Advances in Social Science, Education and Humanities Research, volume 550
Proceedings of the 1st International Conference on Mathematics and Mathematics Education (ICMMED 2020)

Mathematical Modelling of Schistosomiasis Transmission Dynamics in Traditional Cattle Farmer Communities

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ABSTRACT
. In this work, a deterministic mathematical model of schistosomiasis transmission dynamics is discussed. In rural areas, many people work as a cattle farmer. Cattle farmers in endemic areas are very susceptible to schistosoma worm infection. To study the dynamics of schistosomiasis spread in traditional cattle farmer communities, we develop a mathematical model by considering human, cattle, and snail population as well as parasite density in environment. The model is expressed as a system of first order differential equations. Firstly, we verify the non-negativity and boundedness of the solutions of the model. After determining the equilibrium points of the system, we determine the basic reproduction number. Linearization and Routh Hurwitz condition are used to analyze the local stability condition of the disease free equilibrium point. Center manifold theory is used to study the local stability condition of the endemic equilibrium point. We prove global stability condition of the disease free equilibrium point by formulating suitable Lyapunov function and using LaSalle invariance principle. Several numerical simulations are presented. Our results show that the farmer should keep the cattle, water, and food clean. In addition, the farmer should use molluscicide in their farm area and give schistosomiasis drug to the cattle, regularly.

Keywords: *Schistosomiasis model, Stability analysis, Center manifold, Lyapunov function.*

1. INTRODUCTION

Schistosomiasis is the second most socioeconomically disastrous parasitic disease [1]. This disease is caused by schistosoma worm. The disease is also known as snail fever because its spread involves snails and the most common symptom shown by an infected person is fever. Eradication of schistosomiasis is very difficult to be achieved because schistosoma worm can infect animal (mammals) as human substitute [2]. Snails are intermediate host whereas humans and mammals are reservoir hosts [1,3–5]. We formulate the model by considering the life cycle of schistosoma worm. After mating, schistosoma adult worm pairs produce eggs. Some eggs stay in the reservoir host body while the others are secreted to environment thorugh urine and feces. In environment, the eggs hatch and release miracidia that can infect certain snail. Miracidium that successfully infect snail develop and produce cercariae in the snail body. Infected snails can release cercariae to environment. Cercariae have ability to infect reservoir host, e.g., human, cattle, mouse and other mammals. In reservoir host body, some cercariae survive and become adult shistosoma worm [3,6,7]. Several mathematical models are developed to study the spread of schistosomiasis in human, mammals, and snail population [8,9]. However, these two models do not take into account the density of the parasite in environment. In this work, we take into account the fact that the transmission occurs even though the reservoir hosts and snails do not have a direct interaction [3,6,7,10]. For that reason, we propose schistosomiasis model considering human, cattle, snail, and parasite density in environment.

2. MODEL FORMULATION AND BASIC PROPERTIES

2.1. Model Formulation

We use the following assumptions to formulate our model:

1. Human, cattle, and snail population are, respectively, divided into three disjoint compartments, i.e., susceptible, latent, and infectious;
2. Constant recruitment for human, cattle, and snail population;
3. There is no recovery for infectious snail;
4. Infectious human and infectious cattle have a chance to recover from schistosomiasis;
5. There is no vertical transmission.

The description of the parameters used is presented in Table 1.

Table 1. Description of model paratemers

Symbol	Description
Λ_h	Human recruitment rate
Λ_a	Cattle recruitment rate
Λ_v	Snail recruitment rate
β_{ch}	Infection rate of cercariae on human
β_{ca}	Infection rate of cercariae on cattle
β_{mv}	Infection rate of miracidia on snail
$1/\theta_{ei}$	Incubation period of human
$1/\phi_{ei}$	Incubation period of cattle
$1/\varphi_{ei}$	Incubation period of snail
θ_{is}	Human recovery rate
ϕ_{is}	Cattle recovery rate
d_h	Natural death rate of human
d_a	Natural death rate of cattle
d_v	Natural death rate of snail
d_c	Natural death rate of cercariae
d_m	Natural death rate of miracidia
d_r	Molluscicide induced snail death rate
σ	Cercariae production rate of infectious snail
α_1	Miracidia production rate of infectious human
α_2	Miracidia production rate of infectious cattle

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Human, cattle and snail in susceptible compartment may become latent and move to latent compartment due to direct contact with parasites in environment. When the incubation period ends, the human, cattle and snail in latent compartment become infectious and move to infectious compartment. Infectious human and infectious cattle can secrete parasite eggs to environment. Infectious snails secrete cercariae to environment. We assume that there is an intervention by using molluscicide. Hence, there is molluscicide induced snail death. Cercariae which live in environment have the ability to infect human and cattle. Miracidia which live in environment have the ability to infect snail. We assume that human and cattle have a chance to recover. Humans and cattle who recover from schistosomiasis move to susceptible compartment. Appropriate to the assumptions, we obtain schistosomiasis model expressed as a system of first order differential equations as follows:

$$\begin{cases} \frac{dS_h}{dt} = \Lambda_h - \beta_{ch} C S_h - d_h S_h + \theta_{is} I_h, \\ \frac{dE_h}{dt} = \beta_{ch} C S_h - (d_h + \theta_{ei}) E_h, \\ \frac{dI_h}{dt} = \theta_{ei} E_h - (d_h + \theta_{is}) I_h, \\ \frac{dS_a}{dt} = \Lambda_a - \beta_{ca} C S_a - d_a S_a + \phi_{is} I_a, \\ \frac{dE_a}{dt} = \beta_{ca} C S_a - (d_a + \phi_{ei}) E_a, \\ \frac{dI_a}{dt} = \phi_{ei} E_a - (d_a + \phi_{is}) I_a, \\ \frac{dS_v}{dt} = \Lambda_v - \beta_{mv} M S_v - (d_v + d_r) S_v, \\ \frac{dE_v}{dt} = \beta_{mv} C S_v - (d_v + d_r + \varphi_{ei}) E_v, \\ \frac{dI_v}{dt} = \varphi_{ei} E_v - (d_v + d_r) I_v, \\ \frac{dC}{dt} = \sigma I_v - d_c C, \\ \frac{dM}{dt} = (\alpha_1 I_h + \alpha_2 I_a) - d_m M, \end{cases} \quad (1)$$

where all parameters are positive.

2.2. Non-negativity and boundedness of the solutions

In this subsection, we prove that system (1) is well-posed.

