

Fuzzy Left Almost Semihyperring

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ABSTRACT

The left almost semihyperring (LA) is an algebraic hyperstructure satisfying two axioms, these are the left inverse hyperoperation and the distributive axiom between multiplication and addition hyperoperation. In this article, the concept of fuzzy sets is applied to these structures so that we get a new algebraic structure, and it is called fuzzy left almost semihyperring. We show that the set of all fuzzy subsets in LA-semihyperring is also LA-semihyperring. Furthermore, the LA-subsemihyperring and hyperideal fuzzy properties and their relationship to the characteristic function and level set are analyzed.

Keywords: LA-semihyperrings, Subset Fuzzy, Fuzzy Hyperideals.

1. INTRODUCTION

Mathematics as a universal language for the analysis, simulation, optimization and control of an industrial process. Mathematical abstraction is able to provide solutions to practical problems that stimulate new findings. The development of technology and communication in this era grows rapidly and there are many interconnections between algebra and science technology and communication, such as cryptography, algorithmic efficiency analysis, geometric structures, automata theory, and also in the concept of probability. Some of these algebraic fields are concepts of algebraic hyperstructure that introduced by the French mathematician, Marty[1] and fuzzy set theory which were first introduced by Zadeh[2]. Nowadays, Fuzzy set theory has various applications in areas such as computer engineering, artificial intelligence, control engineering, operations research, management science, robots and many more.

A number of articles have applied fuzzy concepts to algebraic structures and algebraic hyperstructures. Rosenfeld[3] applied the fuzzy concept to groups. Kuroki[4] introduced the idea of fuzzy ideal and fuzzy bi-ideal to semigroups. Leoreanu and Davvaz[5] introduced fuzzy hyperring. Ahsan et al.[6] discussed fuzzy semiring and its applications. The fuzzy semihypergroup introduced by Sen et al.[7]. Dutta et al.[8] introduced the idea of fuzzy prime and ideal prime in semihyperring.

Kehayopulu[9] discussed fuzzy semihypergroups. The purpose of this article is to apply the fuzzy concept to a hyperstructure algebra called "left almost (LA) semihyperring" that introduced by Nawaz et al.[10] We define the notion of fuzzy LA-semihyperring and analyze its relationship to LA-semihyperring and discuss its hyperideal properties and its relation to t-levels and characteristic functions.

2. PRELIMINARIES

This research uses literature study in several scientific books and journals related to algebraic hyperstructures and the concept of fuzzy sets. We analyze the definitions and properties of the previous structure to develop the LA-semihyperring fuzzy structure. Some basic terminology from the literature on left Almost (LA) semihyperring and fuzzy sets is presented in this section.

Definition 2.1.[10] Let H be the non-empty set and $\mathcal{P}^*(H)$ the set of all non-empty subsets of H . Binary hyperoperation " \oplus " is a mapping defined as follows.

$$\begin{aligned} \oplus: H \times H &\rightarrow \mathcal{P}^*(H) \\ (a, b) &\rightarrow \oplus(a, b) = a \oplus b \end{aligned}$$

The structure (H, \oplus) is called hypergroupoid.

Definition 2.2.[10] Let $A, B \subseteq H$ are non-empty set and $x \in H$. The hyperoperation " \oplus " on $A, B, \{x\} \in \mathcal{P}^*(H)$ is defined as follows.

$$(i) \quad A \oplus B = \bigcup_{a \in A, b \in B} (a \oplus b)$$

- (ii) $A \oplus x = A \oplus \{x\}$.
- (iii) $x \oplus B = \{x\} \oplus B$

Definition 2.3.[10] A hypergroupoid (H, \oplus) is called LA-semihypergroup if for all $x, y, z \in G$ satisfies the left inverted axiom as follows.

$$(x \oplus y) \oplus z = (z \oplus y) \oplus x$$

Which means that

$$\bigcup_{u \in (x \oplus y)} u \oplus z = \bigcup_{v \in (z \oplus y)} v \oplus x$$

Definition 2.4.[10] An algebraic hyperstructure $(R, \circ, *)$ is said to be a LA-semihyperring if it satisfies the following axioms.

- (i) (R, \circ) is a LA-semihypergroup.
- (ii) $(R, *)$ is a LA-semihypergroup with the absorbing element that is "0" ($0 * x = x * 0 = 0$) for every $x \in R$.
- (iii) The hyperoperation " $*$ " is distributive with respect to the hyperoperation " \circ " for all $x, y, z \in R$, that is
- (iv) $x * (y \circ z) = (x * y) \circ (x * z)$ and $(y \circ z) * x = (y * x) \circ (z * x)$.

