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Student Thinking Levels in Solving Open-Ended Geometric-Function Problem by Algebraic Representation Approach

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Abstract. The mathematical problem mostly has been involved in the measurement of student ability according to study objectives and data needs. In this study, geometric-function problem has been used to data collection, analyzed problem type, and students thinking levels based on their completion. The sample of this study was three student high school in a science programme in the region of West Jakarta who selected purposively. Based on data analyzed, the result was found that students view the problem as the task constructed algebraic function formula. The completion and solution that have been shown by students, categorized the problem in open-ended type. The finding of mathematical thinking levels analysis there was a different complexity of thinking process between students because the number of concepts applicate. According to van Hiele theory, student's geometry thinking levels was on abstraction level minimally, and the problem is not relevant for deduction thinking level criterion. For the analysis result about algebraic thinking levels, by the structure of the observed learning outcome (SOLO) taxonomy, students at the levels between relational and extended abstract. Students in these levels was a student high ability so it is appropriate with the background of the sample selected.

Keywords: *education, geometric function, algebraic.*

1. INTRODUCTION

There are various of the mathematics problem type. Some literature has explained the criterion about the characteristic of the problem type. Based on the criterion that was indicated, there is an intersection between types of the problem, so that the terminology of problem type can more comprehensive. Some of that there was that stated a close or open-ended problem [1]–[4], or others. Researches that involved the mathematical problem likes Bahar & Maker [3], Arsyad et al. [5], and Rahman & Ahmar [6], but there has been not that was attached the explanation about how the student point of view related to problem type before it is used for the research. This condition caused we look at categorizing of the problem type just based on the theoretical foundation and not yet resources from solver perspectives so that the type of problem category was not yet to base on the relativity of solver perspective. Whereas the problem was closely related to the person who solves, in Rahman & Ahmar [6] and Dossey [7] has been stated that there was a subjectivity factor that directly related to the problem, even not stated as a problem when a student who was the solver is familiarity with content and its solving approach. Thus, it is because of necessary to be emphasized about using the terminology of problem type for the sample of the study. Despite, it has been just about a definition of problem type itself, however, there was an advanced analysis study that depends on theanalysis result toward its type for a specific interest. Thus, the relevance between theoretical foundation, using its theory, and the result of the study was in implicative relationships.

In this study, we used the geometric-function problem and not to define the type of its problem earlier, it is because the problem type will be defined according to the analysis result of a student views who solve. Hence, student learning experiences and knowledge will impact a determination of problem type [2]. For the geometric-function problem, its completion is not disjoint with student understanding about function concepts. In mathematics, a function can be stated by some representation, for example algebraic, geometric, table, or the other [8], [9]. Based on the representation, theoretically, there is some possibility approach to solve a function problem. Therefore, we have been using a geometric-function problem term as a specific domain to state a function problem that served in geometry representation form [10]. However, the representation form that was served is not with detail levels accurately. From the problem context that was mean, factually it will open the probability for students find the distinguish solution because the representation that was served as the problem is not detailed. Therefore, the geometric-function problem has a potency as an open-ended problem, caused in according to Kojo et al. [1], Bahar & Maker [3], Munroe [4], and Arsyad et al. [5] that an open-ended problem is a problem which is the open solution and the answer is more then one has. The problem type like that will be guiding a student to think on high levels. However, what the geometric-function problem in this study is an open-ended problem? This is the one question that will be answered based on study finding and students perspective as a solver.

The potential of a problem that causes able to guide a student solve it by high order thinking ability, it becomes an interest that this study will involve student's thinking level analysis. Schoenfeld [10] stated that a problem was needed for thinking to solve. By the geometric-function problem, an occurred activity when solving process is thinking about the shape of geometric-function [11]. In Herbst et al. [12] was stated that the geometry thinking process involves cognitive abstract reasoning, mathematical figure, and manipulation. Then, Herbst et al. [12] added that solving the geometry problem involves the process like shape transformation as a means of object mental from a visual figure. therefore, this citation emphasizes about our assumption related to conceptualization of the completion that there is a geometry thinking process with an object of thought, and this is because just a shape of function graph is given in the problem served to students. The shape of function graph which was extracted and transformed by students is to be an object mental via organization and recognition process [12]. Object mental in students mind was called Van de Walle et al. [13] as an object of thought. Related to solving activity and some of the citation above, there was no denial to state that the geometric-function problem has been a problem that required a student for thinking geometrically. Hence, van Hiele theory is a theory that describes how an individual think when involving in geometry problem [12], [14] so that one of all of analysis indicator student's thinking levels that include in this study refers to van Hiele levels.