Theorem 1. *The solutions of the system (1) with non-negative intitial value are always non-negative.*

Proof. Think that there is t^+ such that $S_h(t^+) < 0$. It means that there is t^* such that $S_h(t^*) = 0$, $S_h(t) > 0$ for $t \in [0, t^*)$, $S_h(t^+) < 0$ for $t^+ > t^*$. From the first equation of system (1), we get

$$\left.\frac{dS_h}{dt}\right|_{t=t^*} = \Lambda_h - \beta_{ch}C(t^*)S_h(t^*) - d_hS_h(t^*) + \theta_{is}I_h(t^*)$$
$$= \Lambda_h + \theta_{is}I_h(t^*)$$
$$> 0.$$

It is a contradiction which implies that there is no $t^+ > t^*$ such that $S_h(t^+) < 0$. Hence, $S_h(t) \geq 0$ for $t \geq 0$. In similar way, contradiction is obtained if we consider that another compartment is zero at t^*.

Theorem 2. *Solutions of system (1) with non-negative intitial value are bounded.*

Proof. Let N_h, N_a, N_v as total number of human, cattle, and snail, respectively. Based on the assumptions that we used, it clear that $N_h = S_h + E_h + I_h$, $N_a = S_a + E_a + I_a$, $N_v = S_v + E_v + I_v$. From system (1), we obtain

$$\begin{cases} \frac{dN_h}{dt} = \Lambda_h - d_hN_h, \\ \frac{dN_a}{dt} = \Lambda_a - d_aN_a, \\ \frac{dN_h}{dt} = \Lambda_v - (d_v + d_r)N_v \end{cases} \quad (2)$$

System (2) is a system of first order differential equations. It is straightforward to show that the solution of the system is

$$N_h(t) = \frac{\Lambda_h}{d_h} + \left(N_h(0) - \frac{\Lambda_h}{d_h}\right)e^{-d_ht},$$
$$N_a(t) = \frac{\Lambda_a}{d_h} + \left(N_a(0) - \frac{\Lambda_a}{d_a}\right)e^{-d_at},$$
$$N_v(t) = \frac{\Lambda_v}{d_v + d_r} + \left(N_v(0) - \frac{\Lambda_v}{d_v + d_r}\right)e^{-d_vt}.$$

It is easy to see that $0 \leq N_h(t) \leq \frac{\Lambda_h}{d_h}$ for $t > 0$ if $0 \leq N_h(0) \leq \frac{\Lambda_h}{d_h}$. Moreover, $N_h(t) \to \frac{\Lambda_h}{d_h}$ as $t \to \infty$. Hence, $N_h(t)$ is bounded. In similar way, it easy to verify that $N_a(t)$ and $N_v(t)$ are bounded. From the last two equations of system (1) and the fact that $I_h \leq N_h(t) \leq \frac{\Lambda_h}{d_h}, I_a \leq N_a(t) \leq \frac{\Lambda_a}{d_a}, I_v \leq N_v(t) \leq \frac{\Lambda_v}{d_v + d_r}$, we get

$$\frac{dC}{dt} \leq \sigma\frac{\Lambda_v}{d_v + d_r} - d_cC,$$
$$\frac{dM}{dt} \leq \left(\alpha_1\frac{\Lambda_h}{d_h} + \alpha_2\frac{\Lambda_a}{d_a}\right) - d_mM.$$

Based on Gronwall's Lemma [11], we obtain

$$C \leq \frac{\sigma\Lambda_v}{(d_v + d_r)d_c},$$
$$M \leq \left(\alpha_1\frac{\Lambda_h}{d_h} + \alpha_2\frac{\Lambda_a}{d_a}\right)\Big/ d_m.$$

Therefore, the solutions of system (1) are bounded .

Based on Theorem 1 and Theorem 2, we have the following invariant region of system (1).

$$\Pi_+ = \left\{ \begin{array}{l} (S_h, E_h, I_h, S_a, E_a, I_a, S_v, E_v, I_v, C, M) \mid S_h, E_h, I_h, S_a, E_a, I_a, S_v, E_v, I_v, C, M \geq 0, 0 \leq N_h \leq \frac{\Lambda_h}{d_h}, \\ 0 \leq N_a \leq \frac{\Lambda_a}{d_a}, 0 \leq N_v \leq \frac{\Lambda_v}{d_v + d_r}, 0 \leq C \leq \frac{\sigma\Lambda_v}{(d_v + d_r)d_c}, 0 \leq M \leq \left(\alpha_1\frac{\Lambda_h}{d_h} + \alpha_2\frac{\Lambda_a}{d_a}\right)\Big/ d_m, t \geq 0 \end{array} \right\}$$

2.3. Equilibrium Point

Equilibrium points are determined by solving system (1) when all derivatives (left hand side) are equal to zero. System (1) has two equilibrium points, namely, disease free equilibrium point and endemic equilibrium point.

- The disease free equilibrium point of system (1) is

$$X_0 = (S_h^*, E_h^*, I_h^*, S_a^*, E_a^*, I_a^*, S_v^*, E_v^*, I_v^*, C^*, M^*)$$
$$= \left(\frac{\Lambda_h}{d_h}, 0, 0, \frac{\Lambda_a}{d_a}, 0, 0, \frac{\Lambda_v}{d_v + d_r}, 0, 0, 0, 0\right).$$

X_0 always exists in $\mathbb{R}^{11}_{+0}$.

- The endemic equilibrium point of system (1) is

$$X_1 = (S_h^{**}, E_h^{**}, I_h^{**}, S_a^{**}, E_a^{**}, I_a^{**}, S_v^{**}, E_v^{**}, I_v^{**}, C^{**}, M^{**}),$$

where

$$S_h^{**} = \frac{\Lambda_h(d_h + \theta_{is})(\theta_{ei} + d_h)d_c}{\beta_{ch}\sigma((d_h + \theta_{is})(\theta_{ei} + d_h) - \theta_{is}\theta_{ei})I_v^{**} + d_h(d_h + \theta_{is})(\theta_{ei} + d_h)d_c},$$
$$E_h^{**} = \frac{\beta_{ch}\sigma S_h^{**}I_v^{**}}{(\theta_{ei} + d_h)d_c},$$
$$I_h^{**} = \frac{\theta_{ei}\beta_{ch}\sigma S_h^{**}I_v^{**}}{(d_h + \theta_{is})(\theta_{ei} + d_h)d_c},$$
$$S_a^{**} = \frac{\Lambda_a(d_a + \phi_{is})(\phi_{ei} + d_a)d_c}{\beta_{ca}\sigma((d_a + \phi_{is})(\phi_{ei} + d_a) - \phi_{is}\phi_{ei})I_v^{**} + d_a(d_a + \phi_{is})(\phi_{ei} + d_a)d_c},$$
$$E_a^{**} = \frac{\beta_{ca}\sigma S_a^{**}I_v^{**}}{(\phi_{ei} + d_a)d_c},$$
$$I_a^{**} = \frac{\phi_{ei}\beta_{ca}\sigma S_a^{**}I_v^{**}}{(d_a + \phi_{is})(\phi_{ei} + d_a)d_c},$$
$$S_v^{**} = \frac{\Lambda_v\varphi_{ei} - (d_v + d_r + \varphi_{ei})(d_v + d_r)I_v^{**}}{(d_v + d_r)\varphi_{ei}},$$
$$E_v^{**} = \frac{(d_v + d_r)I_v^{**}}{\varphi_{ei}},$$
$$C^{**} = \frac{\sigma I_v^{**}}{d_c},$$
$$M^{**} = \left(\frac{\alpha_1\theta_{ei}\beta_{ch}\sigma S_h^{**}}{(d_h + \theta_{is})(\theta_{ei} + d_h)d_c} + \frac{\alpha_2\phi_{ei}\beta_{ca}\sigma S_a^{**}}{(d_a + \phi_{is})(\phi_{ei} + d_a)d_c}\right)\frac{I_v^{**}}{d_m},$$

