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Generalized Method of Moment Estimation Method Lagrange Multiplier Test for Simultaneous Spatial of Dynamic Panel Data

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ABSTRACT
In recent developments, the panel data spatial model has penetrated into simultaneous models that do not only move in single models, where the development of this model also indirectly makes the parameter estimation method move in a more complex direction in terms of mathematical proof, also in terms of proof empirically. This paper presents a Generalized Method of Moment (GMM) Lagrange Multiplier (LM) Test for simultaneous spatial modelling for dynamic panel. This is a spatial dependencies test for spatial models.

Keywords: *GMM, LM Test, Simultaneous, Spatial.*

1. INTRODUCTION

Spatial econometrics tends to start from a specific theory or model and focus on estimation, specification and testing problems when there are spatial effects. Spatial econometrics is a field of analytical engineering designed to combine dependencies between geographically adjacent observations (regions or points in space). Spatial econometrics is the development of a classic regression model with respect to cohorts (closest neighbours) that accommodate dependencies between regions / observations. In general, classical econometric models use time series data, while spatial econometric models use cross sectional data.

Initially the main focus of the spatial econometric model is the spatial lag model or commonly referred to as the Spatial Autoregressive (SAR) model or the Spatial Lag Model [1] and the Spatial Error Model (SEM), where the SAR model has a spatial interaction effect on its endogenous variables and models. SEM has a spatial interaction with its errors. [2] were developing a model with the effect of spatial interaction between endogenous variables and the shape of the error. In addition, [3] also introduced the Spatial Durbin Model (SDM) which included weights on the exogenous variables (X).

The estimation method for the Spatial Panel Data model conducted by [4] uses the MLE method to estimate its parameters. However, this method has problems in its computation if N is large. Meanwhile [5] suggest using the GMM estimation method which is computationally more feasible for large N. Furthermore, [2] generalized the GMM method for large samples of N $\rightarrow \infty$ with a fixed T, and based on the Monte Carlo experiment, it was found that the RMSE of the MLE and weighted GMM methods had relatively the same average value.

Spatial Panel Data Modelling has now developed towards simultaneous modelling no longer for single equations, as has been done by [6] that used a Simultaneous Spatial Data Panel model with a SEM model approach for regional growth models, the estimation method used was Generalized Spatial Three-Stage Least Squares (GS3SLS). Meanwhile [7] used the Simultaneous Spatial Autoregressive model with random effects and the parameter estimation method used was the Error Component 3 Stage Least Squares (EC3SLS) method. [8] have developed a Simultaneous

Spatial Dynamic Data Panel Model, namely the Multivariate Model and Simultaneous Equation Dynamic Panel Spatial Autoregressive Models with the Quasi MLE parameter estimation approach.

Dynamic panel spatial data modelling can explain changes in variables so that short-term and long-term effects will be known. The use of simultaneous equations for dynamic panel spatial data is still rarely used. The last research for modelling used the 2 SLS and 3 SLS estimation methods which had the disadvantages of asymptotic bias and inconsistent standard deviation. Meanwhile, the application of GMM in a single model shows that these problems can be resolved well, and the application of GMM estimation for dynamic panel data spatial simultaneous models with the High order approach was carried out by [9].

As with other spatial models, a test is needed whether the former model requires spatial weight in the SSDPD model. The spatial dependency test was first described by [10] with the Moran's I test, which is more in the direction of testing the spatial autocorrelation in the model. In the development of the spatial dependency test for panel data using Lagrange Multiplier and Likelihood Ratio as has been done by [11], while the spatial dependency test with the GMM approach was carried out by [12] and the spatial dependency test for dynamic panel data with the GMM approach was carried out by [13]. All spatial dependency tests that have been done above are still carried out for a single model. Therefore, this paper will describe the spatial dependency testing for the Simultaneous Spatial of Dynamic Panel Data (SSDPD) with the GMM estimation approach.

2. SPATIAL DEPENDENCIES TEST FOR SPATIAL DYNAMIC PANEL DATA MODEL

The spatial dependency test was first introduced by [10], Moran's I test was used to determine the presence of spatial dependencies on the regression model [14]. For other spatial dependency tests in the Cross Section model, it can be seen in [3], [15].

To test the spatial dependency in the panel data model, it can be seen in [16], [17],[18]. The spatial dependency test using Moran's I above for panel data has been developed by [19], namely by replacing the W weight matrix with the $\mathbf{W}_{NT}$, which is a weight matrix that includes the time element in it. Moran's I test for panel data is as follows:

$$I = \frac{\boldsymbol{e}'\mathbf{W}_{NT}\boldsymbol{e}}{\boldsymbol{e}'\boldsymbol{e}} \qquad (1)$$

Moran's I test statistic for spatial autocorrelation testing of the spatial panel data dynamic models with time invariant spatial weight matrix developed by [20].

$$I = \frac{\boldsymbol{e}'\left(\mathbf{I}_{T-1} \otimes \mathbf{W}_n\right)\boldsymbol{e}}{\boldsymbol{e}'\boldsymbol{e}} \qquad (2)$$

The Spatial panel data dynamic models used is spatial time simultaneous models with a stable trans-formed case approach. The Monte Carlo simulation results show that power Moran's I test for The Spatial panel data dynamic models tend to be large when N and T are bigger, the sign of spatial parameter dependencies δ for endogenous variables also affects the value of test power where the performance power test is better if the parameter value δ positive, so the greater the parameter value the better the power value of Moran's I test of the dynamic model. Moran's I test can measure the intensity of the spatial relations between units i and j positively. A large test power value also indicates that I > E (I) means that each spatial unit tends to be adjacent (grouped) with spatial units having the same attribute (in high-high or low-low relation-ship conditions).

A part from Moran's I method, other methods that are often used for testing spatial dependencies are the Lagrange Multiplier (LM), Likelihood Ratio (LR) and Wald methods. The LR test follows the Maximum Likelihood estimation method [3], which is the ratio between the spatial model likelihood function and the linear regression model likelihood function (where the spatial weight is zero), while the LM test is only based on estimates below H0 where the error is calculated based on estimation OLS on the model and multiplied by the weight of the matrix.

One of the tests of spatial dependency on panel data was carried out by [11], namely the Lagrange Multiplier (LM) and Likelihood Ratio (LR) statistical testing for spatial random effects on the panel data model. For the spatial model of fixed effects panel data, [21] propose two specifications, one with only individual effects (one-way effect), and the other with individual and time effects (two-way effect).