2. RESEARCH METHODOLOGY

This was an analysis study about thinking levels based on problem type from student views respectively. Students who were a respondent in this study were three persons with the initials name AN, NI, and PR, from 11th grade in science program on the one best high school at the region of West Jakarta. These three respondents have been selected students purposively between seven students who were involved in solving geometric-function problem independently because the selection toward the others of four students had an effect of not shown the completion with analysis process comprehensively. On besides, three students who have been selected were students had a good ability according to mathematics teacher at the school. In this study, for collecting the data, the problem which has been served to be solved by students was adapted from Tobin [15] with the task to find the sketch of the first function graph of the graph in Figure 1.

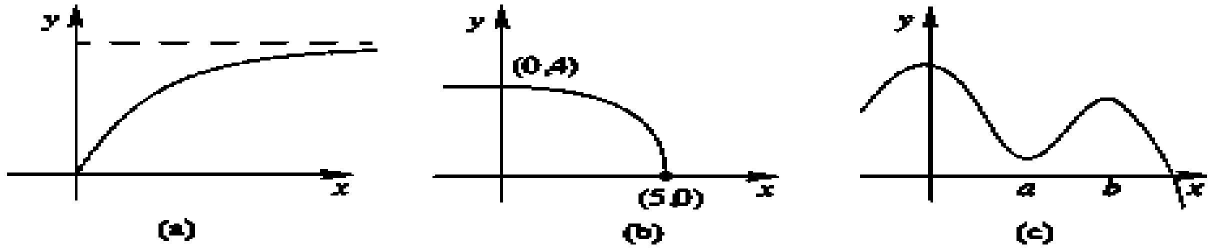

FIGURE 1. Sketch of the graphics function

Data was collected from two resources, it was: (1) the result of student completion toward the mathematical problem, and (2) their detail interpretation about the completion that was made it. Its interpretation contained conception that has been believed truth by the student so that its conceptions as an object or the result from the thinking process in solving the problem [12]. Data that was collected then analyzed to determine the problem type with the benchmark in determining its type based on literature like Kojo et al. [1] and Bahar & Maker [3] that explained the criterion of the problem type. Problem type required a completion that needed in thinking ability so that there is related to student thinking levels. Hence, after problem type has been known so data analysis directed to look at material concepts contained in the completion and then the involvement of mathematical concepts would specify student thinking levels at concepts. While for analyzing thinking levels based on van Hiele theory referred to some literature like Herbst et al. [12], Van de Walle et al. [13], and Luneta [14]. The finally was analyzing algebraic thinking levels referred to some literature like Oflaz & Demircioglu [16], Özdemir et al. [17], [18], and Apawu et al. [19].

3. RESULT AND DISCUSSION

At the following will be serving in the first how is completion process of the geometric-function problem, then it will be a resource in the analysis of problem type and students thinking levels and next to other section.