Definition 2.5.[10] Let R be an LA-hypersemiring. An element $e \in R$ is called

- (i) Left identity (resp., pure left identity) if for all $x \in R, x \in e * x$ (resp., $x = e \circ x$)
- (ii) Right identity (resp., pure right identity) if for all $x \in R, x \in x * e$ (resp., $x = x \circ e$)
- (iii) Identity (resp., pure identity) if for all $x \in R, x \in e * x \cap x * e$ (resp., $x = e \circ x \cap x \circ e$)

3. FUZZY LA SEMIHYPERRING

This section present the definition and some basic properties of fuzzy LA-semihyperring and its hyperideal

Definition 3.1. Let (R, \oplus, \odot) be an LA-semihyperring. A fuzzy subset f of R is a function that defined as follow.

$$f: R \mapsto [0,1]$$

Definition 3.2. Let $F(R)$ be a set of all fuzzy subset on LA-semihyperring (R, \oplus, \odot) . Hyperoperation " $+$ " and " \circ " on $F(R)$ are defined as follows.

$$(f + g)(x) = \begin{cases} \bigvee_{x \in (y \oplus z)} \{f(y) \wedge g(z)\}, & x \in y \oplus z \\ 0, & x \notin y \oplus z \end{cases}$$

and

$$(f \circ g)(x) = \begin{cases} \bigvee_{x \in (\sum_{i=1}^n y_i \odot z_i)} \left\{ \bigwedge_1^n f(y_i) \wedge g(z_i) \right\}, & x \in y \oplus z \\ 0, & x \notin y \oplus z \end{cases}$$

for every $f, g \in F(R)$ and $x, y, z \in R$.

Theorem 3.3. If (R, \oplus, \odot) is an LA-semihyperring then $(F(R), +, \circ)$ is an LA-semihyperring.

Proof. Let $f, g, h \in F(R)$ and $x \in R$ such that $x \notin y \circ z$ for some $y \circ z$ then

$$[(f + g) + h](x) = 0 = [(h + g) + f](x).$$

Let $x \in y \oplus z$ then

$$\begin{aligned} [(f + g) + h](x) &= \bigvee_{x \in y \oplus z} \{f + g\}(y) \wedge h(z) \\ &= \bigvee_{x \in y \oplus z} \left\{ \bigvee_{y \in a \oplus b} \{f(a) \wedge g(b)\} \wedge h(z) \right\} \\ &= \bigvee_{x \in (a \oplus b) \oplus z} \{f(a) \wedge g(b) \wedge h(z)\} \\ &= \bigvee_{x \in (a \oplus b) \oplus z} \{f(a) \wedge g(b) \wedge h(z)\} \\ &= \bigvee_{x \in (z \oplus b) \oplus a} \{h(z) \wedge g(b) \wedge f(a)\} \\ &= \bigvee_{x \in (z \oplus b) \oplus a} \{h(z) \wedge g(b) \wedge f(a)\} \\ &= \bigvee_{x \in p \oplus a} \left\{ \bigvee_{p \in z \oplus b} \{h(z) \wedge g(b)\} \wedge f(a) \right\} \\ &= \bigvee_{x \in p \oplus a} \{(h + g)(p) \wedge f(a)\} \\ &= [(h + g) + f](x). \end{aligned}$$

Therefore, $(F(R), +)$ is an LA-hypersemigroup. Let $x \notin \sum_{i=1}^n y_i \odot z_i$ then

