 = 0$ and $CI_3 = 0.021\% \leq 2\%$. Thus, the UPDHFPR matrices R_1, R_2 hold the multiplicative expected consistency, the R_3 holds the acceptable consistency. Next, the completed UPDHFPR matrices $\tilde{R}_i$ $(i = 1, 2, 3)$ is listed.

Step 4. According to the weight information of three interviewers, $r = \{0.5, 0.3, 0.2\}$, the WUPDHFPRA operator to aggregate the UPDHFPR information $\tilde{R}_i$, then the aggregated UPDHFPR matrix R is got. R is a UPDHFPR matrix.

Step 5. According to Eq. (34), the results of R are calculated

$$
\tilde{R}_1 = \begin{pmatrix}
\left\langle \begin{matrix} \{0.5 \mid 1\}, \\ \{0.5 \mid 1\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \{0.5|\mathbf{0.8308}, 0.8|0.1, 0.9 \mid \mathbf{0.0692}\}, \\ \{0.2 \mid 1\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \{0.6 \mid 1\}, \\ \{0.1|\mathbf{0.7840}, 0.01|\mathbf{0.2160}\} \end{matrix} \right\rangle \\
\left\langle \begin{matrix} \{0.2 \mid 1\}, \\ \{0.5|\mathbf{0.8308}, 0.8|0.1, 0.9 \mid \mathbf{0.0692}\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \{0.5 \mid 1\} \\ \{0.5 \mid 1\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \{0.4|1\}, \\ \{0.2|0.4, 0.1|\mathbf{0.4792}, 0.3 \mid \mathbf{0.1208}\} \end{matrix} \right\rangle \\
\left\langle \begin{matrix} \{0.1|\mathbf{0.7840}, 0.01|\mathbf{0.2160}\}, \\ \{0.5 \mid 1\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \{0.2|0.4, 0.1|\mathbf{0.4792}, 0.3 \mid \mathbf{0.1208}\}, \\ \{0.4 \mid 1\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \{0.5 \mid 1\}, \\ \{0.5 \mid 1\} \end{matrix} \right\rangle
\end{pmatrix}
$$

$$
\tilde{R}_2 = \begin{pmatrix}
\left\langle \begin{matrix} \{0.5 \mid 1\}, \\ \{0.5 \mid 1\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \{0.3 \mid 1\} \\ \{0.4|\mathbf{0.2122}, 0.5|\mathbf{0.7878}\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \{0.1|\mathbf{0.4396}, 0.4|\mathbf{0.2954}, 0.8 \mid \mathbf{0.2650}\}, \\ \{0.1 \mid 1\} \end{matrix} \right\rangle \\
\left\langle \begin{matrix} \{0.4 \mid \mathbf{0.2122}, \{0.5 \mid \mathbf{0.7878}\}, \\ \{0.3 \mid 1\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \{0.5 \mid 1\}, \\ \{0.5 \mid 1\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \{0.2|\mathbf{0.3839}, 0.8|\mathbf{0.6162}\} \\ \{0.1|\mathbf{0.9701}, 0.2|\mathbf{0.0299}\} \end{matrix} \right\rangle \\
\left\langle \begin{matrix} \{0.1 \mid 1\}, \\ \{0.1|\mathbf{0.4396}, 0.4|\mathbf{0.2954}, 0.8 \mid \mathbf{0.2650}\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \{0.1|\mathbf{0.9701}, 0.2|\mathbf{0.0299}\}, \\ \{0.2|0.3839, 0.8|0.6162\} \end{matrix} \right\rangle, \left\langle \begin{matrix} \{0.5 \mid 1\}, \\ \{0.5 \mid 1\} \end{matrix} \right\rangle
\end{pmatrix}
$$

$$\tilde{R}_3 = \begin{pmatrix} \left\langle \begin{array}{c}\{0.5 \mid 1\}, \\ \{0.5 \mid 1\}\end{array} \right\rangle, \left\langle \begin{array}{c}\{0.6 \mid 1\} \\ \{0.4 \mid 1\}\end{array} \right\rangle, \left\langle \begin{array}{c}\{0.2 \mid 1\}, \\ \{0.8 \mid 1\}\end{array} \right\rangle \\ \left\langle \begin{array}{c}\{0.4 \mid 1\}, \\ \{0.6 \mid 1\}\end{array} \right\rangle, \left\langle \begin{array}{c}\{0.5 \mid 1\}, \\ \{0.5 \mid 1\}\end{array} \right\rangle, \left\langle \begin{array}{c}\{0.05|\mathbf{0.7686}, 0.35|\mathbf{0.2314}\}, \\ \{0.3|\mathbf{0}, 0.6|\mathbf{1}\}\end{array} \right\rangle \\ \left\langle \begin{array}{c}\{0.8 \mid 1\}, \\ \{0.2 \mid 1\}\end{array} \right\rangle, \left\langle \begin{array}{c}\{0.01|\mathbf{0}, 0.65|\mathbf{1}\}, \\ \{0.05|\mathbf{0.7686}, 0.35|\mathbf{0.2314}\}\end{array} \right\rangle, \left\langle \begin{array}{c}\{0.5 \mid 1\}, \\ \{0.5 \mid 1\}\end{array} \right\rangle \end{pmatrix}$$