The last use of the LM test for dynamic panel data spatial models was carried out by [13], where [13] used [22] model as a model that tested its spatial dependency, is:

$$\begin{aligned}\boldsymbol{y}_{nt} &= \sum_{j=1}^{p}\lambda_{j0}\mathbf{W}_{nj}\boldsymbol{y}_{nt} + \gamma_0\boldsymbol{y}_{n,t-1} + \sum_{j=1}^{p}\rho_{j0}\mathbf{W}_{nj}\boldsymbol{y}_{n,t-1} \\ &\quad + \mathbf{X}_{nt}\beta_0 + \boldsymbol{c}_{n0} + \alpha_{t0}\boldsymbol{l}_n + \boldsymbol{V}_{nt}\end{aligned} \qquad (3)$$

With the model after the transformation is as follows:

$$\boldsymbol{J}_n\boldsymbol{y}_{nt}^{*} = \sum_{j=1}^{p}\lambda_{j0}\boldsymbol{J}_n\mathbf{W}_{nj}\boldsymbol{y}_{nt}^{*} + \gamma_0\boldsymbol{J}_n\boldsymbol{y}_{n,t-1}^{(*,-1)}$$

$$+\sum_{j=1}^{p}\rho_{j0}\boldsymbol{J}_n\mathbf{W}_{nj}\boldsymbol{y}_{n,t-1}^{(*,-1)}+\boldsymbol{J}_n\mathbf{X}_{nt}^{*}\beta_0+\boldsymbol{J}_n\mathbf{V}_{nt}^{*} \quad (4)$$

The LM test by [13] uses the GMM estimation method which can detect dependencies on spatial lag, time lag and spatial time lag. The test begins with estimating the optimal GMM on a limited model. [13] partitioned $\theta=(\beta',\psi',\phi')$, with ψ and ϕ are k_ψ x 1 and k_ϕx 1 sized vectors until $k_\psi + k_\phi = 2_p + 1$. ψ and ϕ can be a combination of several parameters namely {λ,γ,ρ}, such that the LM test with $H_0{:}\, r(\theta_0) = 0$; ; defined as follows:

$$LM = NC'\left(\hat{\theta}_{n,T,r}\right)B^{-1}\left(\hat{\theta}_{nT,r}\right)C\left(\hat{\theta}_{nT,r}\right) \quad (3)$$

with, $C_a = \boldsymbol{G}_a^{'}(\theta)\hat{\boldsymbol{\Sigma}}_{nT}^{-1}\bar{\boldsymbol{g}}_{nT}(\theta)$ and $G_a = \frac{1}{N}\frac{\partial \boldsymbol{g}_{nT}(\theta)}{\partial a'}$, $a \in \{\beta,\psi,\phi\}$ and $\bar{\boldsymbol{g}}_{nT} = \frac{1}{N}\boldsymbol{g}_{nT}$.

Also $G(\theta) = \left(G_\beta(\theta), G_\psi(\theta), G_\phi(\theta)\right)$ and $C(\theta) = (C'_\beta(\theta), C'_\psi(\theta), C'_\phi(\theta))'$ and the asymptotic distribution of the LM test is $LM \xrightarrow{d} \chi^2_{kr}$.

Moment matrix $\boldsymbol{g}_{nT}$ dan $\boldsymbol{\Sigma}_{\mathbf{nT}}$ that is used [13] follow [22] as in equation (2) and (3). For testing the parameters along with the hypothesis $H_0{:}\lambda_0=0$, $\rho_0=0$, $\gamma_0 = 0$, with H_1: at least one parameter is not equal to zero, then if H_0 is true, the model will be a non-spatial model that can be estimated using the OLS method. If $\hat{\theta}_{nT}$ is estimator GMM constrained optimal under the null hypothesis and $\hat{\theta}_{nT}$ is another consistent estimator of θ_0 under the null hypothesis. As stated in [23], LM test must be formulated based on an estimator GMM constrained optimal. If $\theta = (\lambda',\rho',\gamma)')$, so $\mathbf{B}(\boldsymbol{\theta}) = \mathbf{G}'(\boldsymbol{\theta})\hat{\boldsymbol{\Sigma}}_{\mathbf{nT}}^{-1}\mathbf{G}(\boldsymbol{\theta})$ and considered partitions from **B(θ):**

$$\mathbf{B}(\theta)=\begin{pmatrix} \underbrace{B_\beta(\theta)}_{k_x\times k_x} & \underbrace{B_{\beta\psi}(\theta)}_{k_x\times k_\psi} & \underbrace{B_{\beta\phi}(\theta)}_{k_x\times k_\phi} \\ \underbrace{B_{\psi\beta}(\theta)}_{k_\psi\times k_x} & \underbrace{B_\psi(\theta)}_{k_\psi\times k_\psi} & \underbrace{B_{\psi\phi}(\theta)}_{k_\psi\times k_\phi} \\ \underbrace{B_{\phi\beta}(\theta)}_{k_\phi\times k_x} & \underbrace{B_{\phi\psi}(\theta)}_{k_\phi\times k_\psi} & \underbrace{B_\phi(\theta)}_{k_\phi\times k_\phi} \end{pmatrix} \quad (4)$$

Where ψ and ϕ can be a combination of *{λ,γ,ρ}* then LM test defined as:

$$LM_J(\tilde{\theta}_{nT}) = NC'_\vartheta(\tilde{\theta}_{nT})\left[B_{\vartheta\beta}(\tilde{\theta}_{nT}\right]^{-1}C_\vartheta(\tilde{\theta}_{nT}), \quad (5)$$

with,

$$C'_\vartheta(\tilde{\theta}_{nT}) = \left(C'_\lambda(\tilde{\theta}_{nT}), C'_\rho(\tilde{\theta}_{nT}), C'_\gamma(\tilde{\theta}_{nT})\right)',$$

$$B_{\vartheta\beta}\left(\tilde{\theta}_{nT}\right) = B_\vartheta\left(\tilde{\theta}_{nT}\right) - B_{\vartheta\beta}(\tilde{\theta}_{nT})B_\beta^{-1}(\tilde{\theta}_{nT})B_{\beta\vartheta}(\tilde{\theta}_{nT})$$

$$B_{\vartheta\beta}(\tilde{\theta}_{nT}) = B'_{\beta\vartheta}(\tilde{\theta}_{nT}) = \left(B'_{\lambda\beta}(\tilde{\theta}_{nT}), B'_{\rho\beta}(\tilde{\theta}_{nT}), B'_{\gamma\beta}(\tilde{\theta}_{nT})\right)',$$

and,

$$B_\vartheta(\tilde{\theta}_{nT}) = \begin{pmatrix} B_\lambda(\tilde{\theta}_{nT}) & B_{\lambda\rho}(\tilde{\theta}_{nT}) & B_{\lambda\gamma}(\tilde{\theta}_{nT}) \\ B_{\rho\lambda}(\tilde{\theta}_{nT}) & B_\rho(\tilde{\theta}_{nT}) & B_{\rho\gamma}(\tilde{\theta}_{nT}) \\ B_{\gamma\lambda}(\tilde{\theta}_{nT}) & B_{\gamma\rho}(\tilde{\theta}_{nT}) & B_\gamma(\tilde{\theta}_{nT}) \end{pmatrix} \quad (6)$$

Based on the definition of the LM test above, (13) define several propositions including:

- The null hypothesis for the form of spatial lag *contemporaneous*: $H_0^\lambda{:}\, \lambda_0 = 0$ as are ρ_0 and γ_0.
- The null hypothesis for the form of spatial lag *contemporaneous*: $H_0^\rho{:}\, \rho_0 = 0$ as are λ_0 and γ_0.
- The null hypothesis for the form of spatial lag *contemporaneous*: $H_0^\gamma{:}\, \gamma_0 = 0$ as are ρ_0 and λ_0.