$$[(f \circ g) \circ h](x) = 0 = [(h \circ g) \circ f](x).$$

Let $x \in \sum_{i=1}^n y_i \odot z_i$ then

$$\begin{aligned} [(f \circ g) \circ h](x) &= \bigvee_{x \in \sum_{i=1}^n y_i \odot z_i} \left\{ \bigwedge_1^n (f \odot g)(y_i) \wedge h(z_i) \right\} \\ &= \bigvee_{x \in \sum_{i=1}^n y_i \odot z_i} \left\{ \bigwedge_1^n \left\{ \bigvee_{y_i \in \sum_{j=1}^n a_j \odot b_j} \left\{ \bigwedge_1^n f(a_j) \wedge g(b_j) \right\} \wedge h(z_i) \right\} \right\} \\ &= \bigvee_{x \in \sum_{i=1}^n (\sum_{j=1}^n a_j \odot b_j) \odot z_i} \left\{ \bigwedge_1^n \left\{ \bigwedge_1^n f(a_j) \wedge g(b_j) \right\} \wedge h(z_i) \right\} \\ &= \bigvee_{x \in \sum_{i=1}^n (\sum_{j=1}^n a_j \odot b_j) \odot z_i} \left\{ \bigwedge_1^n \left\{ \bigwedge_1^n f(a_j) \wedge g(b_j) \right\} \wedge h(z_i) \right\} \\ &= \bigvee_{x \in \sum_{i=1}^n (a_i \odot b_i) \odot z_i} \left\{ \bigwedge_1^n (f(a_i) \wedge g(b_i)) \wedge h(z_i) \right\} \\ &= \bigvee_{x \in \sum_{i=1}^n (a_i \odot b_i) \odot z_i} \left\{ \bigwedge_1^n (f(a_i) \wedge g(b_i)) \wedge h(z_i) \right\} \\ &= \bigvee_{x \in \sum_{i=1}^n (z_i \odot b_i) \odot b_i} \left\{ \bigwedge_1^n (h(z_i) \wedge g(b_i)) \wedge f(a_i) \right\} \\ &= \bigvee_{x \in \sum_{i=1}^n (z_i \odot b_i) \odot b_i} \left\{ \bigwedge_1^n (h(z_i) \wedge g(b_i)) \wedge f(a_i) \right\} \\ &= \bigvee_{x \in \sum_{i=1}^n (\sum_{j=1}^n z_j \odot b_j) \odot a_i} \left\{ \bigwedge_1^n \left\{ \bigwedge_1^n h(z_j) \wedge g(b_j) \right\} \wedge f(a_i) \right\} \\ &= \bigvee_{x \in \sum_{i=1}^n p_i \odot a_i} \left\{ \bigwedge_1^n \left\{ \bigvee_{y_i \in \sum_{j=1}^n z_j \odot b_j} \left\{ \bigwedge_1^n h(z_j) \wedge g(b_j) \right\} \wedge f(a_i) \right\} \right\} \\ &= \bigvee_{x \in \sum_{i=1}^n p_i \odot a_i} \left\{ \bigwedge_1^n (h \odot g)(p_i) \wedge f(a_i) \right\} = [(h \circ g) \circ f](x). \end{aligned}$$

Now, we will prove that " $+$ " distributive to " \circ ".

$$\begin{aligned} [f \circ (g + h)](x) &= \bigvee_{x \in \sum_{i=1}^n y_i \odot z_i} \left\{ \bigwedge_1^n (f(y_i) \wedge (g + h)(z_i)) \right\} \\ &= \bigvee_{x \in \sum_{i=1}^n y_i \odot z_i} \left\{ \bigwedge_1^n \left\{ f(y_i) \wedge \bigvee_{z_i \in a \oplus b} \{g(a) \wedge h(b)\} \right\} \right\} \end{aligned}$$

$$\begin{aligned}
 &= \bigvee_{x \in \sum_{i=1}^n y_i \odot (a \oplus b)} \left\{ \bigwedge_1^n f(y_i) \wedge (g(a) \wedge h(b)) \right\} \\
 &= \bigvee_{x \in (\sum_{i=1}^n y_i \odot a) \oplus (\sum_{i=1}^n y_i \odot b)} \left\{ \left\{ \bigwedge_1^n (f(y_i) \wedge g(a)) \right\} \wedge \left\{ \bigwedge_1^n (f(y_i) \wedge h(b)) \right\} \right\} \\
 &= \bigvee_{x \in p \oplus q} \left\{ \bigvee_{p \in \sum_{i=1}^n y_i \odot a} \left\{ \bigwedge_1^n (f(y_i) \wedge g(a)) \right\} \wedge \bigvee_{q \in \sum_{i=1}^n y_i \odot b} \left\{ \bigwedge_1^n (f(y_i) \wedge h(b)) \right\} \right\} \\
 &= \bigvee_{x \in p \oplus q} \{ (f \circ g)(p) \wedge (f \circ h)(q) \} = [(f \circ g) + (f \circ h)](x).
 \end{aligned}$$

With analog way to prove right distributive. $F(R)$ is left inverted and distributive such that $(F(R), +, \circ)$ is an LA-semihyperring. \square

Theorem 3.4. If $(F(R), +, \circ)$ is an LA-semihyperring then the medial law is hold in $F(R)$.

Proof. Straightforward

Theorem 3.5. If (R, \oplus, \odot) is an LA-semihyperring with a pure left identity then the following axioms are hold.

- (i) $(f \circ g) \circ (h \circ k) = (k \circ h) \circ (g \circ f)$
- (ii) $f \circ (g \circ h) = g \circ (f \circ h)$.

For any $f, g, h, k \in F(R)$.

Proof. Straightforward.

Theorem 3.6. An LA-semihyperring $F(R)$ is a semihyperring if only if $f \circ (g \circ h) = (h \circ g) \circ f$ for all $f, g, h, \in F(R)$.

