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Completely Prime Modules Over Path Algebras for Dynkin Quiver of Type A_n and D_n

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ABSTRACT

Let K be a field and Q be a quiver (directed graph), and let $A = KQ$ be the path algebra that corresponds to Q with coefficients in K. An A-module M is a completely prime (c-prime) module in the sense that $rm = 0$ for one $m \in M$ and $r \in A$ implies that either r annihilates all M or $m = 0$. In this paper, we prove that for Dynkin quiver type A_n and D_n , an A-module M is c-prime if and only if it is simple.

.***Keywords:*** *Completely Prime Module, Path Algebra, Dynkin.*

1. INTRODUCTION

The concept of a prime module was proposed as a generalization of the prime ideal structure of a ring [2]. Suppose that M is a left module over the ring R (written R - module). A proper submodule N of M is said to be prime if $rRm = 0$ with $r \in R$, and $m \in M$ implies $m \in N$ or $r \subseteq MN$ [2]. Various different contexts and example have been studied by [9][15][12]. Irawati ([5],[6]) generalized the concept of the hereditary Noether prime ring (HNP) into the concept of the HNP module. In addition, Wardhana [14] and Saleh [11] discussed the characterization of prime submodules. A module M is said to be a completely prime (c-prime) module over R if $rm = 0$ with $r \in R$ and $m \in M$ implies $rM = 0$ or $m = 0$. We prove that if M is a c-prime module, then M is a prime module. If R is a commutative ring, then the prime module is the same as the c-prime module. The aim of this study was to explore the notion of c-prime modules in the setting of path algebras.

The path algebra is part of representation theory. The representation theory is used to represent a complex mathematical object into a simpler one. A path algebra KQ is an algebra over a field K whose basis is the set of all paths in a quiver Q. A quiver $Q = (Q^0, Q^1, r, s)$ consists of two sets Q^0 (whose elements are called vertices), Q^1 (whose elements are called arrows), and two maps , s , $Q^1 \to Q^0$, which associate to each arrow $e \in Q^1$, its source $s(e) \in Q^0$, and its range $r(e) \in Q^0$, respectively. An algebra can be represented in the form of a quiver and a module can be represented as a quiver representation [1]. The path algebra has been widely studied in ([10], [8], [13], [3]).

In Section 2 of this paper, we will give the basic definitions and notation that will be used in this paper. In Section 3, it is shown that indecomposable simple modules over the path algebra of Dynkin quiver type A_n and D_n are c-prime modules.

2. PATH ALGEBRA, REPRESENTATION AND MODULE

This section explains the basic theory of path algebra. We refer to [1] for more details.

A quiver $Q = (Q^0, Q^1, r, s)$ consists of two sets $Q^{0,}Q^1$ and two maps $s: Q^1 \to Q^0$ and

$r: Q^1 \to Q^0$. The elements of Q^0 are called vertices and the elements of Q^1 are called arrows. An arrow α in Q^1 of source $a = s(\alpha)$ and range $b = r(\alpha)$ is usually denoted by $\alpha : a \to b$. A quiver $Q = (Q^0, Q^1, r, s)$ is usually denoted by $Q = (Q^0, Q^1)$ or even simply by Q. A quiver Q is said to be finite if Q^0 and Q^1 are finite sets. If $s^{-1}(v)$ is a finite set for every $v \in Q^0$, then Q is called row finite. A vertex v which emits no arrows is called a sink.

A path in quiver Q is a sequence of arrows $p = e_1 e_2 \ldots e_n$ such that $r(e_i) = s(e_i + 1)$ for all i. A finite path in quiver Q is a finite sequence of arrows $p = e_1 e_2 \ldots e_n$ where $r(e_i) = s(e_i + 1)$ for all i. In this case, the path p is said to have length n, denoted by $l(p) = n$. If p is a path such that $v = s(p) = r(p)$, then p is called a closed path based at v. If $r(p) = s(p)$ and $s(e_i) \neq s(e_j)$ for every $i \neq j$, then p is called a cycle. A quiver without cycles is called acyclic.