3.1. Student Completion of The Problem

There were some of solving steps by respondent respectively before to find a solution. We are showing it in this section for every student who is marked with their initials name, AN, NI, and PR. The first, from the solving process by AN, he determined to solve by estimating the algebraic function model of the graph, its like on the study by Hong & Thomas [20]. The graph in Figure 1 has been defined by AN as a function $y=-e^{-x}+a$ for a element real number. AN constant exponential graph of e^x as the general shape. Then, he obtained its function formula based on the similarity between the shape and the upside down of the exponential graph. Next, AN was used a geometry transformation concept of translation by $\begin{bmatrix}0 & 1\end{bmatrix}$ to the graph of $y=-e^{-x}$ in order to pass the origin $(0,0)$ and defined the example of function asymptote with $\lim_{x\to\infty} -e^{-x}+1=1$. At this process, we have understood the function formula that was mean by AN is $y=-e^{-x}+1$, and we were agreed with his product of thought because there is no anything that can be a contrary data of the process. For the representation of the algebraic function of $y=-e^{-x}+a$, has been derivated by AN to be $y'=e^{-x}$ and the sketch of the gradient function graph was drawn him. We did not agree for $y=-e^{-x}+a$ because not for every a element real number that the shape of the graph corresponded to Figure 1. Here, we don't show the function graph, but based on the solving process by AN could be stated that true for the gradient function graph him in spite of process have wrong. This is a sequence of the process in solving the problem by the graph to algebraic representation and back to a graph representation.

The second will be describing the solving process that was interpreted by NI. He solved the problem also by determining the algebraic function formula of the graph. His result analysis toward Figure 1 was stated with $y=-x^{-1}+a$ for a element real number as the function formula for the graph. NI determined the algebraic representation because of his conception about the graph in Figure lead to the shape of $y=-x^{-1}$. Next, NI defined to there has been a so that $y=-x^{-1}$ was added by a, but there was no interpretation about it and we did not see it as the application of geometry transformation concept because if like that so NI would observe the origin as a coordinate that was passed in the Figure. Here, we also observed the function formula that was constructed by NI and the shape of a pattern, because of the function with the formula is never passed the origin so it is distinguished of the graph in Figure 1. Consequently, a sketch of the gradient function or the first function graph of $y = -x^{-1} + a$ equally $y' = 1/x^2$ is also never to pass the origin. Solving the derivative function problem has been needed knowledge about intervals, differential calculus, asymptote, property, and limit [20], [21], and it looks like NI was missing its concepts.

Third, PR has been confirmed his completion that the function graph in Figure 1 was the model for algebraic function formula of $y = 1 - a^{-\sqrt{x}} + a$ for $a > 1$ and a element real number. The algebraic representation of the graph has been obtained by PR via some exploration process of function and the model of the graph. For example, $y = a^x$ was identified $a = 2$ so that $y = 2^x$ but the shape of the graph was not yet suitable with Figure 1, and then PR used $a = -2$ so $y = -2^x$. This activity was stated by Kop et al. [9] as the process which needs an ability to read algebraic expression and make rough estimates of the pattern that would be emerging in representation. Then, PR used the reflection concept of geometry transformation to the function toward x-axis and defined that because of the curve started with $f(c) = -1$ so in the function formula was added by 1, so that it was determined the shape of the curve that is similar with the Figure 1. Some of suitabilizing did to function formula until the finding of algebraic representation of the graph was generalized by PR used $e = a$ so that $y = 1 - e^{-\sqrt{x}}$. The shape of the function algebraic formula is not contrary to the graph in Figure 1. Next, PR determined the first derivative of its algebraic representation $y' = \sqrt{x} / 2xe^{\sqrt{x}}$ was the result as well as a sketch of the graph. There was no an error about the derivative of function formula $y = 1 - e^{-\sqrt{x}}$, but in Figure 1 has been given that there was an asymptote of the curve and the function domain was a positive real number, while the sketch of the first function graph by PR will increase at the particular domain intervals. Hence, we state that it is a factor disflexibility factor of the completion algebraic representation toward the function graph that was given.

Student solved the geometric-function problem and all three by the way to determined algebraic representation of graph function. Related to its the fact, Hong & Thomas [20] and Tokgoz & Gualpa [21] revealed that a student who has been faced with the function graph for sketching the derivative function graph would be liable to try to find the algebraic function of the graph. Then, this student solving approach shown there was similarity thinking style because there is no one between three student who chose to solve the problem geometrically like in explanation by Tobin [15] in spite of student did not construct the algebraic function formula. Like the statement of Choi & Hong [22] that there was student tend to depend on algebraic thinking style, even though Hong & Thomas [20] stated that student who used his algebraic thinking was disadvantaged in terms of available time and perform less well in solving a problem. Evidently, its statement was not wrong after we saw all three the solving process by sample whom shown at this section on the top. Furthermore, the student completion approach is our base to state that the geometric-function problem was liable seen as an algebraic representation problem. Then, from the three completion, the result of this study states that there is no solution similarity to every algebraic representation that was found by students. However, there is a similarity of the completion approach model that was interpreted by AN and PR, because their analysis is the same applying geometry transformation concept although the concept that they were used is different, AN by translation and PR by reflection.