Proof. Straightforward.

Theorem 3.7. An LA-semihyperring $F(R)$ is a commutative semihyperring if only if $(f \circ g) \circ h = f \circ (h \circ g)$.

Proof. Straightforward.

Definition 3.8. Let (R, \oplus, \odot) be an LA-semihyperring. A fuzzy subset f is a fuzzy LA-subsemihyperring of R if the following axioms are hold.

- (i) $\bigwedge_{z \in x \oplus y} f(z) \geq \min\{f(x), f(y)\}$
- (ii) $\bigwedge_{z \in x \odot y} f(z) \geq \min\{f(x), f(y)\}$.

For all $x, y, z \in R$.

Definition 3.9. Let (R, \oplus, \odot) be an LA-semihyperring. A fuzzy subset f is hyperideal of R if the following axioms are hold.

- (i) $\bigwedge_{z \in x \oplus y} f(z) \geq \min\{f(x), f(y)\}$
- (ii) $\bigwedge_{z \in x \odot y} f(z) \geq \{f(y)\}$ (left hyperideal)
- (iii) $\bigwedge_{z \in x \odot y} f(z) \geq \{f(x)\}$ (right hyperideal)
- (iv) $\bigwedge_{z \in x \odot y} f(z) \geq \max\{f(x), f(y)\}$ (hyperideal).

For all $x, y, z \in R$

An LA-semihyperring R can be considered a fuzzy subset of itself notated as \mathbf{R} and defined as follows.

$$\begin{aligned}
 \mathbf{R} : R &\mapsto [0, 1] \\
 &: x \mapsto \mathbf{R}(x) = 1
 \end{aligned}$$

Lemma 3.10. If (R, \oplus, \odot) be an LA-semihyperring with a pure left identity then $\mathbf{R} \circ \mathbf{R} = \mathbf{R}$.

Proof. Let $x \in R$ and e be a pure left identity in R such that $x \in e \odot x$. Thus,

$$\begin{aligned}
 (\mathbf{R} \circ \mathbf{R})(x) &= \bigvee_{x \in \sum_{i=1}^n a_i \odot b_i} \left\{ \bigwedge_1^n \{ \mathbf{R}(a_i) \wedge f(b_i) \} \right\} \\
 &\geq \mathbf{R}(e) \wedge \mathbf{R}(x) = 1 = \mathbf{R}(x).
 \end{aligned}$$

Theorem 3.11. Let f be a fuzzy subset of an LA-semihyperring R the the following statements are true.

- (i) A fuzzy subset f is a fuzzy LA-subsemihyperring R if only if $f + f \subseteq f$ and $f \circ f \subseteq f$.
- (ii) A fuzzy subset f is a fuzzy left hyperideal of LA-semihyperring R if only if $f + f \subseteq f$ and $\mathbf{R} \circ f \subseteq f$.
- (iii) A fuzzy subset f is a fuzzy right hyperideal of LA-semihyperring R if only if $f + f \subseteq f$ and $f \circ \mathbf{R} \subseteq f$.

Proof. Let f and \mathbf{R} be subset fuzzy of an LA-semihyperring R .

(i) let f be a fuzzy LA-subsemihyperring R .

$$(f + f)(z) = \bigvee_{x \in a \oplus b} \{ f(a) \wedge f(b) \} \leq \bigvee_{x \in a \oplus b} f(a \oplus b) = f(z)$$

and

$$\begin{aligned}
 (f \circ f)(z) &= \bigvee_{x \in \sum_{i=1}^n a_i \odot b_i} \left\{ \bigwedge_1^n f(a_i) \wedge f(b_i) \right\} \\
 &\leq \bigvee_{x \in \sum_{i=1}^n a_i \odot b_i} \left\{ \bigwedge_1^n f(a_i \odot b_i) \right\} = f(z)
 \end{aligned}$$

Conversely, Let $f + f \subseteq f$ and $f \circ f \subseteq f$.

$$\begin{aligned}
 \bigwedge_{z \in x \oplus y} f(z) &\geq \bigwedge_{z \in x \oplus y} (f + f)(z) = (f + f)(x \oplus y) \\
 &= \bigvee_{z \in (x \oplus y) \in (a \oplus b)} f(a) \wedge f(b) \geq f(x) \wedge f(y)
 \end{aligned}$$

It means

$$\bigwedge_{z \in x \oplus y} f(z) \geq \min\{f(x), f(y)\}.$$

Next,

$$\begin{aligned}
 \bigwedge_{z \in x \odot y} f(z) &\geq \bigwedge_{z \in x \odot y} (f \circ f)(z) = (f \circ f)(x \odot y) \\
 &= \bigvee_{z \in (x \odot y) \in \sum_{i=0}^n a_i \odot b_i} \left\{ \bigwedge_1^n (f(a_i) \wedge f(b_i)) \right\} \\
 &\geq f(x) \wedge f(y)
 \end{aligned}$$