I_v^{**} are roots of $P_1(I_v)$

$$\begin{aligned} P_1(I_v) &= p_1 I_v^{\,3} + p_2 I_v^{\,2} + p_3 I_v \\ &= I_v\left(p_1 I_v^{\,2} + p_2 I_v + p_3\right) \\ &= I_v P_2(I_v), \end{aligned}$$

where

$$\begin{aligned} p_1 &= -\beta_{ch}\sigma\left((d_h+\theta_{is})(d_h+\theta_{ei})-\theta_{is}\theta_{ei}\right)\beta_{ca}\sigma\left((d_a+\phi_{is})(d_a+\phi_{ei})-\phi_{is}\phi_{ei}\right)(d_v+d_r)^2(d_v+d_r+\varphi_{ei})d_m \\ &\quad -\beta_{ch}\sigma\left((d_h+\theta_{is})(d_h+\theta_{ei})-\theta_{is}\theta_{ei}\right)\alpha_2\phi_{ei}\beta_{ca}\sigma\Lambda_a(d_v+d_r)(d_v+d_r+\varphi_{ei})\beta_{mv} \\ &\quad -\beta_{ca}\sigma\left((d_a+\phi_{is})(d_a+\phi_{ei})-\phi_{is}\phi_{ei}\right)\alpha_1\theta_{ei}\beta_{ch}\sigma\Lambda_h(d_v+d_r)(d_v+d_r+\varphi_{ei})\beta_{mv}, \\ p_2 &= \beta_{ca}\sigma\left((d_a+\phi_{is})(d_a+\phi_{ei})-\phi_{is}\phi_{ei}\right)d_h(d_h+\theta_{is})(d_h+\theta_{ei})d_c(d_v+d_r)^2(d_v+d_r+\varphi_{ei})d_m\left(\Re_e^{(1)}-1\right) \\ &\quad +\beta_{ch}\sigma\left((d_h+\theta_{is})(d_h+\theta_{ei})-\theta_{is}\theta_{ei}\right)d_a(d_a+\phi_{is})(d_a+\phi_{ei})d_c(d_v+d_r)^2(d_v+d_r+\varphi_{ei})d_m\left(\Re_e^{(2)}-1\right) \\ &\quad -\beta_{mv}\alpha_1\theta_{ei}\beta_{ch}\sigma\Lambda_h d_a(d_a+\phi_{is})(d_a+\phi_{ei})d_c(d_v+d_r)(d_v+d_r+\varphi_{ei})d_m \\ &\quad -\beta_{mv}\alpha_2\phi_{ei}\beta_{ca}\sigma\Lambda_a d_h(d_h+\theta_{is})(d_h+\theta_{ei})d_c(d_v+d_r)(d_v+d_r+\varphi_{ei})d_m, \\ p_3 &= \left(\Re_e-1\right)d_h(d_h+\theta_{is})(d_h+\theta_{ei})d_c d_a(d_a+\phi_{is})(d_a+\phi_{ei})d_c(d_v+d_r)^2(d_v+d_r+\varphi_{ei})d_m, \end{aligned}$$

$$\Re_e = \Re_e^{(1)} + \Re_e^{(2)},$$

$$\Re_e^{(1)} = \frac{\sigma\beta_{mv}\Lambda_v\alpha_1\theta_{ei}\beta_{ch}\Lambda_h\varphi_{ei}}{(d_h+\theta_{ei})(d_h+\theta_{is})d_m(d_v+d_r)^2 d_h(d_v+d_r+\varphi_{ei})d_c},$$

$$\Re_e^{(2)} = \frac{\sigma\beta_{mv}\Lambda_v\alpha_2\phi_{ei}\beta_{ca}\Lambda_a\varphi_{ei}}{(d_a+\phi_{ei})(d_a+\phi_{is})d_m(d_v+d_r)^2 d_a(d_v+d_r+\varphi_{ei})d_c}.$$

It is clear that $P_1(I_v)$ has one zero root $(I_v=0)$. If we substitute $I_v=0$ into X_1, we obtain disease free equilibrium point. The other two roots of $P_1(I_v)$ are the solutions of $P_2(I_v)=0$. Since all parameters are positive, we observe that $\Re_e^{(1)}>0$ and $\Re_e^{(2)}>0$. It means that if $\Re_e<1$ then $\Re_e^{(1)}<1$ and $\Re_e^{(2)}<1$. Notice that $p_1<0$ always holds. It is easy to see that $\frac{p_2}{p_1}>0$ and $\frac{p_3}{p_1}>0$ if $\Re_e<1$. Hence, $P_2(I_v)$ has no positive roots if $\Re_e<1$. On the other hand, $\frac{p_3}{p_1}<0$ if $\Re_e>1$. Thus, $P_2(I_v)$ has one positive root and one negative root if $\Re_e>1$. Hence, we get the following results:

1. $P_1(I_v)$ has no positive roots if $\Re_e<1$;
2. $P_1(I_v)$ has one zero root, one positive root, and one negative root if $\Re_e>1$;
3. $P_1(I_v)$ has two zero roots and one negative root if $\Re_e=1$.

These results show that the endemic equilibrium point X_1 exists in Ω_{+0}^{11} if $\Re_e>1$.

