ATLANTIS PRESS

Advances in Social Science, Education and Humanities Research, volume 529
Proceedings of the International Conference on Engineering, Technology and Social Science (ICONETOS 2020)

Teaching for Understanding Mathematics in Primary School

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ABSTRACT
Conceptual understanding is one indicator of an expert in mathematics. Conceptual understanding is important for students in addition to procedural skills. Students who have a good conceptual understanding will be able to find solutions with new or different procedures, while students who do not have conceptual understanding will only use standard procedures. Teaching is the most important thing in the process of developing students' conceptual understanding. The teacher as a learning designer in the classroom has a role that must be able to direct and guide students to develop conceptual understanding through effective learning. This article aims to discuss the teaching of conceptual understanding in primary schools. This article uses a structured literature review method. The author uses various documents such as national and international indexed journals, quality books that have been written by mathematical figures, especially material on understanding mathematics concepts and teaching. The results showed that the teacher could teach conceptual understanding by using the teaching language carefully and easily understood, making questions that encouraged students to think, avoiding learning shortcuts, avoiding memorizing facts and procedures, helping students make connections between concepts, and develop understanding through CRA (Concrete-Representation-Abstract).

Keywords: *Teaching, Understanding Mathematics, Primary School*

1. INTRODUCTION

Conceptual understanding is one of the six main standards in learning mathematics [1]. The other five standards are problem-solving skills, communication skills, connection skills, reasoning skills and representation skills. Concept understanding is the ability not only to know facts [2] but rather the ability to understand concepts through facts and provide explanations for each step of mathematical procedures. Erkki Pehkonen also said that mathematics is not only concerned with arithmetic operations, but the goal of teaching mathematics is to develop mathematical understanding and thinking [3]. Kilpatrick also explained that conceptual understanding is one of the five important strands as an indicator of an expert in mathematics [4]. Conceptual understanding is an important ability that must be acquired by students in addition to procedural skills [5]. Students who have a good conceptual understanding will be able to find solutions with new or different procedures, while students who do not have a conceptual understanding will only use standard procedures [6].

Students must have a conceptual understanding in learning mathematics because many concepts in mathematics are abstracts. With good conceptual understanding, students will not tend to memorize facts and procedures so that students will be more flexible when solving problems. This is because mastery in problem solving requires conceptual understanding to be adapted to procedures for finding solutions [7]. One that affects students' understanding ability is the type and process of teaching mathematics in the classroom [8]. In the results of the study, Kirschner suggests teaching conceptual and procedural understanding simultaneously and argues that students should not be allowed to carry out procedures without knowing the concept. [9]. However, some also argue that teaching should be focused on understanding concepts [6] because the effect of conceptual understanding on procedural skills is stronger than the opposite.

Many research results show that understanding concepts is very important. One of them is Markku S. Hannula who examined 3057 fifth and seventh grade students, which resulted that understanding the concept of fractions had an effect on understanding the concept of limitations [10].

Not only that, Resnick said that many students learn mathematics only as symbols without meaning. Resnick also shows how to teach mathematics to students by supporting thought processes and constructing meaning in mathematics [11].

In addition, another study was also conducted by Dian Mustika Anggraini showing that there is an influence on student learning outcomes using manipulative things on conceptual understanding. Manipulative things also help students construct meaningful knowledge [12].

Based on these facts, it shows that teaching is an important thing in the process of developing students' conceptual understanding. Teachers as learning designers in the classroom must be able to direct and guide students to develop conceptual understanding through effective learning. Therefore, this article discusses the teaching of conceptual understanding in primary schools.

2. METHOD

This article uses a structured literature review method, which is a detailed and clearly defined systematic discussion to answer more specific research questions. In this article, the author uses primary documents from national and international indexed journals. In addition, the author also uses quality books that have been written by mathematical figures, especially material on understanding concepts and teaching mathematics.