It means

$$\bigwedge_{z \in x \odot y} f(z) \geq \min\{f(x), f(y)\}.$$

(ii) Let f be a fuzzy left hyperideal of LA-semihyperring R and $x \in R$.

$$\begin{aligned}
 (\mathbf{R} \circ f)(z) &= \bigvee_{z \in \sum_{i=1}^n a_i \odot b_i} \left\{ \bigwedge_1^n \{ \mathbf{R}(a_i) \wedge f(b_i) \} \right\} \\
 &= \bigvee_{z \in \sum_{i=1}^n a_i \odot b_i} \left\{ \bigwedge_1^n \{ 1 \wedge f(b_i) \} \right\} = \bigvee_{z \in \sum_{i=1}^n a_i \odot b_i} \left\{ \bigwedge_1^n \{ f(b_i) \} \right\}
 \end{aligned}$$

$$\leq \bigvee_{z \in \sum_{i=1}^n a_i \odot b_i} \left\{ \bigwedge_1^n \{ f(a_i \odot b_i) \} \right\} = f(z)$$

Conversely, let $\mathbf{R} \circ f \subseteq f$ then

$$\begin{aligned}
 \bigwedge_{z \in x \odot y} f(z) &\geq \bigwedge_{z \in x \odot y} (\mathbf{R} \circ f)(z) = (\mathbf{R} \circ f)(x \odot y) \\
 &= \bigvee_{z \in (x \odot y) \in \sum_{i=0}^n a_i \odot b_i} \left\{ \bigwedge_1^n (\mathbf{R}(a_i) \wedge f(b_i)) \right\} \\
 &\geq \mathbf{R}(x) \wedge f(y) = 1 \wedge f(y) \geq f(y).
 \end{aligned}$$

It means that f a fuzzy hyperideal of R . With analog way to prove (iii).

Theorem 3.12. Let (R, \oplus, \odot) be an LA-semihyperring with pure left identity. If $f, g \in F(R)$ are fuzzy LA-subsemihyperring of R then $f \circ g$ is fuzzy LA-subsemihyperring of R .

Proof. Firstly, We will prove that $(f \circ g)(x) \wedge (f \circ g)(y) \leq \bigwedge_{z \in (x \oplus y)} (f \circ g)(z)$.

$$\begin{aligned} & (f \circ g)(x) \wedge (f \circ g)(y) \\ &= \left[\bigvee_{x \in \sum_{i=1}^n a_i \odot b_i} \left\{ \bigwedge_1^n f(a_i) \wedge g(b_i) \right\} \right] \wedge \left[\bigvee_{x \in \sum_{i=1}^n c_i \odot d_i} \left\{ \bigwedge_1^n f(c_i) \wedge g(d_i) \right\} \right] \\ &= \bigvee_{x \in \sum_{i=1}^n a_i \odot b_i} \bigvee_{x \in \sum_{i=1}^n c_i \odot d_i} \left\{ \bigwedge_1^n f(a_i) \wedge g(b_i) \wedge \bigwedge_1^n f(c_i) \wedge g(d_i) \right\} \\ &\leq \bigvee_{x \oplus y \in \sum_{i=1}^n (p_i \odot q_i)} \left\{ \bigwedge_1^n f(p_i) \wedge g(q_i) \right\} = (f \circ g)(x \oplus y) \\ &= \bigwedge_{z \in x \oplus y} (f \circ g)(z) \end{aligned}$$

Secondly, since $F(R)$ is an LA-semihyperring then the medial law is satisfied so that

$$(f \circ g) \circ (f \circ g) = (f \circ f) \circ (g \circ g) \subseteq f \circ g.$$

Theorem 3.13. Let (R, \oplus, \odot) be an LA-semihyperring with a pure identity. If $f, g \in F(R)$ are fuzzy hyperideal of R then $f \circ g$ is a fuzzy hyperideal of R .