3.2. Type of The Problem Based on Student Perspective

This study used the geometric-function problem on data collected. Glance form the problem terminology, there was a condition that must be fulfilled to state that the geometric-function problem which has been served was relatively toward the one of geometric or algebraic representation context, and the condition means was stated in Bahar & Maker [3], it is about sample subjectivity toward the problem. Panaoura et al. [8], Kop et al. [9], Özdemir et al. [17], [18] stated that the function problem could it be seen as a geometric or algebraic representation problem. The completion that student made is one from the representation models. Because of the geometric-function problem in this study has been adapted by Tobin [15], so that one of the alternative completions is also cited from its literature, and solving the problem by the geometric representation that was not involved algebraic representation. The example of concepts which have a role play in solving the derivative problem based on geometric representation is a negative or positive value of the gradient value along the curve, and an increase or decrease [15], [20], [23].

When the geometric-function problem that has been solved by algebraic representation, and to be a views student in their completion, like in discussion at the section of student completion of the problem, some of mathematical concept examples that involved in solving was differential calculus, domain of function, asymptote, sketch of graph and algebraic representation of function, as well as geometry transformation. The finding of this study, students preferred to solve by determining an algebraic function formula. Based on this finding, there is a statement that was delivered by Choi & Hong [22] that actually a student learned differential calculus concept was a skill in an algebraic algorithm or memorized simple mechanical counting process, but had in understanding the concept. Furthermore, Borji et al. [23] stated the specific difficult student have in derivative of graph representation is the basic concept of calculus. We observed its citation as the reason to assume why a student did not solve the problem geometrically like in Tobin [15]. Factually, the trigger of a student to think until the level is how knowledge and understanding were involved together, and from the result of this study was obtained information that applies function and differential calculus concept partially still doing separately.

Actually, the discussion in this section is enough by knowing what is the representation which was used in student solve, because here will explain the type of the problem not only based on theoretical conclusion but also according to students' perspective and completion approach by the representation. Related to problem type, there were three contexts that to be a point of views, the first is the problem itself, the second is the solution, and the third is an approach to find a solution [3]. Furthermore, from all three contexts that was mean, there are some probably of the problem that categorized, it is the problem that the completion close-approach but open-solution, the completion open-approach but close-solution, or the completion and solution is open. However, what is actually emerge an open or close terminology of the problem, and it is about to have some solutions. More then one solution for the problem is a particular characteristic of a problem type. Kojo et al. [1], Bahar & Maker [3], Munroe [4], and Arsyad et al. [5] stated that the problem which more then one solution is an open-ended problem.

In student completion of the problem section, given that there was an analysis that the completion approach by the direct-similarity or indirect-similarity between the product of thought and Figure 1. We used the term direct-similarity when the shape of the curve from algebraic function formula which was served by students is not show an additional process, like a transformation process, to make the shape similar with Figure 1, and this approach has been done by NI. Whereas the term of indirect-similarity for solving approach that was made by AN and PR because there was various of investigating the process that was done and each student made a difference, like a geometry transformation, to make the shape of curve suitable with graphic Figure. The other, based on the solution of a student found, there was the completion by AN and PR which was a similar tendency, but not at all for NI's completion. Here, at least has been found there was a difference between two student's completion so it is stated that the problem solution has more than one. Because all three answers have been different solutions and approaches, its problem was equivalent to the problem that has more than one solution. In other words, it is called an open-ended problem according to some theories cited, for example, Arsyad et al. [5]. Therefore, based on student perspective, geometric-function problem was used in this study is open-ended problem type, and this is an answer to the question that emerged in the introduction section. Furthermore, because of the problem task in this study referred to Tobin [15], and the result found student's completion by algebraic representation, so it is our reason to make a statement that not only the theoretical foundation but also instrument validation is not bad to base on student views, so as correspondent between a student performance and the operational theory.