A path algebra $A = KQ$ is an algebra over a field K whose basis is the set of all paths in quiver Q. Multiplication of two paths is given by concatenation if this is defined, and 0 otherwise. Extending this bilinearly one gets an algebra structure. Note that A has a unit if and only if Q has only a finitely many vertices. Let Q be a finite connected quiver. A two-sided ideal of A built by arrows in Q is called an arrow ideal of A and is denoted by R_Q. A two-sided ideal I of KQ is said to be admissible if there exist $m \geq 2$ such that $R_Q^m \subseteq I \subseteq R_Q^2$. If I is an admissible ideal of A, (Q, I) is said to be a bound quiver. The quotient algebra A/I is called a bound quiver algebra. If A is isomorphic to a bound quiver (Q, I), we visualize any (finite dimensional) A-module M as a K-linear representation of (Q, I), that is, a family of (finite dimensional) K-vector spaces M_a, with $a \in Q^0$ connected by K-linear maps $\varphi\alpha : M_a \to M_b$ corresponding to arrows $\alpha : a \to b$ in Q, and satisfying some relations induced by I.

Let Q be a finite quiver. A representation M of Q is defined as follows: each vertex $a \in Q^0$ associates to a K-vector space M_a and each arrow $\alpha : a \to b$ in Q^1 associates to a K-linear map $\varphi_\alpha : M_a \to M_b$. Such a representation is denoted as $= (M_a, \varphi_\alpha)_{a \in Q^0, a \in Q^1}$, or simply as $M = (M_a, \varphi_\alpha)$. It is called a finite dimensional representation if each vector space M_a is finite dimensional.

Let $M = (M_a, \varphi_\alpha)$ and $M' = (M'_a, \varphi'_\alpha)$ be two representations of Q. A morphism (of representations) $f: M \to M'$ is a family $f = (f_a)_{a \in Q^0}$ of K- linear maps $(f_a : M_a \to M'_a)_{a \in Q^0}$ that are compatible with the structure maps $\varphi\alpha$, that is, for each arrow $\alpha: a \to b$, we have $\varphi'_\alpha f_a = f_b \varphi_a$ or, equivalently, the following square is commutative:

$$\begin{array}{ccc} M_a & \overset{\varphi_\alpha}{\longrightarrow} & M_b \\ \downarrow f_a & & \downarrow f_b \\ M'_a & \overset{\varphi'_\alpha}{\longrightarrow} & M'_b \end{array}$$

Let $f: M \to M'$ and $g: M' \to M"$ be two morphisms of representations of Q, where $f = (f_a)_{a \in Q^0}$ and $g = (g_a)_{a \in Q^0}$. Their composition is defined to be the family $= (g_a f_a)_{a \in Q^0}$, then gf is easily seen to be a morphism from M to $M"$.

The following theorem explains that the category mod A whose objects are finitely generated A-modules and whose morphisms are A-linear maps is equivalent to the category $rep_K(Q, I)$ whose objects are K-linear representations of Q that are bounded by I and finite dimensional.

Theorem 2.1. *(see [1, Theorem 1.6])*

Let $A = KQ/I$ be connected algebra with finite quiver Q and I be an admissible ideal of KQ. There exists a K-linear equivalence of categories

$$F: mod\ A \overset{\simeq}{\to} rep_K(Q, I). \qquad (2.1)$$

By using Theorem 2.1, we can give an interpretation of a simple, projective and injective module as bound representation. Let $a \in Q^0$ define the representation $(S\ (a)_b, \varphi_\alpha)$ of Q, denoted by $S\ (a)$, as follows:

$$S(a)_b = \begin{cases} 0\ if\ b \neq a \\ K\ if\ b = a \end{cases}$$

$$\varphi_\alpha = 0 \text{ for each } \alpha \in Q_1$$

Clearly, $S\ (a)$ is a bound representation of (Q, I). Let (Q, I) be a bound quiver, $A = KQ/I$ and $P\ (a) = e_a A$ where $a \in Q^0$. If $P\ (a) = (P\ (a)_b, \varphi_\beta)$, then $P\ (a)_b$ is the K-vector space with as basis the set of all $\bar{\omega} = \omega + I$, with ω is a path from a to b and for an arrow $\beta : b \to c$, the K-linear map $\varphi_\beta: P(a)_b \to P(a)_c$ is given by right multiplication by $\bar{\beta} = \beta + I$. If $I(a) = (I(a)_b, \varphi_\beta)$, then $I(a)_b$ is the dual of K-vector space with as basis the set of all $\bar{\omega} = \omega + I$, with ω is a path from b to a and for an arrow $\beta : b \to c$, the K-linear map φ_β: $I(a)_b \to I(a)_c$ is given by the dual of the left multiplication $\bar{\beta} = \beta + I$.

3. INDECOMPOSABLE *C*-PRIME MODULES

This section discusses c-prime modules over the path algebra of a Dynkin quiver of type A_n and D_n.

Definition 3.1. Let M be a left R-module. We say that M is a c-prime module if for all $r \in R$, $m \in M$ with $rm = 0$, then $r \in Ann(M)$ or $m = 0$.

Dauns gave a general definition of prime modules[2].

Definition 3.2. Let M be a left R-module, then M is a prime module if for all $r \in R$, $m \in M$ with $rRm = 0$, then $rM = 0$ or $m = 0$.

If R is a commutative ring, then Definition 3.1 is equivalent to Definition 3.2.

Proposition 3.3. If M is a c-prime module, then M is a prime module.

Proof. Suppose M is a c-prime module. Let $r \in R$, $m \in M$, with $rRm = 0$, then it is clear that $rm = 0$. So $rM = 0$ and M is a prime module.

The aim of this section is to show that for Dynkin quiver type A_n and D_n, an indecomposable A-module M is c-prime if and only if it is simple.

Proposition 3.4. Let K be a field and $A = KA_n$ where A_n is the quiver in Figure 1, with $n \geq 1$. Let M be an indecomposable A-module, then M is c-prime if and only if M is simple.

Figure 1. Dynkin Quiver Type A_n

Proof. In the first step we see the subquiver of Figure 1 in 3 cases.

(i)

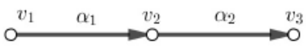

Figure 2. Subquiver A_3

Let A_3 be the quiver in Figure 2. Let $M = I(v_1) = S(v_1)$. We will show that $I(v_1) = S(v_1)$ is a c-prime module. Let $r \in KA_3$, $m = (m_1, 0, 0), m \in M$ with $rm = 0$. If $r = v_1$, then $m_1 = 0$. If $r = v_i$, $i \neq 1$, then $v_i \in Ann\,M$. If $r = \alpha_1$, then $\alpha_1 \in Ann\,M$. If $r = \alpha_2$, then $\alpha_2 \in Ann\,M$. If $r = \alpha_1\alpha_2$, then $\alpha_1\alpha_2 \in Ann\,M$. So $I(v_1) = S(v_1)$ is a c-prime module.

Figure 3. Representation of $I(v_1) = S(v_1)$

Let A_3 be the quiver in Figure 2. We will show that $M = S(v_2)$ is a c-prime module.

Let $r \in KA_3$, $m = (0, m_1, 0), m \in M$ with $rm = 0$. If $r = v_2$, then $m_2 = 0$. If $r = v_1$ $i \neq 2$, then $v_i \in Ann\,M$. If $r = \alpha_1$, then $\alpha_1 \in Ann\,M$. If $r = \alpha_2$, then $\alpha_2 \in Ann\,M$. If $r = \alpha_1\alpha_2$, then $\alpha_1\alpha_2 \in Ann$. So $S(v_2)$ is a c-prime module.

Figure 4. Representation of $S(v_2)$

Let A_3 be the quiver in Figure 2.