Proof. Based on Theorem 3.12, It is clear that $f \circ g$ satisfy the first axiom of Definition 3.9. Next, we will prove $f \circ g$ is a fuzzy hyperideal. Let $x \in \sum_{i=0}^n a_i \odot b_i$ and e is a pure identity of R . By medial law we have

$$\begin{aligned} x \odot y &= \sum_{i=1}^n (a_i \odot b_i) \odot y = \sum_{i=1}^n (a_i \odot b_i) \odot (e \odot y) \\ &= \sum_{i=1}^n (a_i \odot e) \odot (b_i \odot y). \end{aligned}$$

Since f and g are hyperideal of R then

$$\begin{aligned} (f \circ g)(x) &= \bigvee_{x \in \sum_{i=1}^n a_i \odot b_i} \left\{ \bigwedge_1^n \{f(a_i) \wedge g(b_i)\} \right\} \\ &\leq \bigvee_{x \odot y \in \sum_{i=1}^n (a_i \odot e) \odot (b_i \odot y)} \left\{ \bigwedge_1^n \{f(a_i \odot e) \wedge g(b_i \odot y)\} \right\} \\ &\leq \bigvee_{x \odot y \in \sum_{i=1}^n c_i \odot d_i} \left\{ \bigwedge_1^n \{f(c_i) \wedge g(d_i)\} \right\} = (f \circ g)(x \odot y) \\ &= \bigwedge_{z \in x \odot y} (f \circ g)(z) \end{aligned}$$

Thus, $f \circ g$ are a fuzzy right hyperideal of R . With analog way to prove fuzzy left hyperideal of R .

Theorem 3.14. Let (R, \oplus, \odot) be an LA-semihyperring with pure identity. Every fuzzy right hyperideal of R is a fuzzy hyperideal of R .

Proof. Let e be a pure identity of R and f be a fuzzy right hyperideal of R . We will prove that f is a fuzzy left hyperideal of R . By Lemma 3.10 we have

$$R \circ f = (R \circ R) \circ f = (f \circ R) \circ R \subseteq f \circ R \subseteq f.$$

Theorem 3.15. Let (R, \oplus, \odot) be an LA-semihyperring. If $\{f_i\}_{i \in \mathbb{N}}$ are fuzzy left hyperideal of R then $\bigcap_{i \in \mathbb{N}} f_i$ is a fuzzy left hyperideal of R .

Proof. Let f_i be fuzzy left hyperideal of R for all $i \in \mathbb{N}$.

$$\begin{aligned} \bigcap_{i \in \mathbb{N}} f_i(x \oplus y) &= \inf\{f_i(x \oplus y)\} \geq \inf\{\min\{f_i(x), f_i(y)\}\} \\ &= \min\{\inf f_i(x), \inf f_i(y)\} \\ &= \min\{(\cap f_i)(x), (\cap f_i)(y)\} \end{aligned}$$

and

$$\begin{aligned} \bigcap_{i \in \mathbb{N}} f_i(x \odot y) &= \inf\{f_i(x \odot y)\} \geq \inf\{\min\{f_i(y)\}\} \\ &= \min\{\inf f_i(y)\} = \min\{(\cap f_i)(y)\}. \end{aligned}$$

So, $\bigcap_{i \in \mathbb{N}} f_i$ is a fuzzy left hyperideal of R .

Theorem 3.16. Let (R, \oplus, \odot) be an LA-semihyperring with pure left identity. If f and g are fuzzy hyperideal of R then $f \circ g \subseteq f \cap g$.

Proof. Let $f, g \in F(R)$ are fuzzy hyperideal of R .

$$\begin{aligned} (f \circ g)(x) &= \bigvee_{x \in \sum_{i=1}^n a_i \odot b_i} \left\{ \bigwedge_1^n \{f(a_i) \wedge g(b_i)\} \right\} \\ &\leq \bigvee_{x \in \sum_{i=1}^n a_i \odot b_i} \left\{ \bigwedge_1^n \{f(a_i \odot b_i) \wedge g(a_i \odot b_i)\} \right\} \\ &= \bigvee_{x \in \sum_{i=1}^n a_i \odot b_i} \left\{ \bigwedge_1^n \{(f \cap g)(a_i \odot b_i)\} \right\} \\ &= (f \cap g)(x). \end{aligned}$$

Theorem 3.17. Let (R, \oplus, \odot) be an LA-semihyperring. If f is a fuzzy right hyperideal of R then $f \circ f$ is a fuzzy hyperideal of R .

Proof. Let f is a fuzzy right hyperideal of R .

$$\begin{aligned} (f \circ f)(x) &= \bigvee_{x \in \sum_{i=1}^n a_i \odot b_i} \left\{ \bigwedge_1^n \{f(a_i) \wedge f(b_i)\} \right\} \\ &= \bigvee_{x \odot y \in \sum_{i=1}^n (a_i \odot b_i) \odot y} \left\{ \bigwedge_1^n \{f(a_i) \wedge f(b_i)\} \right\} \\ &\leq \bigvee_{x \odot y \in \sum_{i=1}^n (y \odot b_i) \odot a_i} \left\{ \bigwedge_1^n \{f(y_i \odot b_i) \wedge f(a_i)\} \right\} \\ &= \bigvee_{x \odot y \in \sum_{i=1}^n p_i \odot a_i} \left\{ \bigwedge_1^n \{f(p_i) \wedge f(a_i)\} \right\} = (f \circ f)(x \odot y) \\ &= \bigwedge_{z \in x \odot y} (f \circ f)(z). \end{aligned}$$

It means that $f \circ f$ is a fuzzy left hyperideal so that $f \circ f$ is a fuzzy hyperideal.