3. MODEL ANALYSIS

3.1 The basic reproduction number

We use next generation matrix [12] to assess the basic reproduction number. We consider $E_h, I_h, E_a, I_a, E_v, I_v, C, M$ as infected compartments. Consequently, we get

$$F=\begin{pmatrix} 0&0&0&0&0&0&\beta_{ch}S_h^*&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&\beta_{ca}S_a^*&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&\beta_{mv}S_v^*\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix},$$

$$V=\begin{pmatrix} (d_h+\theta_{ei})&0&0&0&0&0&0&0\\ -\theta_{ei}&(d_h+\theta_{is})&0&0&0&0&0&0\\ 0&0&(d_a+\phi_{ei})&0&0&0&0&0\\ 0&0&-\phi_{ei}&(d_a+\phi_{is})&0&0&0&0\\ 0&0&0&0&(d_v+d_r+\varphi_{ei})&0&0&0\\ 0&0&0&0&-\varphi_{ei}&(d_v+d_r)&0&0\\ 0&0&0&0&0&-\sigma&d_c&0\\ 0&-\alpha_1&0&-\alpha_2&0&0&0&d_m \end{pmatrix}.$$

The basic reproduction number of system (1) is the spectral radius of FV^{-1}. It is easy to show that the spectral radius of FV^{-1} is

$$\rho\left(FV^{-1}\right)=\left(\frac{\sigma\beta_{mv}S_v^*\alpha_1\theta_{ei}\beta_{ch}S_h^*\varphi_{ei}}{(d_h+\theta_{ei})(d_h+\theta_{is})d_m(d_v+d_r)(d_v+d_r+\varphi_{ei})d_c}+\frac{\sigma\beta_{mv}S_v^*\alpha_2\phi_{ei}\beta_{ca}S_a^*\varphi_{ei}}{(d_a+\phi_{ei})(d_a+\phi_{is})d_m(d_v+d_r)(d_v+d_r+\varphi_{ei})d_c}\right)^{\frac{1}{2}}.$$

After substituting $S_h^*=\frac{\Lambda_h}{d_h}, S_a^*=\frac{\Lambda_a}{d_a}$, and $S_v^*=\frac{\Lambda_v}{d_v+d_r}$ into $\rho\left(FV^{-1}\right)$, we obtain the basic reproduction number

$$\Re_0=\left(\frac{\sigma\beta_{mv}\Lambda_v\alpha_1\theta_{ei}\beta_{ch}\Lambda_h\varphi_{ei}}{(d_h+\theta_{ei})(d_h+\theta_{is})d_m(d_v+d_r)^2 d_h(d_v+d_r+\varphi_{ei})d_c}+\frac{\sigma\beta_{mv}\Lambda_v\alpha_2\phi_{ei}\beta_{ca}\Lambda_a\varphi_{ei}}{(d_a+\phi_{ei})(d_a+\phi_{is})d_m(d_v+d_r)^2 d_a(d_v+d_r+\varphi_{ei})d_c}\right)^{\frac{1}{2}}.$$

We observe that $\Re_e=\Re_0^{\,2}$. It is clear that $\Re_0$ is always positive. Hence, if $\Re_0>1$ then $\Re_e>1$. In agreement with the existence condition of the endemic equilibrium, X_1 exists in Ω_{+0}^{11} if $\Re_0>1$.

3.2 Local stability of disease free equilibrium point

The Jacobian matrix of system (1) at X_0 is

$$J_{dfe}=\begin{pmatrix} -j_1 & 0 & j_2 & 0 & 0 & 0 & 0 & 0 & 0 & -j_3 & 0 \\ j_4 & -j_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & j_3 & 0 \\ 0 & j_6 & -j_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -j_8 & 0 & j_9 & 0 & 0 & 0 & -j_{10} & 0 \\ 0 & 0 & 0 & j_{11} & -j_{12} & 0 & 0 & 0 & 0 & j_{10} & 0 \\ 0 & 0 & 0 & 0 & j_{13} & -j_{14} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -j_{15} & 0 & 0 & 0 & -j_{16} \\ 0 & 0 & 0 & 0 & 0 & 0 & j_{17} & -j_{18} & 0 & 0 & j_{16} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & j_{19} & -j_{20} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & j_{21} & -j_{22} & 0 \\ 0 & 0 & j_{23} & 0 & 0 & j_{24} & 0 & 0 & 0 & 0 & -j_{25} \end{pmatrix},$$

where

$j_1=d_h$, $j_2=\theta_{is}$, $j_3=\beta_{ch}S_h^*$, $j_4=0$,
$j_5=(d_h+\theta_{ei})$, $j_6=\theta_{ei}$, $j_7=(d_h+\theta_{is})$, $j_8=d_a$,
$j_9=\phi_{is}$, $j_{10}=\beta_{ca}S_a^*$, $j_{11}=0$, $j_{12}=(d_a+\phi_{ei})$,
$j_{13}=\phi_{ei}$, $j_{14}=(d_a+\phi_{is})$, $j_{15}=(d_v+d_r)$, $j_{16}=\beta_{mv}S_v^*$,
$j_{17}=0$ $j_{18}=(d_v+d_r+\varphi_{ei})$, $j_{19}=\varphi_{ei}$, $j_{20}=(d_v+d_r)$
$j_{21}=\sigma$ $j_{22}=d_c$, $j_{23}=\alpha_1$, $j_{24}=\alpha_2$,
$j_{25}=d_m$.

The eigenvalues of J_{dfe} are the solutions of $G(\lambda)=0$,

where

$$G(\lambda)=(\lambda+j_1)(\lambda+j_8)(\lambda+j_{15})G_1(\lambda),$$

$$G_1(\lambda)=\lambda^8+g_1\lambda^7+g_2\lambda^6+g_3\lambda^5+g_4\lambda^4+g_5\lambda^3+g_6\lambda^2+g_7\lambda^1+g_8,$$

$$g_1=\sum_{i_1=1}^{8}k_{i_1},$$

$$g_2=\sum_{1\le i_1<i_2}^{8}k_{i_1}k_{i_2},$$

$$g_3=\sum_{1\le i_1<i_2<i_3}^{8}k_{i_1}k_{i_2}k_{i_3},$$

$$g_4=\sum_{1\le i_1<i_2<i_3<i_4}^{8}k_{i_1}k_{i_2}k_{i_3}k_{i_4},$$

$$g_5=\sum_{1\le i_1<\ldots<i_5}^{8}k_{i_1}k_{i_2}k_{i_3}k_{i_4}k_{i_5},$$

$$g_6=\sum_{1\le i_1<\ldots<i_6}^{8}k_{i_1}k_{i_2}k_{i_3}k_{i_4}k_{i_5}k_{i_6}-j_{13}j_{24}j_{10}j_{16}j_{19}j_{21}-j_{23}j_3j_{16}j_{19}j_{21}j_6,$$

$$g_7=\sum_{1\le i_1<\ldots<i_7}^{8}k_{i_1}k_{i_2}k_{i_3}k_{i_4}k_{i_5}k_{i_6}k_{i_7}-(j_5+j_7)j_{13}j_{24}j_{10}j_{16}j_{19}j_{21}-(j_{12}+j_{14})j_{23}j_3j_{16}j_{19}j_{21}j_6,$$

$$g_8=(1-\Re_0^2)k_1k_2k_3k_4k_5k_6k_7k_8.$$

$k_1=j_5,k_2=j_7,k_3=j_{12},k_4=j_{14},k_5=j_{18},k_6=j_{20},k_7=j_{22},k_8=j_{25}.$