We will show that $M = S(v_3) = P(v_3)$ is a c-prime module.

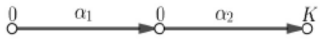

Figure 5. Representation of $S(v_3) = P(v_3)$

Let $r \in KA_3, m = (0, 0, m_1), m \in M$ with $rm = 0$. If $r = v_3$, then $m_1 = 0$. If $r = v_1$ $i \neq 3$, then $v_1 \in Ann\,M$. If $r = \alpha_1$, then $\alpha_1 \in Ann\,M$. If $r = \alpha_2$, then $\alpha_2 \in Ann\,M$. If $r = \alpha_1\alpha_2$ then $\alpha_1\alpha_2 \in Ann\,M$. So $S(v_3) = P(v_3)$ is a c-prime module.

Let A_3 be the quiver in Figure 2. We will show that $M = I(v_2)$ is not a c-prime module.

Figure 6. Representation of $I(v_2)$

Let $m = (m_1, m_2, 0), m \in M$ and $r = \alpha_1$, then $rm = 0$. But $\alpha_1 \notin Ann\,M$, so $I(v_2)$ is not a c-prime module.

Let A_3 be the quiver in Figure 2. We will show that $M = P\,(v_2)$ is not a c-prime module.

Let $m = (0, m_2, m_3), m \in M$ and $r = \alpha_2$, then $rm = 0$. But $\alpha_2 \notin Ann\,M$, so $P\,(v_2)$ is not a c-prime module. So by looking at the subquiver in Figure 2, we can see that M is a c-prime module if and only if M is simple.

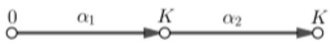

Figure 7. Representation of $P\,(v_2)$

Let A_3 be the quiver in Figure 2. We will show that $M = P\,(v_1) = I(v_3)$ is not a c-prime module.

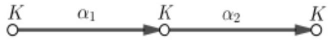

Figure 8. Representation of $P\,(v_1) = I(v_3)$

Let $m = (m_1, m_2, m_3), m \in M$ and $r = \alpha_1$, then $rm = 0$. But $\alpha_1 \notin Ann\,M$, so $P\,(v_1) = I(v_3)$ is not a c-prime module.

Thus, by looking at the subquiver in Figure 2, we can see that M is a c-prime module if and only if M is simple.

(ii)

Figure 9. Subquiver $A_3(1)$

Let A_3 be the quiver in Figure 9. Let $M = I(v_1) = S(v_1)$. We will show that $I(v_1) = S(v_1)$ is a c-prime module. Let $r \in KA_3, m = (m_1, 0, 0), m \in M$ with $rm = 0$. If $r = v_1$, then $m_1 = 0$. If $r = v_i$ $i \neq 1$, then $v_i \in Ann\, M$. If $r = \alpha_1$, then $\alpha_1 \in Ann\, M$. If $r = \alpha_2$, then $\alpha_2 \in Ann\, M$. So $I(v_1) = S(v_1)$ is a c -prime module.

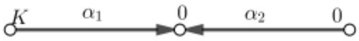

Figure 10. Representation of $I(v_1) = S(v_1)$

Let A_3 be the quiver in Figure 9. Let $M = P(v_2) = S(v_2)$. We will show that $P(v_2) = S(v_2)$ is a c-prime module. Let $r \in KA_3$, $m = (0, m_1, 0)$, $m \in M$ with $rm = 0$. If $r = v_2$, then $m_1 = 0$. If $r = v_i$, $i \neq 2$, then $v_i \in Ann\, M$. If $r = \alpha_i$, $i \in \{1, 2\}$, then $\alpha_i \in Ann\, M$. So $P(v_2) = S(v_2)$ is a c-prime module.