The following is given the test statistics of the above propositions:

- For $H_0^\lambda{:}\, \lambda_0 = 0$ with $\psi = \lambda$ and $\phi = (\rho',\gamma')$ then;

$$LM_\lambda\left(\tilde{\theta}_{nT}\right) = NC_\lambda^{'}\left(\tilde{\theta}_{nT}\right)\left[B_{\lambda\beta}\left(\tilde{\theta}_{nT}\right)\right]^{-1}C_\lambda\left(\tilde{\theta}_{nT}\right) \quad (7)$$

with

$$B_{\lambda\beta}(\tilde{\theta}_{nT}) = B_\lambda(\tilde{\theta}_{nT}) - B_{\lambda\beta}(\tilde{\theta}_{nT})B_\beta^{-1}(\tilde{\theta}_{nT})B_{\beta\lambda}(\tilde{\theta}_{nT}).$$

- For $H_0^\rho{:}\, \rho_0 = 0$ with $\psi = \rho$ and $\phi = (\lambda',\gamma')$ then

$$LM_\rho\left(\tilde{\theta}_{nT}\right) = NC_\rho^{'}\left(\tilde{\theta}_{nT}\right)\left[B_{\rho\beta}\left(\tilde{\theta}_{nT}\right)\right]^{-1}C_\rho\left(\tilde{\theta}_{nT}\right) \quad (8)$$

with $B_{\rho\beta}(\tilde{\theta}_{nT}) = B_\rho(\tilde{\theta}_{nT}) - B_{\rho\beta}(\tilde{\theta}_{nT})B_\beta^{-1}(\tilde{\theta}_{nT})B_{\beta\rho}(\tilde{\theta}_{nT})$

- For $H_0^\rho{:}\, \gamma_0 = 0$ with $\psi = \gamma$ and $\phi = (\lambda',\rho')$ then

$$LM_\gamma\left(\tilde{\theta}_{nT}\right) = NC_\gamma^{'}\left(\tilde{\theta}_{nT}\right)\left[B_{\gamma\beta}\left(\tilde{\theta}_{nT}\right)\right]^{-1}C_\gamma\left(\tilde{\theta}_{nT}\right) \quad (9)$$

with $B_{\gamma\beta}(\tilde{\theta}_{nT}) = B_\rho(\tilde{\theta}_{nT}) - B_{\gamma\beta}(\tilde{\theta}_{nT})B_\beta^{-1}(\tilde{\theta}_{nT})B_{\beta\gamma}(\tilde{\theta}_{nT})$.

3. METHODS

The spatial dependency identification test method for the SSDPD equation follows [13] with the following steps:

i. Determine the SSDPD model, where the SAR model is adapted from the [22] with the number of equations m = 2 as follows:

$$\boldsymbol{y}_{n1,t} = \delta_1\mathbf{W}_{n1}\boldsymbol{y}_{n1,t} + \eta_1\mathbf{W}_{n2}\boldsymbol{y}_{n1,t-1} + \tau_1\boldsymbol{y}_{n1,t-1} + k_1\boldsymbol{y}_{n2,t} + \beta_1\mathbf{X}_{n1,t} + \boldsymbol{c}_{n1} + \boldsymbol{v}_{n1,t} \quad (10)$$

$$\boldsymbol{y}_{n2,t} = \delta_2\mathbf{W}_{n1}\boldsymbol{y}_{n2,t} + \eta_2\mathbf{W}_{n2}\boldsymbol{y}_{n2,t-1} + \tau_2\boldsymbol{y}_{n2,t-1} + k_2\boldsymbol{y}_{n1,t} + \beta_2\mathbf{X}_{n1,t} + \boldsymbol{c}_{n2} + \boldsymbol{v}_{n2,t} \quad (11)$$

$$|\tau| + |\delta| + |\eta| < 1$$

with, $\boldsymbol{y}_{nm,t}$ is the column vector for the response variable of size *n x 1*, where $\boldsymbol{y}_{n1,t} = \left(y_{11t},\cdots,y_{n1,t}\right)'$

, $\boldsymbol{y}_{n2,t} = \left(y_{12t},\cdots,y_{n2,t}\right)'$, $\boldsymbol{y}_{nm,t-1}$ is a predetermined lag 1 variable vector of size *n x 1,* where $\boldsymbol{y}_{n1,t-1} = \left(y_{11t},\cdots,y_{n1,t-1}\right)'$, $\boldsymbol{y}_{n2,t-1} = \left(y_{12t},\cdots,y_{n2,t-1}\right)'$, $\boldsymbol{v}_{nt} = \left(v_{1t},\cdots v_{nt}\right)'$ is the column vector of error of size *n x* 1 and $\boldsymbol{v}_{nt} \sim N\left(0,\mathbf{I}\sigma_v^2\right)$, $\mathbf{W}_{nm}$ = spatial weight matrix of size *n x n*, $\mathbf{X}_{nmt}$ = explanatory variable matrix of size *n x p*, $\boldsymbol{c}_{nm}$ = fixed effect vector *n x 1* , *m* = 1, 2, n is the amount of panel data consisting of individual i and t time

ii. Determine the $\mathbf{W}_r$ matrix, which is a row-normalized weight matrix, the weight matrix is customized accordingly.