It is clear that $G(\lambda)$ has three negative eigenvalues, i.e., $\lambda_1=-j_1,\lambda_2=-j_8,\lambda_3=-j_{15}$. The other eigenvalues are roots of $G_1(\lambda)$. It is easy to see that $k_i>0$ for $i=1,\ldots,8$ since all parameters are positive. It is clear that if $\Re_0=1$ then $G_1(\lambda)$ has one zero root. Next, we use Routh Hurwitz array to investigate the local stability condition of X_0.

Table 2. Routh Hurwitz array

λ^8	1	g_2	g_4	g_6	g_8
λ^7	g_1	g_3	g_5	g_7	0
λ^6	$rh_1=\frac{g_1g_2-g_3}{g_1}$	$rh_2=\frac{g_1g_4-g_5}{g_1}$	$rh_3=\frac{g_1g_6-g_7}{g_1}$	g_8	0
λ^5	$rh_4=\frac{g_3rh_1-g_1rh_2}{rh_1}$	$rh_5=\frac{g_5rh_1-g_1rh_3}{rh_1}$	$rh_6=\frac{g_7rh_1-g_1g_8}{rh_1}$	0	0
λ^4	$rh_7=\frac{rh_2rh_4-rh_1rh_5}{rh_4}$	$rh_8=\frac{rh_3rh_4-rh_1rh_6}{rh_4}$	g_8	0	0
λ^3	$rh_9=\frac{rh_5rh_7-rh_4rh_8}{rh_7}$	$rh_{10}=\frac{rh_6rh_7-rh_4g_8}{rh_7}$	0	0	0
λ^2	$rh_{11}=\frac{rh_8rh_9-rh_7rh_{10}}{rh_9}$	g_8	0	0	0
λ	$rh_{12}=\frac{rh_{10}rh_{11}-rh_98_8}{rh_{11}}$	0	0	0	0
λ^0	g_8	0	0	0	0

Based on Routh Hurwitz condition [13], all roots of $G_1(\lambda)$ have negative real part if the following conditions are satisfied

$g_1>0,$ $rh_1>0,$ $rh_4>0,$ $rh_7>0,$
$rh_9>0,$ $rh_{11}>0,$ $rh_{12}>0,$ $g_8>0.$

We observe that $g_1>0$ and $rh_1>0$ always hold. Moreover, $g_8>0$ if $\Re_0<1$. Notice that if $\Re_0>1$ then at least one of the above conditions is not hold, e.g., $g_8<0$. This result implies that there is at least one eigenvalue which has positive real part if $\Re_0>1$. Hence, we obtain the following theorem.

Theorem 3. X_0 *is locally asymptotically stable If* $\Re_0<1$ *and the following conditions are fulfilled*

$rh_4>0,$ $rh_7>0,$ $rh_9>0,$
$rh_{11}>0,$ $rh_{12}>0,$

X_0 *is unstable If* $\Re_0>1$.

Analytically, we are not able to prove that $rh_4>0,\ rh_7>0,\ rh_9>0,\ rh_{11}>0, rh_{12}>0$ if $\Re_0<1$. However, our numerical studies show that if $\Re_0<1$ then $rh_4>0,\ rh_7>0,\ rh_9>0,\ rh_{11}>0, rh_{12}>0.$ In section 3.4., we prove that the stability condition of X_0 is only and totally dependent on $\Re_0$.

3.3 Local stability of endemic equilibrium point

Theorem 4. X_1 *is locally asymptotically stable if* $\Re_0 > 1$.

Proof. Center manifold theory [15] is used to prove this theorem. We have proved that J_{dfe} has one zero eigenvalue if $\Re_0 = 1$. We investigate the existence of forward bifurcation at $\Re_0 = 1$ to prove the local stability condition of the endemic equilibrium point. We set β_{mv} as bifurcation parameter. Hence, the bifurcation point is

$$\beta_{mv}^{*} = \left(\frac{\sigma \Lambda_h \alpha_1 \theta_{ei} \beta_{ch} \Lambda_h \varphi_{ei}}{(d_h + \theta_{ei})(d_h + \theta_{is}) d_m (d_v + d_r)^2 d_h (d_v + d_r + \varphi_{ei}) d_c} + \frac{\sigma \Lambda_v \alpha_2 \phi_{ei} \beta_{cv} \Lambda_v \varphi_{ei}}{(d_a + \phi_{ei})(d_a + \phi_{is}) d_m (d_v + d_r)^2 d_a (d_v + d_r + \varphi_{ei}) d_c} \right)^{-1}.$$

It is clear that $J_{dfe}\left(\beta_{mv}^{*}\right)$ has simple zero eigenvalue. The right eigenvector $\vec{w}^{r}$ and left eigenvector $\vec{w}^{L}$ of $J_{dfe}\left(\beta_{mv}^{*}\right)$ corresponding to zero eigenvalue are

$$\vec{w}^{r} = \begin{pmatrix} w_1^{r} \\ w_2^{r} \\ w_3^{r} \\ w_4^{r} \\ w_5^{r} \\ w_6^{r} \\ w_7^{r} \\ w_8^{r} \\ w_9^{r} \\ w_{10}^{r} \\ w_{11}^{r} \end{pmatrix} = \begin{pmatrix} \frac{\left(j_2 j_6 j_3 j_{21} j_{19} j_{16} - j_7 j_5 j_3 j_{21} j_{19} j_{16}\right)}{j_1 j_7 j_5 j_{20} j_{18} j_{22}} w_{11}^{r} \\ \frac{j_3 j_{21} j_{19} j_{16}}{j_5 j_{20} j_{18} j_{22}} w_{11}^{r} \\ \frac{j_6 j_3 j_{21} j_{19} j_{16}}{j_7 j_5 j_{20} j_{18} j_{22}} w_{11}^{r} \\ \frac{\left(j_9 j_{13} j_{10} j_{21} j_{19} j_{16} - j_{14} j_{12} j_{10} j_{21} j_{19} j_{16}\right)}{j_{14} j_{12} j_{20} j_{18} j_{22} j_8} w_{11}^{r} \\ \frac{j_{10} j_{21} j_{19} j_{16}}{j_{12} j_{20} j_{18} j_{22}} w_{11}^{r} \\ \frac{j_{13} j_{10} j_{21} j_{19} j_{16}}{j_{14} j_{12} j_{20} j_{18} j_{22}} w_{11}^{r} \\ \frac{-j_{16}}{j_{15}} w_{11}^{r} \\ \frac{j_{16}}{j_{18}} w_{11}^{r} \\ \frac{j_{19} j_{16}}{j_{20} j_{18}} w_{11}^{r} \\ \frac{j_{21} j_{19} j_{16}}{j_{20} j_{18} j_{22}} w_{11}^{r} \\ w_{11}^{r} \end{pmatrix},$$