Figure 11. Representation of $P(v_2) = S(v_2)$

Let A_3 be the quiver in Figure 9. Let $M = I(v_3) = S(v_3)$. We will show that $I(v_3) = S(v_3)$ is a c-prime module. Let $r \in KA_3, m = (0, 0, m_1), m \in M$ with $rm = 0$. If $r = v_3$, then $m_1 = 0$. If $r = v_i, i \neq 3$, then $v_i \in Ann\, M$. If $r = \alpha_i, i \in \{1, 2\}$, then $\alpha_i \in Ann\, M$. So $I(v_3) = S(v_3)$ is a c-prime module.

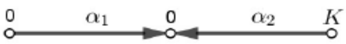

Figure 12. Representation of $I(v_3) = S(v_3)$.

Let A_3 be the quiver in Figure 9. We will show that $M = P(v_1)$ is not a c-prime module.

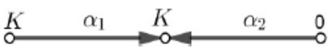

Figure 13. Representation of $P(v_1)$

Let $r \in KA_3, m = (m_1, m_2, 0), m \in M$ and $r = \alpha_1$, then $rm = 0$. But $\alpha_1 \notin Ann\, M$. So $P(v_1)$ is not a c-prime module.

Let A_3 be the quiver in Figure 9. We will show that $M = P(v_3)$ is not a c-prime module.

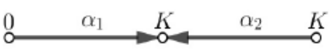

Figure 14. Representation of $P(v_3)$

Let $r \in KA_3, m = (0, m_1, m_2), m \in M$ and $r = \alpha_2$, then $rm = 0$. But $\alpha_2 \notin Ann\, M$. So $P(v_3)$ is not a c-prime module.

Let A_3 be the quiver in Figure 9. We will show that $M = I(v_2)$ is not a c-prime module.

Figure 15. Representation of $I(v_2)$

Let $r \in KA_3, m = (m_1, m_2, m_3), m \in M$ and $r = \alpha_1$, then $rm = 0$. But $\alpha_1 \notin Ann\, M$. So $I(v_2)$ is not a c-prime module. So by looking at the subquiver in Figure 9, we can see that M is a c-prime module if and only if M is simple.

(iii)

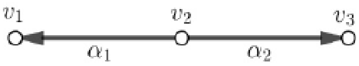

Figure 16. Subquiver $A_3(2)$

Let A_3 be the quiver in Figure 16. Let $M = P(v_1) = S(v_1)$. We will show that $P(v_1) = S(v_1)$ is a c-prime module.

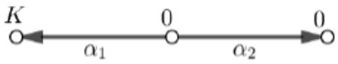

Figure 17. Representation of $P(v_1) = S(v_1)$

Let $r \in KA_3, m = (m_1, 0,0), m \in M$ with $rm = 0$. If $r = v_1$, then $m_1 = 0$. If $r = v_i$ $, i \neq 1$, then $v_i \in Ann\, M$. If $r = \alpha_i$, then $\alpha_i \in Ann\, M$. If $r = \alpha_2$, then $\alpha_2 \in Ann\, M$. So $(v_1) = S(v_1)$ is a c-prime module.

Let A_3 be the quiver in Figure 16. We will show that $M = I(v_2) = S(v_2)$ is a c-prime module.

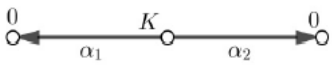

Figure 18. Representation of $I(v_2) = S(v_2)$

Let $r \in KA_3, m = (0, m_1, 0), m \in M$ with $rm = 0$. If $r = v_2$, then $m_1 = 0$. If $r = v_i$ $i \neq 2$, then $v_i \in Ann\, M$. If $r = \alpha_1$, then $\alpha_1 \in Ann\, M$. If $r = \alpha_2$, then $\alpha_2 \in Ann\, M$. So $I(v_2) = S(v_2)$ is a c-prime module.

Let A_3 be the quiver in Figure 16. We will show that $M = S(v_3) = P(v_3)$ is a c-prime module.