Theorem 3.18. Let (R, \oplus, \odot) be an LA-semihyperring with left identity. If f is a fuzzy left hyperideal of R then $f \cup (f \circ R)$ is a fuzzy hyperideal.

Proof. We have f is a fuzzy left hyperideal of R . Since R is an LA-semihyperring then

$$\begin{aligned} (f \cup (f \circ R)) \circ R &= (f \circ R) \cup ((f \circ R) \circ R) \\ &= (f \circ R) \cup (R \circ R) \circ f \\ &= (f \circ R) \cup (R \circ f) \subseteq (f \circ R) \cup f \\ &\subseteq f \cup (f \circ R). \end{aligned}$$

With analog way to prove that $f \cup (f \circ R)$ is a fuzzy left hyperideal.

Definition 3.19. Let (R, \oplus, \odot) be an LA-semihyperring and $f \in F(R)$. For all $t \in [0,1]$, we have level set of R as follows.

$$f_t = \{x | x \in R, f(x) \geq t\}$$

Definition 3.20. Let (R, \oplus, \odot) be an LA-semihyperring and A be a non empty subset of R . The characteristic function of A is defined as follows.

$$\chi_A : R \mapsto [0,1]$$

$$: x \mapsto \chi_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

Lemma 3.21. Let (R, \oplus, \odot) be an LA-semihyperring and $A, B \in P^*(R)$ then the properties are satisfied.

- If $A \subseteq B$ then $\chi_A \subseteq \chi_B$
- $\chi_A + \chi_B = \chi_{A+B}$
- $\chi_A \circ \chi_B = \chi_{A \circ B}$
- $\chi_A \cup \chi_B = \chi_{A \cup B}$
- $\chi_A \cap \chi_B = \chi_{A \cap B}$

Theorem 3.22. Let (R, \oplus, \odot) be an LA-semihyperring and f be a subset fuzzy of R . Then Subset fuzzy f is a fuzzy LA-subsemihyperring (resp., left, right hyperideal) of R if only if for every $t \in [0,1]$, level set f_t is a fuzzy LA-subsemihyperring (resp., left, right hyperideal) of R

Proof. Let f be a fuzzy left hyperideal of R and $f_t \neq \emptyset$. Let $x, y \in f_t$ and $r \in R$. By Definition 3.19 we have $f(y) \geq t$ for any $t \in [0,1]$ such that

$$\bigwedge_{z \in x \oplus y} f(z) = f(x \oplus y) \geq \min\{f(x), f(y)\} \geq t$$

and

$$\bigwedge_{z \in r \odot y} f(z) = f(r \odot y) \geq f(y) \geq t.$$

By Definition 3.19 we have $x \oplus y \in f_t$ and $x \odot y \in f_t$. Thus f_t is a fuzzy left hyperideal of R .

Conversely, suppose f_t is a fuzzy left hyperideal of R for every $t \in [0,1]$. Let $x, y \in R$. Assume

$$\bigwedge_{z \in x \oplus y} f(z) = f(x \oplus y) < \min\{f(x), f(y)\}$$

choose $t \in [0,1]$ such that

$$\bigwedge_{z \in x \oplus y} f(z) < t \leq \min\{f(x), f(y)\}.$$

Then $x, y \in f_t$ so it must be $x \oplus y \in f_t$ but $\bigwedge_{z \in x \oplus y} f(z) < t$ so that $x \oplus y \notin f_t$. That is contradiction so that the assumption is wrong. Thus, $\bigwedge_{z \in x \oplus y} f(z) \geq \min\{f(x), f(y)\}$. Furthermore, we will prove that $\bigwedge_{z \in x \odot y} f(z) \geq f(y)$. Assume

$$\bigwedge_{z \in x \odot y} f(z) = f(x \odot y) < f(y).$$

For $t \in [0,1]$, we have

$$\bigwedge_{z \in x \odot y} f(z) < t \leq f(y).$$

So that $y \in f_t$. Since f_t is fuzzy left hyperideal of R then for any $x \in R$, $x * y \in R \odot f_t \subseteq f_t$ but $\bigwedge_{z \in x \odot y} f(z) < t$ so that $x \odot y \notin f_t$. It is contradiction, so that the assumption is wrong. Hence, $\bigwedge_{z \in x \odot y} f(z) \geq f(y)$. So, f is a fuzzy left hyperideal of R .