3.3. Student Thinking Based on Applying Mathematical Concept

The problem can be solved to be based on ability, learning experiences, and interpretation of the problem [2], [4], [7], [10]. When there is a student, who will solve the problem, is clearly that an approach and the solution have a relation with the understanding factor toward materials content in problem, or a strategic factor to generate an approach [2], [3]. Then, in the framework of the open-ended approach by Munroe [4] has been stated that its two factors was an understanding and applying mathematical knowledge. In this section, we cited some theory to be combined with research findings. Finding in this study, AN and PR had solved by some concepts and it was a function, geometry transformation, and differential. At least have three concepts that they applied in the solving process. Whereas NI's completion approach has been by applying function and differential calculus concepts, and more there was no anything to reveal the mathematical concept that was applied. Here, there have been applying to distinguish concepts to be used by all three students in solving the problem so that it was clear that the completion approach also distinguishes [7]. Despite AN and PR was stated the completion approach with indirect-similarity, but it has been included in student completion of the problem section that a distinguishing of transformation concept was applied both of students. Related to student thinking levels that discussed in this section, the thinking level means was defined to a thinking level based on a complexity of applying concept when solving a problem, and we assumed that more concepts were involved a student in solving so it is more complexitive into their thinking process. This assumption is proportional to the statement of Rahman & Ahmar [6] and Choi & Hong [22] that an ability to read served information just shapes in graphics needed reasoning of complexity properties. Then, affirmed by Luneta [14] that additional concepts make geometry to be more complexitive that required ways of thinking.

Thus, based on the result of analysis about some mathematical concepts was applied by a sample in the completion, AN and PR still on thinking complexity at the same level relatively, but it is different with NI. NI has shown the simple process of thinking than AN and PR. However, simplifying in the thinking process is not a benchmark to determine which thinking level is subsets the other or which is better than others. Hence, stated that our assumption about thinking levels was based on the complexity of concepts are applied in the solving process. In besides, each student has shown a wrong way in the process. Algebraic function formula of by AN and because of not for all of is element real number will form suitable function graph in Figure 1, but if only saw his sketch of the first derivative function graph so it could be stated is true. While the function formula was founded by NI is , and student's wrong in determining its algebraic representation was a sketch of the graph which never passes the point of , so that the graph is different with graphics model in the problem, and the effect is also on the shape of derivative function. Whereas to PR, sketch for derivative function with the formula of is in positive real number domains, will increase at the particular intervals, even though there was an asymptote that is included in Figure 1. Furthermore, all three students have missed that there was a positive real number of domain intervals function in Figure 1, while students have not yet shown them to it, and generally that student's wrong at the end was also caused the concepts related to the domain.

3.4. Student Thinking Level Based on van Hiele Theory

Based on some conceptions which loaded in the interpretation of the solving process, there is a direction of mathematical communication between an object of thought and a product of thought. We observed its communication in according to NI's interpretation, it is the sketch of the graph in Figure 1 as an object of thought, to algebraic representation of, and object mental in graphic form which is a product of thought. While PR's interpretation on his completion was the sketch of the graph as an object of thought, to a model a function formula of, and to an algebraic expression of as a product of though. Whereas the thinking process by AN, it was from the sketch of the graph to algebraic function model of as the product of though, and to object mental of graphic form that was a product of thought of algebraic function model. An object of thought and product of thought has been explained in the thinking levels of van Hiele theory [13]. In the theory, there were had the five of geometry thinking levels and it is visualization (recognition), analysis, abstraction (informal deduction), deduction, rigor [12], [13], [14]. Thus, the thinking process which has been shown by AN, NI, and PR confirmed the theory that has been cited by Herbst et al. [12], Van de Walle et al. [13], and Luneta [14] that the geometry thinking levels of van Hiele was hierarchy, and this confirmation is to the thinking levels of visualization to analysis. Furthermore, this confirmation was based on the indication which has been stated in Van de Walle et al. [13] that the object of thought or the product of though in levels of visualization to analysis was shaped-- the class of shape-- the property of shape. Whereas, the correspondence to context in the solving of geometric-function problem on this study is graph function--model of algebraic function--the algebraic representation of the graph.