iii. Enter the weights $\mathbf{W}_r$ as a weight matrix $\mathbf{W}_{nm,t} = \mathbf{I}_t \otimes \mathbf{W}_r$ in equation (12)

iv. Transforming the model in step (c) with an orthonormal matrix $\mathbf{F}_{\mathrm{T,T-1}}, \mathbf{F}_{\mathrm{n,n-1}}$ to eliminate individual effect variables c_{n0} and time effect α_{t0} , in a way:

a. Form a matrix $\mathbf{J}_T = \left(\mathbf{I}_T - \frac{1}{T}\boldsymbol{\iota}_T\boldsymbol{\iota}_T'\right)$

b. Find the eigenvector matrix $\mathbf{F}_{T,T-1}$ from $\mathbf{J}_T$

c. Form an orthonormal matrix $\left[\mathbf{F}_{T,T-1}, \frac{1}{\sqrt{T}}\boldsymbol{l}_T\right]$

d. Forming the endogenous variable transformation matrix and endogenous lag matrix $\mathbf{Y}_{n,T}^* = \left[\boldsymbol{y}_{n,1}^*, \boldsymbol{y}_{n,2}^*, ..., \boldsymbol{y}_{n,T-1}^*\right] = \left[\boldsymbol{y}_{n,1}, \boldsymbol{y}_{n,2}, ..., \boldsymbol{y}_{n,T}\right]\mathbf{F}_{T,T-1}$

$$\mathbf{Y}_{nT-1}^{(*-1)} = \left[\boldsymbol{y}_{n,0}^{(*,-1)}, \boldsymbol{y}_{n,1}^{(*,-1)}, ..., \boldsymbol{y}_{n,T-2}^{(*,-1)}\right]$$
$$= \left[\boldsymbol{y}_{n,0}, \boldsymbol{y}_{n,1}, ..., \boldsymbol{y}_{n,T-1}\right]\mathbf{F}_{T,T-1}$$

e. Forms a transformation matrix for the predetermined variable $\mathbf{X}_{nT}^*$, $\mathbf{V}_{nT}^*$ and $\boldsymbol{c}_n^*$ by multiplying each by $\mathbf{F}_{T,T-1}$

v. Form a reduced form equation from the variables that were transformed in the previous step by:

- Calculating the matrix $\mathbf{S}_{nT}(\delta) = \mathbf{I}_n - \sum_{j=1}^{p}\delta_j\mathbf{W}_{nj}$,

  $\mathbf{S}_n \equiv \mathbf{S}_n(\delta_0)$ with $\boldsymbol{\delta} = \left(\delta_1,\delta_2\right)'; \boldsymbol{\eta} = \left(\eta_1,\eta_2\right)'$
- Calculating the matrix
  $$\mathbf{A}_n = \mathbf{S}_n^{-1}\left(\tau_0\mathbf{I}_n + \sum_{j=1}^{p}\eta_{j0}\mathbf{W}_{nj}\right)$$
- Create a reduced form matrix from the equations (12) as:
  $$\mathbf{Y}_{N,t}^* = \mathbf{A}_n\mathbf{Y}_{\mathrm{N},t-1}^{(*,-1)} + \mathbf{S}_n^{-1}(\mathbf{X}_{\mathrm{N},t}^*\boldsymbol{\beta}_0 + \boldsymbol{V}_{N,t}^*) \qquad (12)$$

vi. Estimate the reduced from equation (14) with OLS estimation methods to get $\hat{\mathbf{Y}}_{N,t}^*, \hat{\mathbf{Y}}_{\mathrm{N},t-1}^{(*,-1)}$ as initial values.

vii. Forms the moment function vector $\boldsymbol{g}_{nT}(\theta)$ on model (12) with $\hat{\mathbf{Y}}_{N,t}^*, \hat{\mathbf{Y}}_{\mathrm{N},t-1}^{(*,-1)}$ in the model with the following manner:

- Form the matrix $\mathbf{J}_{n,T-1}$
- Form the matrix $\mathbf{P}_{nm,T-1}^s$, *m* =1,2
- Form vector $\boldsymbol{Q}_{n,T-1}$
- Form vector $\boldsymbol{g}_{nT}(\theta)$

viii. Determine the LM dependency testing hypothesis for:

- Test the existence of spatial contemporaneous lag with hypotheses: H_0: $\delta_0 = 0$ and in the presence of η_0 and τ_0
- Test spatial lag in t-1 with hypotheses: H_0: $\eta_0 = 0$ and in the presence of δ_0 and τ_0
- Test the existence of time lag with hypotheses: H_0: $\tau_0 = 0$ and in the presence of η_0 and δ_0

ix. Perform an LM testing for each hypothesis with the following steps:

- counting vectors $\boldsymbol{G}\delta\ (\theta)$, $\boldsymbol{G}\tau\ (\theta)$, $\boldsymbol{G}\eta\ (\theta)$
- form matrix $\boldsymbol{\Sigma}_{nT}^{-1}$
- count $\bar{\boldsymbol{g}}_{nT} = \frac{1}{n}\boldsymbol{g}_{nT}(\theta)$
- counting values $C\delta\ (\theta)$, $C\tau(\theta)$ and $C\eta\ (\theta)$
- counting values $B\delta\ (\theta)$, $B\tau(\theta)$,and $B\eta\ (\theta)$
- counting values $B\delta\beta\ (\theta\ )$, $B\eta\beta\ (\theta\ )$, $B\tau\beta\ (\theta\ )$
- counting values $C\delta\ (\theta\ nT\ )$, $C\eta\ (\theta\ nT\ )$, $C\tau(\theta\ nT\ )$
- for each of the above hypotheses, the LM value is calculated, reject Ho if LM $>\chi_p^2(\vartheta)$

x. Perform steps (ii) to (ix) for equation (13).

4. RESULT AND DISCUSSION

Spatial dependency testing in this sub-chapter applies [13] steps on SSDPD models where LM test with GMM estimation used has advantages over using other LM test methods whose asymptotic distribution tends not to be focused on Chi-Square distribution if applied to original data generated resulting in excessive rejection of zero hypothesis.

The LM GMM method from [13] can be used to test dependencies on:

- spatial shape lag
- time lag form
- spatial form of time lag

The discussion in this sub-chapter will be divided into two parts, namely in the first part discussing the GMM Estimation on a single model SSDPD equation and in the second part about LM GMM test on a single model of SSDPD equation.

4.1 GMM Estimation on a Single Model of SSDPD Equation

GMM estimation on a single model of SSDPD equation is done on each model (12) and (13). Furthermore, the equation was formed into a model of High Order Spatial Autoregressive as formed by [9] as follows:

$$\boldsymbol{Y}_{N,t} = \sum_{j=1}^{P}\delta_{j0}\mathbf{W}_{Nj}\boldsymbol{Y}_{N,t} + \sum_{j=1}^{P}\eta_{j0}\mathbf{W}_{Nj}\boldsymbol{Y}_{N,t\text{-}1} + \mathbf{Z}_{N,t}\boldsymbol{\beta}_0$$
$$+\tau_0\boldsymbol{Y}_{N,t\text{-}1} + \boldsymbol{C}_{N0} + \boldsymbol{V}_{N,t} \quad (13)$$