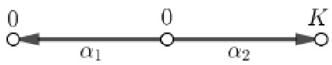

Figure 19. Representation of $S(v_3) = P(v_3)$

Let $r \in KA_3, m = (0, 0, m_1), m \in M$ with $rm = 0$. If $r = v_3$, then $m_1 = 0$. If $r = v_i$ $, i \neq 3$, then $v_i \in Ann\, M$. If $r = \alpha_1$, then $\alpha_1 \in Ann\, M$. If $r = \alpha_2$, then $\alpha_2 \in Ann\, M$. So $S(v_3) = P(v_3)$ is a c-prime module.

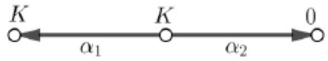

Figure 20. Representation of $I(v_1)$

Let A_3 be the quiver in Figure 16. We will show that $M = I(v_1)$ is not a c-prime module.

Let $r \in KA_3$, $m = (m_1, m_2, 0)$, $m \in M$ and $r = \alpha_1$, then $rm = 0$. But $\alpha_1 \notin Ann\, M$. So $I(v_1)$ is not a c-prime module.

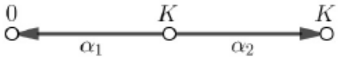

Figure 21. Representation of $I(v_1)$

Let A_3 be the quiver in Figure 16. We will show that $M = P(v_3)$ is not a c-prime module.

Let $r \in KA_3, m = (0, m_1, m_2), m \in M$ and $r = \alpha_2$, then $rm = 0$. But $\alpha_2 \notin Ann\, M$. So $I(v_3)$ is not a c-prime module.

Let A_3 be the quiver in Figure 16. We will show that $M = P(v_2)$ is not a c-prime module.

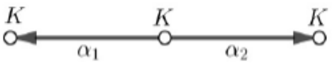

Figure 22. Representation of $P(v_2)$

Let $m = (m_1, m_2, m_3), m \in M$ and $r = \alpha_2$, then $rm = 0$. But $\alpha_2 \notin Ann\, M$. So $P(v_2)$ is not a c-prime module.

So by looking at the subquiver in Figure 16, we can see that M is a c-prime module if and only if M is simple.

Next, we can prove for v_i with $i = \{1, 2, \ldots, n\}$. Let A_n with $n \geq 1$. Let $I(v_1) = S(v_1)$. We will show that $M = I(v_1) = S(v_1)$ is a c-prime module. Let $r \in KA_n$, $m = (m_1, 0, \cdots, 0), m \in M$ with $rm = 0$, $rm = r(m_1, 0, \cdots, 0) = (0, 0, \cdots, 0)$. If $r = v_1$, then $m_1 = 0$. If $r = v_i, i \neq 1$, then $v_i \in Ann\ M$. If $r = \alpha_j$, $j \in \{1, \cdots, n-1\}$, then $\alpha_j \in Ann\ M$.

Figure 23. Representation of $S(v_1)$

If $r = \alpha_j \alpha_{j+i} \ldots . \alpha_k$ with $j, k \in \{1, \cdots, n-1\}, j < k$, then $\alpha_j \alpha_{j+i} \ldots . \alpha_k \in Ann\, M$. So $I(v_1) = S(v_1)$ is a c-prime module.

We will show that $M = S(v_i)$, for $i = 1, 2, \ldots, n-1$ is a c-prime module.

Figure 24. Representation of $S(v_i)$

Let $r \in KA_n$, $m = (0, 0, \cdots, m_1, \cdots, 0), m \in M$ with $rm = 0, rm = r(0, 0, \cdots\ m_1, \cdots, 0) = (0, 0, \cdots, 0)$. If $r = v_j, j \neq i$, then $v_j \in Ann\, M$. If $r = v_i$, then $m_i = 0$. If $r = \alpha_j$, then $\alpha_j \in Ann\, M$. If $r = \alpha_j \alpha_{j+i} \ldots \alpha_k$ with $j, k \in \{1, \cdots, n-2\}, j < k$, then $\alpha_j \alpha_{j+i} \ldots \alpha_k \in Ann\, M$. So $S(v_i)$ is a c-prime module.

Next, we will show that $M = P(v_n) = S(v_n)$ for $i = 1, 2, \cdots, n$ is a c-prime module.