$$R = \begin{pmatrix} \left\langle \begin{array}{c}\{0.5 \mid 1\}, \\ \{0.5 \mid 1\}\end{array} \right\rangle, \left\langle \begin{array}{c}\{0.4449|0.8308, 0.5627|0.1, 0.5969 \mid 0.0692\}, \\ \{0.3072|0.2122, 0.3441|0.7878\}\end{array} \right\rangle, \\ \left\langle \begin{array}{c}\{0.2814|0.4396, 0.4265|0.2954, 0.5251 \mid 0.2650\}, \\ \{0.3338|0.7840, 0.3013|0.2160\}\end{array} \right\rangle \\ \left\langle \begin{array}{c}\{0.3072|0.2122, 0.3441|0.7878\}, \\ \{0.4449|0.8308, 0.5627|0.1, 0.5969 \mid 0.0692\}\end{array} \right\rangle, \left\langle \begin{array}{c}\{0.5 \mid 1\}, \\ \{0.5 \mid 1\},\end{array} \right\rangle, \\ \left\langle \begin{array}{c}\{0.2144|0.2951, 0.3163|0.0888, 0.3249|0.4736, 0.4795|0.1426\}, \\ \left\{ \begin{array}{c} 0.1931|0, 0.2798|0.3880, 0.2211|0, 0.3036|0.0120, 0.1441|0, 0.2347|0.4649, \\ 0.1738|0, 0.2613|0.0143, 0.2452|0, 0.3251|0.1172, 0.2714|0, 0.3485|0.0036 \end{array} \right\} \end{array} \right\rangle, \\ \left\langle \begin{array}{c}\{0.3338|0.7840, 0.3013|0.2160\}, \\ \{0.2814|0.4396, 0.4265|0.2954, 0.5251 \mid 0.2650\}\end{array} \right\rangle, \\ \left\langle \begin{array}{c}\left\{ \begin{array}{c} 0.1931|0, 0.2798|0.3880, 0.2211|0, 0.3036|0.0120, 0.1441|0, 0.2347|0.4649, \\ 0.1738|0, 0.2613|0.0143, 0.2452|0, 0.3251|0.1172, 0.2714|0, 0.3485|0.0036 \end{array} \right\} \\ \{0.2144|0.2951, 0.3163|0.0888, 0.3249|0.4736, 0.4795|0.1426\}\end{array} \right\rangle, \left\langle \begin{array}{c}\{0.5 \mid 1\}, \\ \{0.5 \mid 1\}\end{array} \right\rangle \end{pmatrix}$$

$$\begin{aligned} w_1 &= (0.3219, 0.4074), \\ w_2 &= (0.2080, 0.4116), \\ w_3 &= (0.1742, 0.4751). \\ \psi_1 &= (0.3219, 0.5926, 0.7293), \\ \psi_2 &= (0.2080, 0.5884, 0.6196), \\ \psi_3 &= (0.1742, 0.5249, 0.6493). \end{aligned}$$

Calculate the positive and negative deviations of R as follows

$$\begin{array}{c} x_{12}^+ = x_{12}^- = y_{12}^+ = y_{12}^- = x_{13}^+ = x_{13}^- = y_{13}^+ = y_{13}^- = 0 \\ x_{23}^+ = x_{23}^- = y_{23}^+ = y_{23}^- = x_{21}^+ = x_{21}^- = y_{21}^+ = y_{21}^- = 0 \\ x_{31}^+ = x_{31}^- = y_{31}^+ = y_{31}^- = x_{32}^+ = x_{32}^- = y_{32}^+ = y_{32}^- = 0. \end{array}$$

Step 6. By Eq. (11), the CI value of R is obtained, $CI = 0 < 5\%$. Thus, the aggregated UPDHFPR matrix R holds the acceptable expected consistency. The priority vectors are applicable and receivable. By Definition 10, we can get $\Delta(w_1) = -0.2707$, $\Delta(w_2) = -0.3804$, $\Delta(w_3) = -0.3507$, $w_2 \leq w_3 \leq w_1$.

Step 7. The greater the values of ψ_i $(i = 1, 2, 3)$, the better the candidate v_i. Eventually, v_1 is the optimal candidate.