$Y_{N,t} = \left(y_{11,t}, y_{21,t}, ..., y_{n1,t}, y_{12,t}, y_{22,t}, ..., y_{n2,t}\right)'$ or can be form to $\boldsymbol{Y}_{N,t} = \left(Y_{11,t}, Y_{21,t}, ..., Y_{N,t}\right)'$ is N x 1 vector endogenous variable and $\mathbf{Z}_{N,t}$ is an exogenous variable matrix of N x K size where $\mathbf{Z}_{n1,t} = \left(\mathbf{X}_{n1,t}, \boldsymbol{y}_{n1,t}\right)$ and $\mathbf{Z}_{n2,t} = \left(\mathbf{X}_{n2,t}, \boldsymbol{y}_{n2,t}\right)$, $\mathbf{Z}_{\mathrm{N},t} = \left(\mathbf{Z}_{n1,t}, \mathbf{Z}_{n2,t}\right)$

$\boldsymbol{V}_{N,t} = (v_{11,t}, v_{21,t}, ..., v_{n1,t}, v_{12,t}, v_{22,t}, ..., v_{n2,t})'$ is a Nx1 sized vector of disturbance term where $v_{it} \sim N(0, \sigma_0^2)$. While $\mathbf{W}_{nj}\boldsymbol{Y}_{n,t}$ and $\mathbf{W}_{nj}\boldsymbol{Y}_{n,t-1}$ are the spatial lag dependent variabels, where $\mathbf{W}_{nj}$ is a symmetrical spatial weight matrix of N x N which are non stochastic and are formed based on y_{it} between spatial units by $j = 1,2,...,p$. Matrix $\mathbf{W}_{nj}$ can be a normalized matrix of rows or not.

$\delta_{j0} = (\delta_{10}, \delta_{20}, ..., \delta_{p0})'$, $\eta_{j0} = (\eta_{10}, \eta_{20}, ..., \eta_{p0})'$ are the spatial parameter autoregressive, while $C_{j0} = (c_{10}, c_{20}, ..., c_{p0})'$ is an individual fixed effect vector.

If $p \geq 2$ then the model (12) is a high order SAR model, to avoid indexical variables then individual effect variables $\boldsymbol{C}_{N0}$ have to eliminate in way by transforming it with a matrix $\mathbf{F}_{T,T\text{-}1}$ obtained from the matrix $[\mathbf{F}_{T,T\text{-}1}, \frac{1}{\sqrt{T}}\boldsymbol{\iota}_T]$ which is the orthonormal matrix of the Eigen vector matrix $\mathbf{J}_T = (\mathbf{I}_T - \frac{1}{T}\boldsymbol{\iota}_T\boldsymbol{\iota}_T')$. $\mathbf{F}_{T,T\text{-}1}$ is an Eigen matrix of vector-sized eigenvalues of one and $\boldsymbol{l}_T$ is a single vector with dimensions related to the eigenvalues of one and T. Variable dependent matrix $[Y_{n1}, Y_{n2}, ..., Y_{nt}]$ can be transformed into a matrix $Nx(T-1)$, so that it becomes:

$[Y_{n1}^*, Y_{n2}^*, ..., Y_{nt\text{-}1}^*] = [Y_{n1}, Y_{n2}, ..., Y_{nt}]\mathbf{F}_{T,T\text{-}1}$ and,

$[Y_{n1}^{(*,-1)}, Y_{n2}^{(*,-1)}, ..., Y_{n,t\text{-}1}^{(*,-1)}] = [Y_{n0}, Y_{n1}, ..., Y_{n,t\text{-}1}]\mathbf{F}_{T,T\text{-}1}$

$\boldsymbol{Y}_{nt\text{-}1}^* \neq \boldsymbol{Y}_{n,t\text{-}1}^{(*,-1)}$

Multiplication with $\mathbf{F}_{T,T\text{-}1}$ also applies to variables v_{nt}^* and $\mathbf{Z}_{nt}^*$, because $\boldsymbol{l}_T'\mathbf{F}_{T,T\text{-}1} = 0$ then $[c_{n0}, ..., c_{n0}]\mathbf{F}_{T,T\text{-}1} = 0$ so that individual effects can be eliminated by the orthonormal matrix transformation. After the transformation process, equation or model (12) will become,

$$\boldsymbol{Y}_{N,t}^* = \sum_{j=1}^{P}\delta_{j0}\mathbf{W}_{Nj}\boldsymbol{Y}_{N,t}^* + \sum_{j=1}^{P}\eta_{j0}\mathbf{W}_{Nj}\boldsymbol{Y}_{N,t\text{-}1}^{(*,-1)} + \mathbf{Z}_{N,t}^*\boldsymbol{\beta}_0$$
$$+\tau_0\boldsymbol{Y}_{N,t\text{-}1}^{(*,-1)} + \boldsymbol{V}_{N,t}^*; t = 1, 2, \cdots .T-1 \quad (14)$$

with, $\boldsymbol{V}_{N,t}^*; t = \left(\frac{T-t}{T-t+1}\right)^{\frac{1}{2}}\left[\boldsymbol{V}_{n,t} - \frac{1}{T\text{-}t}\sum_{h=t+1}^{T}\boldsymbol{V}_{nh}\right]$ and

$\boldsymbol{Y}_{n,t\text{-}1}^{(*,-1)} = \left(\frac{T-t}{T-t+1}\right)^{1/2}\left[\mathbf{Y}_{n,t\text{-}1} - \frac{1}{T\text{-}t}\sum_{h=t}^{T-1}\mathbf{Y}_{nh}\right]$

Time effect elimination is not done on this model because the model used is a fixed effect model so there is no time effect variable on this model.

Estimation using the MLE method for the model (12) cannot be done according to [22] on the grounds that:

- If $\mathbf{W}_{nj}$ cannot be rows normalized then SAR structure cannot be defined
- The existence of variables with time lag as explanatory variables, those variables will correlate with the form of errors if transformed by $\mathbf{F}_{T,T\text{-}1}$.