Next, there has been a process to determine the algebraic function of the graph, and its process involved the relationship between graph properties like shape, the domain of a function, the coordinate was passed, it was view together in conceptualization solving scheme. By the base theory in literature, Luneta [14] that the indicator or abstraction level was a student could have combined the shape and property to give an exact definition as well as related to the shape with the other. All three student, AN, NI, and PR, shown thinking ability geometrically at its level, or in Herbst et al. [12], its level was stated by order. However, not ideally to states that the geometry thinking level for a sample this study at informal deduction or abstraction level. Hence, an analysis has been continued to deduction of thinking level. In Van de Walle et al. [13], the product of though for deduction level is a deductive system of properties, whereas Herbst et al. [12] and Luneta [14] stated the indicator its level is student apply formal deductive argument like in proof. Both of the indicators in the citations have been closely related to problem context which was solved and its solving approach. Because of the geometric-function problem has been served was not lead in order to students thinking process proposed to a proof deductively, so we conclude there is not have relevant data to state sample is not at the deduction thinking level. This is the example of our state at the introduction section that advanced analysis was depended on the result of the analysis. Here, problem type was not for analyzing of deduction thinking level, but we used to conclude about the minimal level of student geometry thinking level, because the problem type that served has been relevant with the context of deduction level in geometry thinking van Hiele theory.

3.5. Algebraic Level of Student Thinking

In this study, the geometric-function problem has been served is to sketch of derivative function graph, and this problem invited a student to think algebraically than geometrically. Van de Walle et al. [13], Oflaz & Demircioglu [16], Özdemir et al. [17], [18], and Chimoni & Pitta-Pantazi [24] stated that algebraic thinking is mathematical thinking related to reveal of pattern and investigate a mathematical relationship of geometrical shape, generalized, and used symbol to problem-solving. Based on its statement, the sample in this study have used a symbol as the result of their investigation toward function graph in the problem, and all three students found the completion that was constructed at a base on the symbol commonly said by algebraic representation. Construction of the algebraic function formula involved a student to think algebraically within processing object of though mentally and product of thought that contained the functional relationship graphically [20]. Its construction process was explained Chimoni & Pitta-Pantazi [24] and Warren et al. [25] as an algebraic thinking ability. Furthermore, generalization has done by NI, AN, and PR in determining process the function formula of the graph. All of the function that was constructed by the student is made on the basis of the explicit relationship between two variables, x and y, from the properties of shape, Oflaz & Demircioglu [16] and Warren et al. [25] categorized its generalization process through a combination of explicit and visual thinking. However, there was a conversion process of the representation from geometric to algebraic as the consequence in solving the problem by the way. Students choose it's the solving ways because of there is a factor related to thinking style [22], or their understanding was on basic concepts of calculus [23]. Furthermore, in Özdemir et al. [17], [18] and Hong & Thomas [20] said that they had more experiences in algebraic problem-solving, and students felt confident to solve the problem by determining the algebraic representation of function because they more understood a basic idea in solving method.

A study by Apawu et al. [19] used the SOLO taxonomy as the framework theory to state algebraic thinking level. Then, there were five levels in its theory, it is: prestructural, uni-structural, multi-structural, relational, and extended abstract. From this five's levels, we revealed the criterion for relational level, it is a student able to make a relationship between pattern, generalization, and representation. Hong & Thomas [20] and Warren et al. [25] has been stated that algebraic thinking involved relational thinking. Here, the student of this sample study loaded obviously the relational level criterion so that furthermore of our analysis was observed the characteristic that is shown for the extended abstract level. For its level, criticized that a student can to generalize the relationship between something new and more abstract situation [19]. Next, we compared this extended abstract level criterion with the condition that occurred in solving and the interpretation by a sample of this study. We translated an algebraic function formula which was a student's completion on extended abstract level context was a new and loaded more abstract situation, like a domain of function concept that corresponded to depend on the graphic figure of the geometric-function problem. Has been the result analysis at the section of student thinking based on applying a mathematical concept that all three students passed their attention toward intervals domain of function which was just on positive real number. Hence, its result analysis was the reason to state that student algebraic thinking level in according to the framework theory of SOLO taxonomy stay on between the relational and extended abstract level. For a student who has been its level, they were a student in high ability [19], so was true that mathematics teacher recommendation of three students about their cognitive ability.