$$\left(\vec{w}^{L}\right)^{T} = \begin{pmatrix} w_1^{L} \\ w_2^{L} \\ w_3^{L} \\ w_4^{L} \\ w_5^{L} \\ w_6^{L} \\ w_7^{L} \\ w_8^{L} \\ w_9^{L} \\ w_{10}^{L} \\ w_{11}^{L} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{j_6 j_{23} j_{16}}{j_5 j_{25} j_7} w_8^{L} \\ \frac{j_{23} j_{16}}{j_{25} j_7} w_8^{L} \\ 0 \\ \frac{j_{13} j_{24} j_{16}}{j_{12} j_{25} j_{14}} w_8^{L} \\ \frac{j_{24} j_{16}}{j_{25} j_{14}} w_8^{L} \\ 0 \\ w_8^{L} \\ \frac{j_{21}}{j_{20} j_{22}} \left(\frac{j_3 j_6 j_{23} j_{16}}{j_5 j_{25} j_7} + \frac{j_{10} j_{13} j_{24} j_{16}}{j_{12} j_{25} j_{14}} \right) w_8^{L} \\ \frac{1}{j_{22}} \left(\frac{j_3 j_6 j_{23} j_{16}}{j_5 j_{25} j_7} + \frac{j_{10} j_{13} j_{24} j_{16}}{j_{12} j_{25} j_{14}} \right) w_8^{L} \\ \frac{j_{16}}{j_{25}} w_8^{L} \end{pmatrix},$$

where

$j_1 = d_h$, $j_2 = \theta_{is}$, $j_3 = \beta_{ch} S_h^{*}$, $j_5 = (d_h + \theta_{ei})$,
$j_6 = \theta_{ei}$, $j_7 = (d_h + \theta_{is})$, $j_8 = d_a$, $j_9 = \phi_{is}$,
$j_{10} = \beta_{ca} S_a^{*}$, $j_{12} = (d_a + \phi_{ei})$, $j_{13} = \phi_{ei}$, $j_{14} = (d_a + \phi_{is})$,
$j_{15} = (d_v + d_r)$, $j_{16} = \beta_{mv}^{*} S_v^{*}$, $j_{18} = (d_v + d_r + \varphi_{ei})$, $j_{19} = \varphi_{ei}$,
$j_{20} = (d_v + d_r)$ $j_{21} = \sigma$ $j_{22} = d_c$, $j_{23} = \alpha_1$,
$j_{24} = \alpha_2$, $j_{25} = d_m$.

w_{11}^{r} is arbitrary positive. w_8^{L} is computed, such that $\vec{w}^{L} \square w^{r} = 1$. It is clear that $w_8^{L} > 0$ and $w_1^{r}, w_4^{r}, w_7^{r} < 0$; $w_2^{r}, w_3^{r}, w_5^{r}, w_6^{r}, w_8^{r}, w_9^{r}, w_{10}^{r} > 0$; $w_2^{L}, w_3^{L}, w_5^{L}, w_6^{L}, w_9^{L}, w_{10}^{L}, w_{11}^{L} > 0.$

Now, we set

$$x_1 = S_h, x_2 = E_h, x_3 = I_h, x_4 = S_a, x_5 = E_a,$$
$$x_6 = I_a, x_7 = S_v, x_8 = E_v, x_9 = I_v, x_{10} = C, x_{11} = M,$$
$$f_1 = \frac{dS_h}{dt}, f_2 = \frac{dE_h}{dt}, f_3 = \frac{dI_h}{dt}, f_4 = \frac{dS_a}{dt}, f_5 = \frac{dE_a}{dt},$$
$$f_6 = \frac{dI_a}{dt}, f_7 = \frac{dS_v}{dt}, f_8 = \frac{dE_v}{dt}, f_9 = \frac{dI_v}{dt}, f_{10} = \frac{dC}{dt}, f_{11} = \frac{dM}{dt}.$$

Thus, we obtain the following results

$$w_2^{\,L}w_1^{\,r}w_{10}^{\,r}\frac{\partial^2 f_2(\mathrm{X}_0,\beta_{mv}^{\,*})}{\partial x_1\partial x_{10}}=\beta_{ch}\left(\frac{j_6j_{23}j_{16}}{j_5j_{25}j_7}w_8^{\,L}\right)\left(\frac{(j_2j_6j_3j_{21}j_{19}j_{16}-j_7j_5j_3j_{21}j_{19}j_{16})}{j_1j_7j_5j_{20}j_{18}j_{22}}w_{11}^{\,r}\right)\left(\frac{j_{21}j_{19}j_{16}}{j_{20}j_{18}j_{22}}w_{11}^{\,r}\right)<0,$$

$$w_2^{\,L}w_{10}^{\,r}w_1^{\,r}\frac{\partial^2 f_2(\mathrm{X}_0,\beta_{mv}^{\,*})}{\partial x_{10}\partial x_1}=\beta_{ch}\left(\frac{j_6j_{23}j_{16}}{j_5j_{25}j_7}w_8^{\,L}\right)\left(\frac{(j_2j_6j_3j_{21}j_{19}j_{16}-j_7j_5j_3j_{21}j_{19}j_{16})}{j_1j_7j_5j_{20}j_{18}j_{22}}w_{11}^{\,r}\right)\left(\frac{j_{21}j_{19}j_{16}}{j_{20}j_{18}j_{22}}w_{11}^{\,r}\right)<0,$$

$$w_5^{\,L}w_4^{\,r}w_{10}^{\,r}\frac{\partial^2 f_5(\mathrm{X}_0,\beta_{mv}^{\,*})}{\partial x_4\partial x_{10}}=\beta_{ca}\left(\frac{j_{13}j_{24}j_{16}}{j_{12}j_{25}j_{14}}w_8^{\,L}\right)\left(\frac{(j_9j_{13}j_{10}j_{21}j_{19}j_{16}-j_{14}j_{12}j_{10}j_{21}j_{19}j_{16})}{j_{14}j_{12}j_{20}j_{18}j_{22}j_8}w_{11}^{\,r}\right)\left(\frac{j_{21}j_{19}j_{16}}{j_{20}j_{18}j_{22}}w_{11}^{\,r}\right)<0,$$