Theorem 3.23. Let (R, \oplus, \odot) be an LA-semihyperring. The following statements are true.

- (i) A non empty subset S is an LA-subsemihyperring of R if only if χ_S is a fuzzy LA-subsemihyperring of R .
- (ii) A non empty subset I is a left hyperideal (resp., right hyperideal) of R if only if χ_I is an fuzzy left hyperideal (resp., right hyperideal) of R .

Proof. i) (\Rightarrow) Let S be an LA-subsemihyperring of R . Let $x, y \in R$. If $x \oplus y \notin S$ and $x \odot y \notin S$ then $\chi_S(x) = 0$, $\chi_S(y) = 0$, $\chi_S(x \oplus y) = 0$ and $\chi_S(x \odot y) = 0$ such that

$$\bigwedge_{z \in x \oplus y} \chi_S(z) = \chi_S(x \oplus y) \geq 0 = \min\{\chi_S(x), \chi_S(y)\}$$

and

$$\bigwedge_{z \in x \odot y} \chi_S(z) = \chi_S(x \odot y) \geq 0 = \min\{\chi_S(x), \chi_S(y)\}.$$

If $x \oplus y \in S$ and $x \odot y \in S$ then $\chi_S(x) = 1$ and $\chi_S(y) = 1$ such that

$$\bigwedge_{z \in x \oplus y} \chi_S(z) = 1 \geq \min\{\chi_S(x), \chi_S(y)\}$$

and

$$\bigwedge_{z \in x \odot y} \chi_S(z) = 1 \geq \min\{\chi_S(x), \chi_S(y)\}.$$

Therefore, χ_S is a fuzzy LA-subsemihyperring of R . (\Leftarrow) Let χ_S be a fuzzy LA-subsemihyperring of R . Let $x, y \in R$ such that $\chi_S(x) = 1$ and $\chi_S(y) = 1$ then

$$\bigwedge_{z \in x \oplus y} \chi_S(z) = \chi_S(x \oplus y) \geq \min\{\chi_S(x), \chi_S(y)\} = 1$$

and

$$\bigwedge_{z \in x \odot y} \chi_S(z) = \chi_S(x \odot y) \geq \min\{\chi_S(x), \chi_S(y)\} = 1$$

Hence, $\chi_S(x \oplus y) = 1$ and $\chi_S(x \odot y) = 1$ such that $x \oplus y \in S$ and $x \odot y \in S$. Thus, S is an LA-subsemihyperring of R .

ii) (\Rightarrow) Let I be a left hyperideal of R . Let $x, y \in R$. If $y \notin I$ then $\chi_I(y) = 0$ such that

$$\bigwedge_{z \in x \odot y} \chi_I(z) = \chi_I(x \odot y) \geq 0 = \chi_I(y).$$

If $y \in I$ then $x * y \in I$. Since I is a left hyperideal of R then

$$\bigwedge_{z \in x \odot y} \chi_I(z) = \chi_I(x \odot y) \geq 1 = \chi_I(y).$$

So, χ_I is a fuzzy LA-subsemihyperring of R .

(\Leftarrow) Let χ_I is a fuzzy LA-subsemihyperring of R . Let $r \in R$ and $y \in I$. Since χ_I is a fuzzy LA-subsemihyperring of R then

$$\bigwedge_{z \in r \odot y} \chi_I(z) = \chi_I(r \odot y) \geq \chi_I(y) = 1.$$

This implies $\chi_I(r \odot y) = 1$ such that $r \odot y \in R \odot I \subseteq I$. Hence, I is a hyperideal of R .

4. CONCLUSION

The set of all fuzzy subsets of LA-semihyperring is also LA-semihyperring so that the algebraic hyperstructure satisfies all the axioms on the left almost

structure. A fuzzy subset of an LA-semihyperring can be considered as fuzzy sub LA-semihyperring, fuzzy left hyperideal, or fuzzy right hyperideal so that all sub fuzzy and hyperideal fuzzy properties are fulfilled. A fuzzy subset is fuzzy sub LA-semihyperring (fuzzy hyperideal) if and only if the level set is also fuzzy sub LA-semihyperring (fuzzy hyperideal). A subset of LA-semihyperring (hyperideal) is sub LA-semihyperring (hyperideal) if and only if the characteristic function is also sub LA-semihyperring (hyperideal).

ACKNOWLEDGMENTS

The authors gratefully acknowledge the reviewer who has been willing to review our article. Your comments and suggestions have really helped in the development of our article.

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