4. CONCLUSION

The geometric-function problem has been an instrument to collect data. Data was the completion and its process that was followed the interpretation its completion. Based on data analyzed, all three students who were a responden shown that they look the problem as the task to construct algebraic function formula of the graphic Figure, so it views directed to a type of algebraic representation problem. Moreover, because of student's completion approach obtained different solutions, although there was a similarity of mathematical concept was used in solving, this made a difference in the solution of each student. By its fact, stated that the geometric-function problem in this study is a type of open-ended problem, which required the students solved to think at a high level. There has been a basic analysis to student thinking level, it was according to mathematical concepts applied, van Hiele theory of geometry thinking level, and algebraic thinking level by SOLO taxonomy. Based on the result of analysis students thinking to apply mathematical concepts, at least there were two concepts which were applied in solving process, it was function and diferential calculus, and founded that there was the difference of complexity level in thinking process between students. While the result of analysis students thinking according to van Hiele Theory, founded that students' geometry thinking level until at abstraction level, but the analysis to deduction level was not continued because the geometric-function problem has not been relevant with the context of its level. Whereas the result of analysis to algebraic thinking level founded that student stayed on between the relational and extended abstract level.

Based on this study result, at least has been obtained a knowledge that students in the same of geometry thinking or algebraic level not yet course to apply a number of concepts same also to solve the problem. However, this was limited to open-ended geometric-function problem type. One of the limitations that show was a like to analysis on deduction of geometry thinking level according to van Hiele theory, it was caused the problem context is not relevant with its level characteristic. Whereas the other was there has been the limitation in obtaining data to analyze each problem types and thinking level based on the solving process by geometric representation, so its limitation is the recommendation to the next study.

REFERENCES

[1] A. Kojo, A. Laine, and L. Näveri, "How did you solve it? – Teachers' approaches to guiding mathematics problem solving," *Lumat*, vol. 6, no. 1, pp. 22–40, 2018.

[2] P. Intaros, M. Inprasitha, and N. Srisawadi, "Students' Problem Solving Strategies in Problem Solving-mathematics Classroom," *Procedia - Soc. Behav. Sci.*, vol. 116, pp. 4119–4123, 2014.

[3] A. Bahar and C. J. Maker, "Cognitive backgrounds of problem solving: A comparison of open-ended vs. closed mathematics problems," *Eurasia J. Math. Sci. Technol. Educ.*, vol. 11, no. 6, pp. 1531–1546, 2015.

[4] L. Munroe, "The Open-Ended Approach Framework," *Eur. J. Educ. Res.*, vol. 4, no. 3, pp. 97–104, 2015.

[5] N. Arsyad, A. Rahman, and A. S. Ahmar, "Developing a self-learning model based on open-ended questions to increase the students' creativity in calculus," *Glob. J. Eng. Educ.*, vol. 19, no. 2, pp. 143–147, 2017.

[6] A. Rahman and A. S. Ahmar, "Exploration of Mathematics Problem Solving Process Based on The Thinking Level of Students in Junior High School," *Int. J. Environ. Sci. Educ.*, vol. 11, no. 14, pp. 7278–7285, 2016.

[7] J. A. Dossey, "Problem Solving from a Mathematical Standpoint," in *Educational Research and Innovation: The Nature of Problem Solving Using Research to Inspire 21st Century Learning*, C. Benő and F. Joachim, Eds. Paris: OECD Publishing, 2017, pp. 59–72.

[8] A. Panaoura, P. Michael-Chrysanthou, A. Gagatsis, I. Elia, and A. Philippou, "A Structural Model Related to the Understanding of the Concept of Function: Definition and Problem Solving," *Int. J. Sci. Math. Educ.*, vol. 15, no. 4, pp. 723–740, 2017.