3. RESULT AND DISCUSSION

3.1. Conceptual Understanding

Kilpatrick in his book "Helping Children Learn Mathematics" defines conceptual understanding as the ability to understand mathematical concepts, operations and relationships [4]. Thus, students' conceptual understanding will relate to each other, depending on the number and extent of connections that students know. Students will be able to know facts in mathematics as a whole with understanding the concept. They will be able to organize this knowledge as a whole to gain new mathematical knowledge and connect with existing knowledge. In addition, conceptual understanding involves understanding how procedures work in an algorithm [13] so that students will be more confident in solving math problems. As explained in the National Council Teaching Mathematics that the combination of factual knowledge, conceptual understanding, and procedural fluency can provide a source of strength in mathematics learning [1]. In other words, students will be successful in mathematics if they master these three knowledge.

Conceptual understanding is very important in learning mathematics. Conceptual understanding has a big influence on procedural fluency. With a good conceptual understanding students will be able to explain why and how the procedure can work. *National Council Teaching Mathematics* explains that students will demonstrate a conceptual understanding of mathematics learning when they are able to provide evidence by [1]:

a. Recognize, label and provide examples of concepts.
b. Using and connecting models, diagrams, manipulatives to represent concepts in various ways.
c. Identify and apply principles.
d. Know and apply facts.
e. Comparing, differentiating and integrating related concepts and principles.
f. Recognizing, interpreting and applying symbols used to represent concepts.

The following are some examples that can explain the meaning of conceptual understanding:

a. Using zeroes in place values is simple, but important to understand. “Is it 30 + 40? What is the result”? A student who lacks understanding of the concept of place values may only perform a standard procedure of adding the two together to get the result. But there will be difficulty when asked to explain effectively that, “30 is 3 tens and 40 is 4 tens. So, 3 tens and 4 tens are 9 tens. Meanwhile, 9 tens is equal to 90. "
b. The commutative nature of adding whole numbers. "How much is 12 + 7? Do we need to recalculate for 7 + 12? ”. Students who do not understand the properties of addition in whole numbers may recalculate 7 + 12. While students with good conceptual understanding do not need to do it anymore because they know that 12 + 7 = 7 + 12.
c. Recognizing the sum symbol. "12 + 8 = ... + 10". It is important to understand the meaning of the symbol equal (=) as equality. This will minimize students writing 20 as a result. This happens if students do not understand the meaning of the same symbol as equality between the right and left sides.
d. The multiplication operation on decimal is a common topic. When students are asked to estimate the result of 8, 345 x 5, 45. Students with conceptual understanding will be able to explain that the result of the multiplication is between 40 and 54 obtained from that one factor is greater than 8 and less than 9, while the second factor is greater than 5 and less than 6 so the result should be between 40 and 54.

However, in some situations, Bartell's research shows that when students find conceptual and procedural based questions, students show conceptual and procedural understanding solutions simultaneously or procedurally only [14]. Students with a good conceptual understanding will do the procedure correctly and be able to explain every step in the procedure. Students with poor conceptual understanding will tend to have misconceptions.

The following is an example of student responses when given the decimal sum of 63.7 + 49.8 taken from Bartell [14]. The first student response is to show a conceptual understanding with the correct answer as follows: answering questions 63.7 + 49.8 by changing shapes. By taking 0.2 from 63.7, it becomes 63.5. Then add 0.2 to 49.8 so that it becomes 50. So that the form of the addition becomes 63.5 + 50. Then, 63.5 + 50 means 6 tens plus 5 tens to produce 11 tens, which consists of 1 hundreds and 1 tens. Next we will add 3 units and 5 tenths. So the result is simply adding 1 hundreds, 1 tens, 3 units and 5 tenths which is equal to 113.5. "

The second student's response by showing procedural, correct answers with explanations that may seem more conceptual, are as follows:

1	1	1	
	6	3,	7
	4	9,	8 +
1	1	3,	5

The first is to add 7 to 8 which is 15, then write 5 at the bottom and group 1 in the ones column in front of it. Then add 1 to 3 with 9 which is 13, then write 3 and group 1 in the tens column in front of it. Then add 1 to 6 with 4 and produce 11 and then the result is written at the bottom which is 113.5.

Shows procedural, wrong answers, no good explanations and misconceptions in the calculation process, as follows:

1	1	1	
	6	3,	7
	4	9,	8 +
1	1	4,	5

3.2. How to Teach Understanding Mathematical Concepts?

Conceptual understanding is one of the focuses in learning mathematics. The point is to make efficient arithmetic lessons. However, with mathematics learning that prioritizes conceptual understanding, it is important for teachers to be able to develop a more conceptual understanding.