$$w_5^{\,L}w_{10}^{\,r}w_4^{\,r}\frac{\partial^2 f_5(\mathrm{X}_0,\beta_{mv}^{\,*})}{\partial x_{10}\partial x_4}=\beta_{ca}\left(\frac{j_{13}j_{24}j_{16}}{j_{12}j_{25}j_{14}}w_8^{\,L}\right)\left(\frac{(j_9j_{13}j_{10}j_{21}j_{19}j_{16}-j_{14}j_{12}j_{10}j_{21}j_{19}j_{16})}{j_{14}j_{12}j_{20}j_{18}j_{22}j_8}w_{11}^{\,r}\right)\left(\frac{j_{21}j_{19}j_{16}}{j_{20}j_{18}j_{22}}w_{11}^{\,r}\right)<0,$$

$$w_8^{\,L}w_7^{\,r}w_{11}^{\,r}\frac{\partial^2 f_8(\mathrm{X}_0,\beta_{mv}^{\,*})}{\partial x_7\partial x_{11}}=\beta_{mv}^{\,*}w_8^{\,L}\left(\frac{-j_{16}}{j_{15}}w_{11}^{\,r}\right)w_{11}^{\,r}<0,$$

$$w_8^{\,L}w_{11}^{\,r}w_7^{\,r}\frac{\partial^2 f_8(\mathrm{X}_0,\beta_{mv}^{\,*})}{\partial x_{11}\partial x_7}=\beta_{mv}^{\,*}w_8^{\,L}\left(\frac{-j_{16}}{j_{15}}w_{11}^{\,r}\right)w_{11}^{\,r}<0,$$

$$w_8^{\,L}w_{11}^{\,r}\frac{\partial^2 f_8(\mathrm{X}_0,\beta_{mv}^{\,*})}{\partial\beta_{mv}\partial x_{11}}=w_8^{\,L}w_{11}^{\,r}\frac{\Lambda_v}{d_v+d_r}>0.$$

Hence, we get

$$a=w_2^{\,L}w_{10}^{\,r}w_1^{\,r}\left(\frac{\partial^2 f_2(\mathrm{X}_0,\beta_{mv}^{\,*})}{\partial x_1\partial x_{10}}+\frac{\partial^2 f_2(\mathrm{X}_0,\beta_{mv}^{\,*})}{\partial x_{10}\partial x_1}\right)$$
$$+w_5^{\,L}w_{10}^{\,r}w_4^{\,r}\left(\frac{\partial^2 f_5(\mathrm{X}_0,\beta_{mv}^{\,*})}{\partial x_4\partial x_{10}}+\frac{\partial^2 f_5(\mathrm{X}_0,\beta_{mv}^{\,*})}{\partial x_{10}\partial x_4}\right)$$
$$+w_8^{\,L}w_7^{\,r}w_{11}^{\,r}\left(\frac{\partial^2 f_8(\mathrm{X}_0,\beta_{mv}^{\,*})}{\partial x_7\partial x_{11}}+\frac{\partial^2 f_8(\mathrm{X}_0,\beta_{mv}^{\,*})}{\partial x_{11}\partial x_7}\right)$$
$$<0,$$

$$b=w_8^{\,L}w_{11}^{\,r}\frac{\partial^2 f_8(\mathrm{X}_0,\beta_{mv}^{\,*})}{\partial\beta_{mv}\partial x_{11}}>0.$$

Based on Theorem 4.1 in [15], forward bifurcation occurs at $\beta_{mv}=\beta_{mv}^{\,*}(\Re_0=1)$. According to the theorem, there is a positive equilibrium point which is locally asymptotically stable if $\beta_{mv}>\beta_{mv}^{\,*}$. Note that $\beta_{mv}>\beta_{mv}^{\,*}$ implies $\Re_0>1$. Therefore, the unique endemic equilibrium point that exists when $\Re_0>1$ is locally asymptotically stable if $\Re_0>1$.

3.4. Global stability of disease free equilibrium point

Theorem 5. *X_0 is globally asymptotically stable if $\Re_0\le 1$.*

Proof. Lyapunov function and LaSalle invariance principle are used to prove this theorem. Consider the candidate of Lyapunov function as follows

$$V=v_1\left(S_h-S_h^{\,*}-S_h^{\,*}\ln\frac{S_h}{S_h^{\,*}}\right)+v_2\left(S_a-S_a^{\,*}-S_a^{\,*}\ln\frac{S_a}{S_a^{\,*}}\right)+v_3\left(S_v-S_v^{\,*}-S_v^{\,*}\ln\frac{S_v}{S_v^{\,*}}\right)$$
$$+v_4E_h+v_5I_h+v_6E_a+v_7I_a+v_8E_v+v_9I_v+v_{10}C+v_{11}M,$$

where

$$v_1=v_4=\frac{\theta_{ei}\alpha_1(d_a+\phi_{is})}{\alpha_2(d_h+\theta_{is})(d_h+\theta_{ei})},\qquad v_2=v_6=\frac{\phi_{ei}}{(d_a+\phi_{ei})},$$
$$v_3=v_8=\frac{(d_a+\phi_{is})d_m}{\alpha_2\beta_{mv}S_v^{\,*}},v_5=\frac{\alpha_1(d_a+\phi_{is})}{\alpha_2(d_h+\theta_{is})},\qquad v_9=\frac{(d_a+\phi_{is})d_m(d_v+d_r+\varphi_{ei})}{\varphi_{ei}\alpha_2\beta_{mv}S_v^{\,*}},$$
$$v_{10}=\frac{(d_v+d_r)(d_a+\phi_{is})d_m(d_v+d_r+\varphi_{ei})}{\sigma\varphi_{ei}\alpha_2\beta_{mv}S_v^{\,*}},\qquad v_{11}=\frac{(d_a+\phi_{is})}{\alpha_2}.$$