[9] P. M. G. M. Kop, F. J. J. M. Janssen, P. H. M. Drijvers, M. V. J. Veenman, and J. H. Van Driel, "Identifying a framework for graphing formulas from expert strategies," *J. Math. Behav.*, vol. 39, pp. 121–134, 2015.

[10] A. H. Schoenfeld, *Mathematical Problem Solving*. USA: Academic Press INC, 2014.

[11] F. Alghadari, Y. Yuni, and A. Wulandari, "Conceptualization in solving a geometric-function problem : An effective and efficient process," in *Journal of Physics: Conf. Series*, 2019, p. 012004.

[12] P. Herbst, T. Fujita, S. Halverscheid, and M. Weiss, *The learning and teaching of geometry in secondary schools: A modeling perspective*. 2017.

[13] J. . Van de Walle, K. . Karp, and J. . Bay-Williams, *Elementary and Middle School Mathematics: Teaching Developmentally*, 9th ed. USA: Pearson Education, 2017.

[14] K. Luneta, "Understanding students ' misconceptions : An analysis of final Grade 12 examination questions in geometry Understanding students ' misconceptions : An analysis of final Grade 12 examination questions in geometry," *Pythagoras*, vol. 36, no. 1, pp. 1–11, 2015.

[15] P. Tobin, *International Baccalaureate Mathematics Standard Level*, 3rd ed. Australia: IBID Press, 2007.

[16] G. Oflaz and H. Demircioğlu, "Determining ways of thinking and understanding related to generalization of eighth graders," *Int. Electron. J. Elem. Educ.*, vol. 11, no. 2, pp. 99–112, 2018.

[17] A. Özdemir, S. Kaş, and E. Bahadır, "The Effect of Teaching Carried Out by Using Worksheets on the Algebraic Thinking Levels of Primary School 8th Grade Students," *Br. J. Educ. Soc. Behav. Sci.*, vol. 10, no. 3, pp. 1–17, 2015.

[18] A. Ş. Özdemir, S. Kaş, and E. Bahadır, "The Effect Of Teaching Algebra By Using Worksheets On The Problem Solving Skills Of The Primary School 8th Grade Students," *Eurasian Educ. Lit. J.*, vol. 2, no. 2, pp. 21–36, 2015.

[19] J. Apawu, N. Akosua Owusu-Ansah, and P. Akayuure, "A study on the algebraic working processes of senior high school students in Ghana," *Eur. J. Sci. Math. Educ.*, vol. 6, no. 2, pp. 62–68, 2018.

[20] Y. Y. Hong and M. O. J. Thomas, "Graphical construction of a local perspective on differentiation and integration," *Math. Educ. Res. J.*, vol. 27, no. 2, pp. 183–200, 2015.

[21] E. Tokgoz and G. Gualpa, "STEM Majors' Cognitive Calculus Ability to Sketch a Function Graph," in *American Society for Engineering Education*, 2015, pp. 26.1396.1-26.1396.13.

[22] Y. J. Choi and J. K. Hong, "On the students thinking of the properties of derivatives," *J. Korean Soc. Math. Educ.*, vol. 53, no. 1, pp. 25–40, 2014.

[23] V. Borji, V. Font, H. Alamolhodaei, and A. Sánchez, "Application of the Complementarities of Two Theories, APOS and OSA, for the Analysis of the University Students ' Understanding on the Graph of the Function and its Derivative," *EurasiaJournal Math. Sci. Technol. Educ.*, vol. 14, no. 6, pp. 2301–2315,

2018.
[24] M. Chimoni and D. Pitta-Pantazi, "Parsing the notion of algebraic thinking within a cognitive perspective," *Educ. Psychol.*, vol. 37, no. 10, pp. 1186–1205, 2017.
[25] E. Warren, M. Trigueros, and S. Ursini, "Research on the learning and teaching of Algebra," *Second Handb. Res. Psychol. Math. Educ. Journey Contin.*, pp. 73–108, 2016.