Figure 25. Representation of $S(v_n)$

Let $r \in KA_n, m \in M$ with $rm = 0$, $m = (0, 0, 0, \cdots, m_n)$. If $r = v_i, i \neq n$, then $v_i \in Ann\, M$. If $r = v_n$, then $m_n = 0$. If $r = \alpha_j$, then $\alpha_j \in Ann\ M$. If $r = \alpha_j \alpha_{j+1} \ldots . \alpha_k$, then $\alpha_j \alpha_{j+1} \ldots . \alpha_k \in Ann\, M$. So $P(v_n) = S(v_n)$ is a c-prime module for $i = 1, 2, , n-1$.

Next we will show that $M = I(v_n) = P(v_1)$ is not a c-prime module.

Let $r = \alpha_1$, $m = (0, m_2, m_3, \ldots, m_n) \neq 0$, then $rm = 0$ but $\alpha_1 \notin Ann\, M$. So $I(v_n) = P(v_1)$ is not a c-prime module.

Figure 26. Representation of $I(v_n)$

Next we will show that $M = I(v_i)$ is not a c-prime module, for $i = 2, 3, \cdots, n-1$.

Let $m = (m_1, m_2, \ldots . m_{i-1}, 0, 0, \ldots, 0)$, and $r = \alpha_i$, then $rm = 0$. But $\alpha_i \notin Ann\, M$. So $I(v_i)$ is not a c-prime module.

Figure 27. Representation of $I(v_i)$

Next we will show that for another indecomposable module M over path algebra KA_n, M is not a c-prime module. We can see the representation of M in the Figure 28.

Figure 28. Representation of M

Suppose that r in KA_n, $m \in M$, where $m = (m_1, m_2, \ldots . m_l)$ with $rm = 0$.

Let $m_1 = \left(m_{(1,1)}, m_{(2,1)}, \ldots . m_{(a_{1,1})}\right)$,

$m_2 = (m_{(2,1)}, m_{(2,2)}, \ldots . m_{(a_{2,2})})$, and

$m_l = (m_{(1,l)}, m_{(2,l)}, \ldots . m_{(a_l,l)})$.

Without loss of generality, if $r = \alpha$ then $\varphi_\alpha(m_1) = 0$. Thus $m_1 \in Ker\ \varphi_\alpha$. If φ_α is an injective, then $m_1 = 0$. Suppose that $m = (0, m_2, \ldots. m_l) \neq 0$. Therefore $r \notin Ann\, M$. If φ_α is not injective, $0 \neq m_1 \in Ker\ \varphi_\alpha$. Suppose that $m' = (m_1', m_2', \ldots, m_l')$ with $m_1'\ Ker\ \varphi_\alpha$, then $rm' \neq 0$, such that $r \in Ann\ M$. So M is not a c-prime module.

Proposition 3.5. Let K be a field and $A = KD_n$, where D_n is the quiver in Figure 28, with $n \geq 4$. Let M be an indecomposable A-module, then M is c-prime if and only if M is simple.

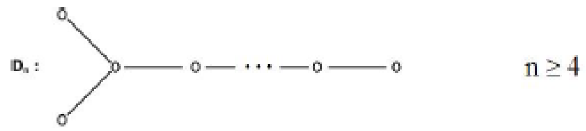

Figure 29. Dynkin Quiver Type D_n

Proof. The proof is similar as that for Proposition 3.4.

4. CONCLUSION

In this paper, we have discussed about c-prime modules over path algebra for Dykin quiver type A_n and D_n. The conclusion is as follows:

a. Let K be a field and $A = KA_n$ where A_n is the quiver in Figure 1, with $n \geq 1$. Let M be an indecomposable A-module, then M is c-prime if and only if M is simple.

b. Let K be a field and $A = KD_n$, where D_n is the quiver in Figure 29, with $n \geq 4$. Let M be an indecomposable A-module, then M is c-prime if and only if M is simple.

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