The time derivative of V is

$$\frac{dV}{dt}=v_1\left(1-\frac{S_h^{\,*}}{S_h}\right)\frac{dS_h}{dt}+v_2\left(1-\frac{S_a^{\,*}}{S_a}\right)\frac{dS_a}{dt}+v_3\left(1-\frac{S_v^{\,*}}{S_v}\right)\frac{dS_v}{dt}+v_4\frac{dE_h}{dt}$$
$$+v_5\frac{dI_h}{dt}+v_6\frac{dE_a}{dt}+v_7\frac{dI_a}{dt}+v_8E_v+v_9\frac{dI_v}{dt}+v_{10}\frac{dC}{dt}+v_{11}\frac{dM}{dt}$$
$$=\frac{-d_h\theta_{ei}\alpha_1(d_a+\phi_{is})}{S_h\alpha_2(d_h+\theta_{is})(d_h+\theta_{ei})}(S_h-S_h^{\,*})^2-\frac{d_a\phi_{ei}}{S_a(d_a+\phi_{ei})}(S_a-S_a^{\,*})^2$$
$$-\frac{(d_v+d_r)(d_a+\phi_{is})d_m}{S_v(d_a+\phi_{ei})\alpha_2\beta_{mv}S_v^{\,*}}(S_v-S_v^{\,*})^2+\frac{\theta_{ei}\alpha_1(d_a+\phi_{is})\theta_{is}}{\alpha_2(d_h+\theta_{is})(d_h+\theta_{ei})}I_h\left(\frac{S_h-S_h^{\,*}}{S_h}\right)$$
$$+\frac{\phi_{ei}\phi_{is}}{(d_a+\phi_{ei})}I_a\left(\frac{S_a-S_a^{\,*}}{S_a}\right)+(\Re_0^2-1)\frac{(d_v+d_r)(d_a+\phi_{is})d_m(d_v+d_r+\varphi_{ei})d_c}{\sigma\varphi_{ei}\alpha_2\beta_{mv}S_v^{\,*}}C.$$

Note that $S_h\le S_h^{\,*},S_a\le S_a^{\,*},S_v\le S_v^{\,*}$ always hold. It is clear that $\frac{dV}{dt}\le 0$ if $\Re_0\le 1$. Moreover, $\frac{dV}{dt}=0$ if and only if $S_h=S_h^{\,*},S_a=S_a^{\,*},S_v=S_v^{\,*},C=0$. It follows that the largest invariant set contained in $\left\{(S_h,E_h,I_h,S_a,E_a,I_a,S_v,E_v,I_v,C,M)\,|\,\frac{dV}{dt}=0\right\}$ is a singleton set that is $\{\mathrm{X}_0\}$. Thus, X_0 is globally asymptotically stable if $\Re_0\le 1$.

4. NUMERICAL SIMULATIONS

In this section, we present two numerical simulations. The first and the second simulation show schistosomiasis prevalence when $\Re_0>1$ and $\Re_0<1$, respectively. All parameter values are chosen for illustrative objective only. The parameter values used are given in table 3.

After subtituting parameter values given in the second column of Table 3 into $\Re_0$, we get $\Re_0=1.4479>1$. Based on theorem 3 and Theorem 4, the disease free equilibrium point is unstable and the endemic equilibrium point is locally asymptotically stable. It means that all infected compartments, i.e., $E_h,I_h,E_a,I_a,E_v,I_v,C,M$ converge to positive equilibrium. Figure 1 shows that the solution curves of infected compartments tend to positive value. Hence, our theoretical result is similar to our numerical result.

Table 3. Parameter value

Symbol	Simulation with $\Re_0 > 1$	Simulation with $\Re_0 < 1$
Λ_h	2	2
Λ_a	0.5	0.5
Λ_v	0.75	0.75
β_{ch}	0.7	0.7
β_{ca}	0.7	0.3
β_{mv}	0.5	0.5
θ_{ei}	0.4	0.4
ϕ_{ei}	0.4	0.4
φ_{ei}	0.2	0.2
θ_{is}	0.9	0.9
ϕ_{is}	0.1	0.9
d_h	0.3	0.3
d_a	0.6	0.6
d_v	0.4	0.4
d_c	0.3	0.3
d_m	0.2	0.2
d_r	0.4	0.7
σ	0.7	0.7
α_1	0.6	0.6
α_2	0.6	0.6

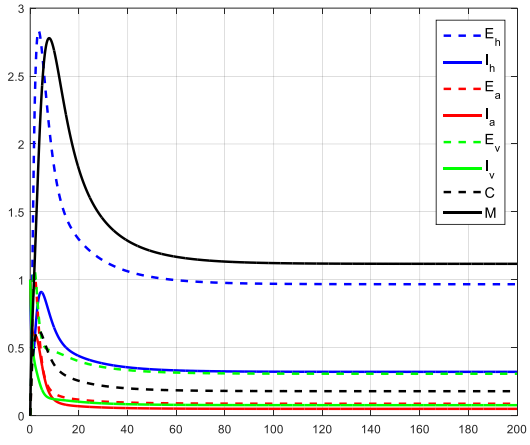

Figure 1. Schistosomiasis prevalence with $\Re_0 > 1$

For the second simulation, we set β_{ca} and ϕ_{is}, d_r less than the values used in the first simulation and higher than the values used in the first simulation, respectively. It means that there are interventions which reduce transmission on the cattle and increase recovery rate of the cattle. After subtituting parameter values given in the the third column of table 3 into $\Re_0$, we get $\Re_0 = 0.8740 < 1$. Based on Theorem 5, the disease free equilibrium point is globally asymptotically stable. It means that all infected compartments, i.e., $E_h, I_h, E_a, I_a, E_v, I_v, C, M$ tend to zero.

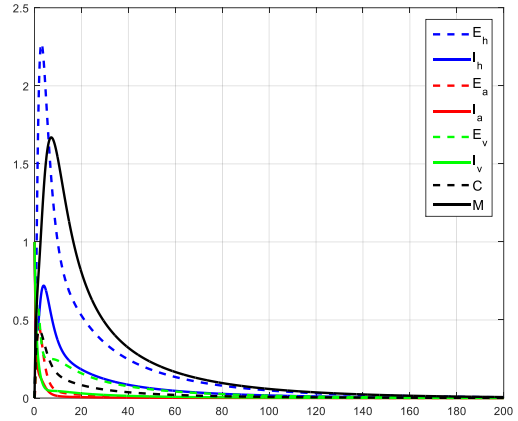

Figure 2. Schistosomiasis prevalence with $\Re_0 < 1$

Figure 2 shows that the solution curves of infected compartments tend to zero. Hence, our theoretical result is similar to our numerical result. In addition, intervention on the cattle population can reduce schistosomiasis prevalence in human, cattle and snail population.

5. CONCLUSION

In this work, we discuss a deterministic mathematical model to study schistosomiasis transmission dynamics in traditional cattle farmer communities. We observe that intervention on the cattle can reduce schistosomiasis prevalence in human and cattle population. The farmer should keep the cattle, water, and food clean. In addition, the farmer should use molluscicide in their farm area and give schistosomiasis drug to the cattle, regularly.

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