Teaching understanding is how to do things that can encourage students to think. So the teacher must be able to move from helping students memorize to facilitating students to understand mathematical concepts more deeply.

The following will describe some insights on how to teach math concepts to primary school students:

1. Use the language of instruction carefully and be easy to understand.

In mathematics learning, there are several terms that have different meanings [15]. The mention of these terms can make students confused in the learning process, especially for primary school students. In its delivery, the terms in mathematics learning must be adjusted to the age and understanding of students' language so that it is easily understood by students.

For example, when the teacher wrote fractions $\frac{8}{12}$, and ask students to make the fraction smaller. Delivery in such language will mislead students' understanding. By following the calculation process assisted by the teacher, students will arrive at the answer $\frac{2}{3}$. This will make students confused and think that $\frac{2}{3}$ smaller than $\frac{8}{12}$, because 2 and 3 are smaller than 8 and 12. If students think like this, then there will be difficulties in convincing students that the two fractions are the same. This is what later in the new text describes the process of change as a simplification, and the teacher can show that the two fractions are the same or equal by using various forms of representation that are easily understood by students. The use of language and terms that have multiple or ambiguous meanings will have an impact on students' understanding, giving rise to misconceptions.

2. Create questions that encourage students to think.

Developing students' conceptual understanding is basically encouraging students to think. Teachers need tools to encourage students to think, namely with a question. Not an ordinary question but a question that can encourage deep and deep thinking (thought-provoking). The teacher must be able to do it according to the topic being studied. These questions are not only given when interacting with students but can also be given in the form of questions in mathematical assignments. Quoted from Moore, questions that can encourage students to think such as explaining, finding evidence through examples, generalizing, applying, making analogies and representing concepts in new ways [16].

The following are examples of questions that can encourage students to think.

a. 45 + 39 = 84, can you find the result of 39 + 45 without counting back? Give your explanation! [17].
b. Explain why it is possible to borrow in the reduction process? [18].
c. Do all fractions only represent values less than one?
d. Draw a variety of pictures that represent fractions $\frac{3}{5}$.

3. Avoid learning shortcuts first.

Mathematics learning that focuses on algorithms and shortcuts without ensuring conceptual understanding is still firmly entrenched today [15]. This kind of learning practice is still a tradition by teachers. Mathematics learning that focuses on shortcuts will eliminate meaningful math learning. Teaching those shortcuts is not always a bad thing, but they are only helpful. The teacher must first ensure that the students' conceptual understanding is embedded correctly and deeply. It can be indicated by students being able to show all the steps in detail in a problem solving process. Then students are able to explain and justify why each of these steps can occur by relating the process to the related concept. After students are able to do all of this, the teacher can provide knowledge to students in more efficient ways to carry out the same process.

For example, the material compares two fractions that have different denominators, are $\frac{2}{3}$ and $\frac{3}{4}$. Consider the comparison of the two examples in the table 1.

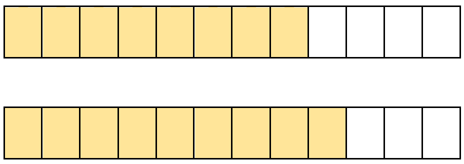
4. Avoid memorizing facts and procedures.

Traditional teaching still often leads to memorizing methods of facts and completion procedures in practice. Students will more easily forget the meaning of numbers or operations. It is different if the facts and procedures are presented in context. This will help students understand the concepts. For example, when students are asked to memorize multiplication. Just by presenting numbers 3 x 3 , 5 x 6 , 7 x 8 maybe students will be able to answer correctly through memorizing. However, this cannot guarantee that if the facts are presented in another form the students are still able to answer correctly. Suppose, what is the meaning of 5 x 6? It is necessary to have a supporting context that accompanies these numbers. In his study also argued that the presentation of natural numbers without description and context would limit students' conceptual understanding [15].

5. Helping students make connections between concepts.

Teachers do not have to teach new knowledge to students, but teachers can help students to connect new knowledge with other previous knowledge by finding and exploiting the relationship between mathematical concepts and ideas by utilizing the fact that one knowledge is part of other knowledge. The ability of this connection depends on a deep understanding of mathematical concepts.