Based on this reason, [22] used the GMM method to estimate the model (12), because the method did not ask for a SAR form for $\mathbf{J}_n\boldsymbol{y}_{nt}^*$ and free from asymptotic forms of bias. To be able to estimate the parameters of the model (12) is required reduced form of the equation (16) the following:

$$\boldsymbol{Y}_{nt}^* = \mathbf{A}_n\boldsymbol{Y}_{nt}^{(*,-1)} + \mathbf{S}_n^{-1}(\mathbf{Z}_{nt}^*\boldsymbol{\beta}_0 + \boldsymbol{V}_{nt}^*) \quad (15)$$

with, $\mathbf{S}_n(\delta) = \mathbf{I}_n - \sum_{j=1}^{P}\delta_j\mathbf{W}_{nj}, \mathbf{S}_n \equiv \mathbf{S}_n(\delta_0)\mathbf{A}_n$

$$= \mathbf{S}_n^{-1}\left(\tau_0\mathbf{I}_n + \sum_{j=1}^{\mathrm{p}}\eta_j\mathbf{W}_{nj}\right)$$

For any spatial lag $\mathbf{W}_{nj}\boldsymbol{y}_{nt}^*$ for $j = 1,2,...,p$ by defining $\mathbf{G}_{nj} = \mathbf{W}_{nj}\mathbf{S}_n^{-1}$, then obtained,

$$\mathbf{W}_{nj}\boldsymbol{Y}_n^* = \mathbf{G}_{nj}(\mathbf{R}_{nt}^*\boldsymbol{\lambda}_0) + \mathbf{G}_{nj}\boldsymbol{V}_{nt}^*, \quad (16)$$

with $\lambda_0 = (\tau_0, \eta_0, \beta_0)'$ and $\mathbf{R}^*_{nt} = \left[\boldsymbol{Y}^{(*,-1)}_{n,t\text{-}1}, \mathbf{W}_n\boldsymbol{Y}^{(*,-1)}_{n,t\text{-}1}, \boldsymbol{Z}^*_{nt}\right]$ is the predetermined variable in equation (18) with $\mathbf{W}_n\boldsymbol{Y}^{(*,-1)}_{n,t\text{-}1} = \left(\mathbf{W}_{n1}\boldsymbol{Y}^{(*,-1)}_{n,t\text{-}1}, \cdots, \mathbf{W}_{np}\boldsymbol{Y}^{(*,-1)}_{n,t\text{-}1}\right)$.

For a linear moment matrix, data is compiled and formed moment conditions. An Instrumental Variable (IV) matrix takes shape $\mathbf{J}_n\boldsymbol{Q}_{nt}$ where $\boldsymbol{J}_n = \mathbf{I}_n - \frac{1}{n}\boldsymbol{l}_n\boldsymbol{l}'_n$ because $\boldsymbol{J}_n\boldsymbol{l}_n = 0$ and $\mathbf{Q}_{nt}$ has a fixed dimension column q is greater than or equal to $k_x + 2_p + 1.Q_{nt}$ selected are as follows: $\boldsymbol{Q}_{nt} = [\boldsymbol{y}_{n,t\text{-}1}, \mathbf{W}_n\boldsymbol{y}_{n,t\text{-}1}, \mathbf{W}^2_n y_{n,t\text{-}1}, \boldsymbol{Z}^*_{nt}, \mathbf{W}_n\boldsymbol{Z}^*_{nt}$

$$, \mathbf{W}^2_n\boldsymbol{Z}^*_{nt}] \quad (17)$$

To estimate equation (16) it should be noted that between $\boldsymbol{Y}^{(*,-1)}_{n,t\text{-}1}$ and $\boldsymbol{V}^*_{nt}$ correlated, for that it is necessary to IV for $\boldsymbol{Y}^{(*,-1)}_{n,t\text{-}1}$, $\mathbf{W}_{nk}\boldsymbol{Y}^{(*,-1)}_{n,t\text{-}1}$ and $\mathbf{W}_{nj}\boldsymbol{Y}^*_{n,t}$ at each t. Therefore, IV is needed for the explanatory variables as follows:

$$\mathbf{J}_n[\mathbf{W}_n, \boldsymbol{Y}^*_{nt}, \boldsymbol{Y}^{(*,-1)}_{n,t-1}, \mathbf{W}_n\boldsymbol{Y}^{(*,-1)}_{n,t-1}] \quad (18)$$

Take $\boldsymbol{V}^*_{n,T-1}(\theta) = (V^{*'}_{n1}(\theta), \cdots, V^{*'}_{n,T-1}(\theta))'$ where $\boldsymbol{V}^*_{nt}(\theta) = S_{nt}(\lambda)\boldsymbol{Y}^*_{nt} - \boldsymbol{R}^*_{nt}\delta$ with $\boldsymbol{\theta} = (\delta_0, \lambda'_0)'$ and $\lambda_0 = (\tau_0, \eta'_0, \beta'_0)'$. The IV estimate with respect to the linear moment is, $\mathbf{Q}'_{n,T-1}\mathbf{J}_{n,T-1}\boldsymbol{V}^*_{n,T-1}(\theta)$ with $\mathbf{Q}'_{n,T-1} = (\mathbf{Q}'_{n1}, \ldots, \mathbf{Q}'_{n,T-1})'$ $\mathbf{J}_{n,T-1} = \mathbf{I}_{T-1} \otimes \mathbf{J}_n$

Vector $\mathbf{P}_{nt}\mathbf{V}^*_{nt}$ cannot be correlated with $\mathbf{J}_n\mathbf{V}^*_{nt}$ in equation (16), for no stochastic matrices $\mathbf{P}_{nl}$ sized *n* x *n* according to the contents of $tr(\mathbf{P}_{nl}\mathbf{J}_n) = 0$, here the value $\mathbf{P}_{n1} = \mathbf{W}_n - tr(\mathbf{W}_n\mathbf{J}_n)/(\mathbf{W}_n\mathbf{J}_n)/(n-1)$ and $\mathbf{P}_{n2} = \mathbf{W}^2_n - tr(\mathbf{W}^2_n\mathbf{J}_n)/(\mathbf{W}_n\mathbf{J}_n)/(n-1)$ while vector $\mathbf{P}_{nt}\mathbf{V}^*_{nt}$ may be correlated with $\mathbf{G}_{nj}\mathbf{V}^*_{nt}$ on the equation (17). Defined $\mathbf{P}_{nl,T-1} = \mathbf{I}_{T-1} \otimes \mathbf{P}_{nl}$ then the quadratic moment is,

$$\mathbf{V}^*_{n,T-1}(\theta)\mathbf{J}_{n,T-1}\mathbf{P}_{nl,T-1}\mathbf{J}_{n,T-1}\mathbf{V}^*_{n,T-1}(\theta), l = 1, 2, \cdots, m$$

and the moment of condition with the approximate number of finite moments is,

$$\boldsymbol{g}_{nt}(\theta) = \begin{pmatrix} \mathbf{V}^{*'}_{n,T\text{-}1}(\theta)\mathbf{J}_{n,T\text{-}1}\mathbf{P}_{n1,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{V}^{*'}_{n,T\text{-}1}(\theta) \\ \vdots \\ \mathbf{V}^{*'}_{n,T\text{-}1}(\theta)\mathbf{J}_{n,T\text{-}1}\mathbf{P}_{nm,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{V}^{*'}_{n,T\text{-}1}(\theta) \\ \mathbf{Q}^{*'}_{n,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{V}^{*'}_{n,T\text{-}1}(\theta) \end{pmatrix} \quad (19)$$