5.2. Analysis

Based on the above results, we can get the following conclusions,

1. The UPDHFN can be applied to depict the uncertain feature of factors in the DHFE through utilizing the occurrence and uncertain occurrence probabilities. Therefore, the UPDHFPRs are more flexible to express the dual hesitant fuzzy preference situations. This result is produced by the three interviewers in the above calculating example.
2. Under this environment, the key role of this probability-gaining method is that the consistency of the UPDHFPRs can be enhance, which is important for the UPDHFPRs.
3. Based on our method, the unknown probabilities are calculated. Another some factors given by DMs could be useless, the reason is that the computed probabilities are zero. Some invalid factors in the UPDHFPRs can be described and removed by using our method.
4. On account of all probabilities are known, thus the integrated UPDHFPRs matrix is a PDHFPR matrix. It is worth noting that the aggregation operator can be replaced with other type of operators. But other operators may complicate the calculation process.

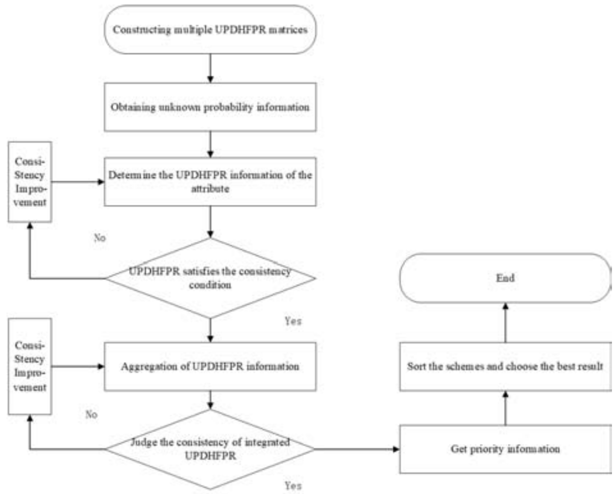

Figure 1 Flow chart of group decision-making in uncertain probabilistic dual hesitant fuzzy preference relation (UPDHFPR) environment.

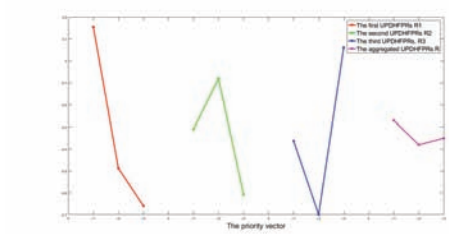

Figure 2 List of the consequences of the four uncertain probabilistic dual hesitant fuzzy preference relations (UPDHFPRs) about the above example.

5. According to Figure 1, the optimal candidate for the first and second interviewer is the third candidate. Then for the third interviewer, the optimal result is the second candidate. For the company, the optimal result is the first candidate.

From Figure 2, although the method in Zhou and Xu [43] select the different desirable candidate with our process, Zhou and Xu's method and our method choice different optimal candidate. But candidate w_2 is the worst option. The main reason for the above results is our method traces and improves the consistency with original uncertain information, and the UPDHFN is utilized to retain the preference information of DMs as much as possible. Besides, from the established multiplicative consistency, it is interpreted the evaluation information of w_1 is preferred to w_3. Therefore, the proposed model is more accurate than Zhou and Xu's [43] idea.

It is obvious that the model in our method, Zhou and Xu's [43] generated the different ranking result. In decision-making system with our method, the uncertainty probabilistic hesitant fuzzy nonmembership degrees are considered. To establish a new UPDHFPR, our method requires DMs to provide more original information to modify the expected UPDHFPR, which indicates that more random information is considered. Meanwhile, based on the construction method of improved multiplicative consistent UPDHFPR, one can find that it is causing a high degree of iteration algorithm complexity.

6. CONCLUSIONS

The objective of this paper is to generalize the DHFS to uncertain probabilistic hesitant fuzzy environment and develop goal programming models based on the UPDHFPR. While, we propose the (accepted) expected consistency, consistent examination and probability-calculating approach of uncertainty probability information. Additionally, to apply the UPDHFPRs in practical fields, (acceptable) group consistency under the UPDHFPR situations was investigated. Finally, a consistency-developing iterative algorithm was designed, and by utilizing text and figures, the group decision-making course were explained.

Investigate on the UPDHFPRs, aggregation operators, and group decision-making approach are only in their early projects. Next, the expected consistency validate and development methods are completed. E.g., (1)the threshold value ε can be further given. In this paper, *CI* subjectively determined. (2)Utilizing global optimization approach to develop the expected consistency of UPDHFPRs. Toward high quality progress [44], the problem of large-scale decision-making needs to be addressed [45–47]. Then, to combine the UPDHFPRs with other PR and deal with group decision-making situations under multiplicative fuzzy PR situation, granular computing approaches (fuzzy information) should be investigated in our future research under the UPDHFPRs. Also, the established model with UPDHFPRs can applied in other fields of production [48,49], like risk evaluation, plant location, etc.

7. COMPLIANCE WITH ETHICAL STANDARDS

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

AUTHORS' CONTRIBUTIONS

All authors have contributed equally to this paper.

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