Table 1 Comparison of Two Fractions

• Compares two fractions by cross multiplying. • The left side is the result of 2 x 4 = 8 and the right side is the result of 3 x 3 = 9 So it can be concluded that $\frac{2}{3} \ldots \frac{3}{4}$ $2 \times 4 \ldots 3 \times 3$ $8 < 9$ So, $\frac{2}{3} < \frac{3}{4}$	• Compares by equating the denominator first. $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ $\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$ So,  $\frac{2}{3} < \frac{3}{4}$

The easiest example is the opportunity to make a connection between the concept of multiplication and repeated addition. Some teachers explain that multiplication is a repeated addition. However, without realizing it, this definition limits the progress of children's understanding of the concept of mean. There is a connection between the concepts of addition, multiplication, division and average. For this reason, it is necessary to provide a broader and deeper and critical understanding that it is true that multiplication is a repeated addition, but it should be noted that multiplication is the repeated addition of groups of the same size. Multiplication expression 3 x 5 can be written 5 + 5 + 5, which can be interpreted that there is 3 groups that each group consists of 4. Next is to make a sharing connection. From this concept it will be known that there is 3 the main components, namely: the total, the number of groups and the size of each group. When viewed from the three components, multiplication and division are only about which parts are known and which parts are not known. In division, the focus lies on the known total, while on the average, the total is unknown. To find the average, you must combine / add groups that have different sizes into a total (as opposed to multiplying the groups that have the same size). Furthermore, after that, is in the position of the distribution context. By doing this the teacher provides opportunities for students to deepen connections in understanding concepts.

6. Develop Understanding through CRA (Concrete-Representation-Abstract)

Several studies, stated that teaching using manipulatives can improve mathematics achievement [19], [20]. By using manipulatives, students can model or represent concepts to understand abstract mathematical ideas. This is because by using mathematical manipulatives, students can interact directly using their physical object to learn a specific target.

According to Paul, mathematical manipulative things are objects that can be touched/held directly by someone where mathematical thinking will consciously and unconsciously develop [21]. For this reason, manipulative things are well designed so that they have the potential to develop mathematical concepts and ideas. For example, at the primary school level, the teacher can use play money to help students learn basic arithmetic, using dienes blocks to help students learn the value of place, addition, subtraction, multiplication and division. Providing opportunities for students to use manipulatives can develop mathematical understanding [1]. Teaching with the manipulation of concrete objects can encourage several students 'abilities including: (1) encouraging the development of abstract reasoning abilities, (2) encouraging students' real world knowledge [22], (3) helping students to find mathematical concepts through explorations by students.

These manipulated things are concrete objects that can help students represent abstract mathematical ideas. The ability to represent concepts is one of the indicators of conceptual understanding [1], hence representation has an important role in developing mathematical conceptual understanding [23]. So that the teaching method using the CRA (concrete-representation-abstract) sequence is considered effective. The CRA (concrete-representation-abstract) sequence is adapted from the Bruner stage in Milton which describes three stages, namely enactive, iconic and symbolic [24]. The enactive step is the steps where students are directly involved with the object of manipulation. Then, the iconic step where the children can develop mental images and can visualize concepts in their minds. Next, the symbolic step, where all information relating to the results of the representation is saved in symbol form. Several studies have shown that CRA can be used to facilitate students' conceptual understanding [25][26][27].

4. CONCLUSION

Conceptual understanding is the ability to understand mathematical concepts, operations and relations by students. Conceptual understanding can make students think structurally so as to increase confidence in solving math problems. Conceptual understanding is very important in learning mathematics. Conceptual understanding has a big influence on procedural fluency.

There are several ways to teach conceptual understanding of mathematics, which can be used by teachers such as using teaching language carefully and easily understood, making questions that encourage students to think, avoiding learning shortcuts, avoiding memorizing facts and procedures, helping students make relationship between concepts, and develop understanding through CRA (Concrete-Representation-Abstract)

ACKNOWLEDGMENT

Researchers thanks to UIN Maulana Malik Ibrahim Malang who continues to support researchers by providing opportunities and facilities so that researchers can complete this research.

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