For GMM estimation, a covariance matrix of the moment function is needed $E\left(g'_{nt}(\theta_0)g_{nt}(\theta)\right)$ who can be approached with,

$$\sum_{nt} = \begin{pmatrix} \frac{1}{n(T-1)}\Delta_{\text{nm,T}} & \mathbf{0}_{qxm} \\ \mathbf{0}_{qxm} & \frac{1}{\sigma^2_0}\frac{1}{n(T-1)}\boldsymbol{Q}_{n,T-1}\mathbf{J}_{n,T-1}\boldsymbol{Q}_{n,T-1} \end{pmatrix} + \frac{1}{n(T-1)}\begin{pmatrix} \left(\mu_4 - 3\sigma^4_0\right)\omega'_{nm,T}\omega_{nm,T} & \mathbf{0}_{mxq} \\ \mathbf{0}_{qxm} & \mathbf{0}_{qxq} \end{pmatrix} \quad (20)$$

with,

$\omega_{nm,T} = [vec_D(\mathbf{J}_{n,T\text{-}1}\mathbf{P}_{n1,T\text{-}1}\mathbf{J}_{n,T\text{-}1}), \ldots, vec_D(\mathbf{J}_{n,T\text{-}1}\mathbf{P}_{nm,T\text{-}1}\mathbf{J}_{n,T\text{-}1})]$,

$\Delta_{\text{nm,T}} = [vec_D(\mathbf{J}_{n,T\text{-}1}\mathbf{P}'_{n1,T\text{-}1}\mathbf{J}_{n,T\text{-}1}), \ldots, vec_D(\mathbf{J}_{n,T\text{-}1}\mathbf{P}'_{nm,T\text{-}1}\mathbf{J}_{n,T\text{-}1})]x$

$[vec_D(\mathbf{J}_{n,T\text{-}1}\mathbf{P}^s_{n1,T\text{-}1}\mathbf{J}_{n,T\text{-}1}), \ldots, vec_D(\mathbf{J}_{n,T\text{-}1}\mathbf{P}^s_{nm,T\text{-}1}\mathbf{J}_{n,T\text{-}1})]$

Operator $vec_D(.)$ is a column vector formed from the diagonal element of the rectangular matrix inputted, and a matrix $\mathbf{A}^s_n = \mathbf{A}_n + \mathbf{A}'_n$ for all square matrices. The optimal estimate from the GMM is derived from

$$\boldsymbol{\theta}_{o,nT} = argmin_{\theta\in\Theta}\boldsymbol{g}'_{nt}(\theta)\,\Sigma^{-1}_{\text{nT}}\boldsymbol{g}_{\text{nt}}(\theta) \quad (21)$$

has an asymptotic distribution,

$$\sqrt{n}(\theta_{0,nT} - \theta) \xrightarrow{d} N(\mathbf{0}, plim_{n\to\infty}\tfrac{1}{T-1}(\mathbf{D}'_{nT}\Sigma^{-1}_{nT}\mathbf{D}_{nT})^{-1} \quad (22)$$

4.2 Lagrange Multiplier (LM) Spatial Dependencies Test for SDPD model

In [22] it is also stated that the resulting GMM estimator is consistent at $\sqrt{nT}$ asymptotically normal, as well as efficient. The use of Instrumental Variable (IV) aims to get the best linear and quadratic moment conditions, in which the number of IVs will increase over time.

Based on [13] LM testing for spatial dependencies can be carried out in the High order model, and by using the moment function, variable instrumental matrix and covariance matrix obtained in the previous step, followed by calculating vectors $\boldsymbol{G}_a(\theta) = (\boldsymbol{G}_\delta(\theta), \boldsymbol{G}_\tau(\theta), \boldsymbol{G}_\eta(\theta))$ in accordance with the null hypothesis to be tested.

Take $\boldsymbol{V}^*_{n,T-1} = (\boldsymbol{V}^{*'}_{n1}, \ldots, \boldsymbol{V}^{*'}_{n,T-1})'$ with $\boldsymbol{V}^*_{nt}(\theta) = S_{nt}(\lambda)\boldsymbol{Y}^{*'}_{nt} - \boldsymbol{R}^*_{nt}\delta$, $\mathbf{Y}^*_{nT\text{-}1} = (Y^{*'}_{n1}, \ldots, Y^{*'}_{n,T\text{-}1})'$, $\mathbf{Y}^{*,\text{-}1}_{n,T\text{-}1} = (Y^{*'}_{n0}, \ldots, Y^{*'}_{n,T\text{-}2})'$, $\mathbf{Z}^*_{nT\text{-}1} = (Z^{*'}_{n1}, \ldots, Z^{*'}_{n,T\text{-}1})'$, and $\mathbf{W}_{nj,T-1} = \mathbf{I}_{T-1} \otimes \mathbf{W}_{nj}$ then $\boldsymbol{G}_a(\theta)$ for each null hypothesis is defined as follows:

$$G_{\delta}(\theta)=\frac{1}{N}\begin{pmatrix} Y^{*'}_{n,T\text{-}1}\mathbf{W}'_{nj,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{P}^{s}_{n1,T\text{-}1}\mathbf{J}_{n,T\text{-}1}V^{*'}_{n,T\text{-}1}(\theta) \\ Y^{*'}_{n,T\text{-}1}\mathbf{W}'_{nj,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{P}^{s}_{n2,T\text{-}1}\mathbf{J}_{n,T\text{-}1}V^{*'}_{n,T\text{-}1}(\theta) \\ \vdots \\ Y^{*'}_{n,T\text{-}1}\mathbf{W}'_{nj,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{P}^{s}_{nm,T\text{-}1}\mathbf{J}_{n,T\text{-}1}V^{*'}_{n,T\text{-}1}(\theta) \\ Q'_{n,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{W}_{nj,T\text{-}1}Y^{*'}_{n,T\text{-}1} \end{pmatrix} \quad (23)$$

$$G_{\delta}(\theta)=\frac{1}{N}\begin{pmatrix} V^{*'}_{n,T\text{-}1}(\theta)\mathbf{J}_{n,T\text{-}1}\mathbf{P}^{s}_{n1,T\text{-}1}\mathbf{J}_{n,T\text{-}1}Y^{(*,-1)}_{n,T\text{-}1} \\ V^{*'}_{n,T\text{-}1}(\theta)\mathbf{J}_{n,T\text{-}1}\mathbf{P}^{s}_{n2,T\text{-}1}\mathbf{J}_{n,T\text{-}1}Y^{(*,-1)}_{n,T\text{-}1} \\ \vdots \\ V^{*'}_{n,T\text{-}1}(\theta)\mathbf{J}_{n,T\text{-}1}\mathbf{P}^{s}_{nm,T\text{-}1}\mathbf{J}_{n,T\text{-}1}Y^{(*,-1)}_{n,T\text{-}1} \\ Q'_{n,T\text{-}1}\mathbf{J}_{n,T\text{-}1}Y^{(*,-1)}_{n,T\text{-}1} \end{pmatrix} \quad (24)$$

$$G_{\delta}(\theta)=\frac{1}{N}\begin{pmatrix} V^{*'}_{n,T\text{-}1}(\theta)Y^{*'}_{n,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{P}^{s}_{n1,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{W}'_{nj,T\text{-}1}Y^{(*,-1)}_{n,T\text{-}1} \\ V^{*'}_{n,T\text{-}1}(\theta)Y^{*'}_{n,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{P}^{s}_{n2,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{W}'_{nj,T\text{-}1}Y^{(*,-1)}_{n,T\text{-}1} \\ \vdots \\ V^{*'}_{n,T\text{-}1}(\theta)Y^{*'}_{n,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{P}^{s}_{nm,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{W}'_{nj,T\text{-}1}Y^{(*,-1)}_{n,T\text{-}1} \\ Q'_{n,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{W}'_{nj,T\text{-}1}Y^{(*,-1)}_{n,T\text{-}1} \end{pmatrix} \quad (27)$$

$$G_{\delta}(\theta)=\frac{1}{N}\begin{pmatrix} V^{*'}_{n,T\text{-}1}(\theta)\mathbf{J}_{n,T\text{-}1}\mathbf{P}^{s}_{n1,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{Z}^{*}_{n,T\text{-}1} \\ V^{*'}_{n,T\text{-}1}(\theta)\mathbf{J}_{n,T\text{-}1}\mathbf{P}^{s}_{n2,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{Z}^{*}_{n,T\text{-}1} \\ \vdots \\ V^{*'}_{n,T\text{-}1}(\theta)\mathbf{J}_{n,T\text{-}1}\mathbf{P}^{s}_{nm,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{Z}^{*}_{n,T\text{-}1} \\ Q'_{n,T\text{-}1}\mathbf{J}_{n,T\text{-}1}\mathbf{Z}^{*}_{n,T\text{-}1} \end{pmatrix} \quad (25)$$

Then look for the following components:

$$C_{\delta}(\theta)=G'_{\delta}(\theta)\hat{\Sigma}^{-1}_{nT}\,\bar{g}_{nT}(\theta) \quad (26)$$

$$C_{\tau}(\theta)=G'_{\tau}(\theta)\hat{\Sigma}^{-1}_{nT}\,\bar{g}_{nT}(\theta) \quad (27)$$

$$C_{\eta}(\theta)=G'_{\eta}(\theta)\hat{\Sigma}^{-1}_{nT}\,\bar{g}_{nT}(\theta) \quad (28)$$

and components $B(\theta)$

$$\begin{aligned} B_{\delta}(\theta)&=(\theta)=G'_{\delta}(\theta)\hat{\Sigma}^{-1}_{nT}G_{\delta}(\theta),B_{\delta\eta}(\theta)\\ &=B'_{\delta\eta}(\theta)=G'_{\delta}(\theta)\hat{\Sigma}^{-1}_{nT}G_{\eta}(\theta) \end{aligned} \quad (29)$$

$$\begin{aligned} B_{\delta\tau}(\theta)&=B_{\delta\tau}(\theta)=G'_{\delta}(\theta)\hat{\Sigma}^{-1}_{nT}G_{\tau}(\theta),B_{\delta\beta}(\theta)\\ &=B'_{\beta\delta}(\theta)=G'_{\delta}(\theta)\hat{\Sigma}^{-1}_{nT}G_{\beta}(\theta) \end{aligned} \quad (30)$$

$$B_{\eta}(\theta)G'_{\eta}(\theta)\hat{\Sigma}^{-1}_{nT}G_{\eta}(\theta),$$

$$B_{\eta\tau}(\theta)=B'_{\eta\tau}(\theta)=G'_{\eta}(\theta)\hat{\Sigma}^{-1}_{nT}G_{\tau}(\theta) \quad (31)$$

$$\begin{aligned} &B_{\eta\beta}(\theta)=B_{\beta\eta}(\theta)=G'_{\eta}(\theta)\hat{\Sigma}^{-1}_{nT}G_{\beta}(\theta),\\ &B_{\tau}(\theta)=G'_{\tau}(\theta)\hat{\Sigma}^{-1}_{nT}G_{\tau}(\theta) \end{aligned} \quad (32)$$

$$\begin{aligned} &B_{\tau\beta}(\theta)=B_{\beta\tau}(\theta)=G'_{\tau}(\theta)\hat{\Sigma}^{-1}_{nT}G_{\beta}(\theta),\\ &B_{\beta}(\theta)=G'_{\beta}(\theta)\hat{\Sigma}^{-1}_{nT}G_{\beta}(\theta) \end{aligned} \quad (33)$$

So that the LM value for each hypothesis is obtained as follows:

a. For $\mathrm{H}_0:\delta_0=0$

$$LM_{\delta}=(\tilde{\theta}_{nT})=NC'_{\delta}(\tilde{\theta}_{nT})[B_{\delta\beta}(\tilde{\theta}_{nT}]^{-1}C_{\delta}(\tilde{\theta}_{nT}), \quad (37)$$

b. For $\mathrm{H}_0:\eta_0=0$

$$LM_{\eta}=(\tilde{\theta}_{nT})=NC'_{\eta}(\tilde{\theta}_{nT})[B_{\eta\beta}(\tilde{\theta}_{nT}]^{-1}C_{\eta}(\tilde{\theta}_{nT}), \quad (34)$$

c. For $\mathrm{H}_0:\tau_0=0$

$$LM_{\tau}=(\tilde{\theta}_{nT})=NC'_{\tau}(\tilde{\theta}_{nT})[B_{\tau\beta}(\tilde{\theta}_{nT}]^{-1}C_{\tau}(\tilde{\theta}_{nT}), \quad (35)$$

Reject H_0 if $\mathrm{LM}>\chi^2_p$

The LM test method that has been done above is also applied to equation (17), where the initial values $\boldsymbol{v}_{N,T}$, $\boldsymbol{y}_{n,T}$ and $\boldsymbol{y}_{N,\mathrm{T}-1}$ are obtained from the OLS equation from the reduced form equation.

5. CONCLUSION

Lagrange Multiplier testing using the GMM estimation approach on the SSDPD model, can be done on each single model, where the initial values of the vectors $\boldsymbol{v}_{N,T}$, $\boldsymbol{y}_{n,T}$ and $\boldsymbol{y}_{N,\mathrm{T}-1}$ are searched using the OLS value in the reduced form equation, then proceed to look for the estimated parameter value using GMM.

This paper is still in the stage of refinement which will be proven in the form of data simulations and applications to secondary data.

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