Advances in Social Science, Education and Humanities Research, volume 550
Proceedings of the 1st International Conference on Mathematics and Mathematics Education (ICMMED 2020)

Ordered Left Almost Hyperring

A M Muftirridha[1], A R Alghofari[2], and N Hidayat[2]

[1] *Student of Master Mathematics Study Program, Brawijaya University*
[2] *Department of Mathematics, Brawijaya University*
Email: ammufti@yahoo.com

ABSTRACT

In this paper, we introduce the concept of left almost hyperring $(R,\oplus,\otimes)$ together with a partial relation order $\leq$ such that satisfies some conditions. This structure $(R,\oplus,\otimes,\leq)$ is called by ordered left almsot hyperring (LA-hyperring). Further, we study some useful contiditons for ordered LA-hyperring to become an ordered hyperring. Also, we notice the notion of hyperideal, bi-hyperideal, and quasi-hyperideal of ordered LA-hyperring and their properties are investigated.

Keywords: *Ordered Hyperstructure, Ordered LA-hyperring, Hyperideal, LA-hyperring.*

1. INTRODUCTION

The algebraic hyperstructure which is a generalization theory of ordinary algebraic structures was first introduced in 1934 by a French mathematician, Marty [1]. In ordinary algebraic structure, the composition of two elements is an element, while in algebraic hyperstructure the composition of two elements is a set. Many mathematicians studied hyperstucture theory because hyperstructures have a lot of applications to many subjects of mathematics and computer science. Corsini and Leoreanu [2] discussed some applications of hyperstructure theory to geometry, hypergraphs, lattices, fuzzy sets, automata, cryptography, codes, artificial intelligence and probabilities.

In 1972, Kazim and Naseerudin [3] introduced the notion of left almost semigroup (LA-group) as a non associative group. A groupoid $(G,+)$ is said to be an LA-semigroup if satisfy $(a+b)+c=(c+b)+a$, for all $a,b,c\in G$. It is known as invertive law. Kamran [4] discussed some properties of LA-group, substructures of LA-group and the quotient structures. Later, the concept of LA-ring was introduced by Yusuf [5]. Basically, LA-ring correspond to ring. An algebraic structures $(R,+,\cdot)$ is a non-empty set R with the binary operations "$+$" and "$\cdot$" such that $(R,+)$ is an LA-group, $(R,\cdot)$ is an LA-semigroup, both left and right distributive laws hold. There are several authors who studied LA-ring and explored some useful properties of LA-ring. Shah and Rehman [6] studied LA-ring of finitely non-zero functions which is generalize the structure of commutative semigroup ring. Hussain and Firdous [7] characterized LA-ring by the properties of their direct product.

Recently, the concept of hyperstructure was applied to LA-semigroups. It was introduced by Hila and Dine [8] as a generalization of semihypergroups and LA-semigroup. Yaqoob and Gulistan [9] introduced the notion of partially ordered LA-semihypergroup. Yaqoob et al. [10] studied intra-regular LA-semihypergroup and characterized it by using their hyperideal properties. Further, Rehman et al. [11] extended the work of Hila and Dine to algebraic hyperstructure which has two hyperoperation, that is LA-hyperring. They also characterized LA-hyperrings throught their hyperideals and hypersystems.

In this paper, we introduced the notion of ordered LA-hyperring. We established some elementary properties of ordered LA-hyperring and studied some useful conditions for ordered LA-hyperring to become an ordered hyperring. Also, we introduced some type of hyperideal of ordered LA-hyperring.

2. PRELIMINARIES

In this section, we recall some definitions and notions of an LA-semihypergroup, an ordered LA-

Copyright © 2021 The Authors. Published by Atlantis Press SARL.
This is an open access article distributed under the CC BY-NC 4.0 license -http://creativecommons.org/licenses/by-nc/4.0/.

semihypergroup, an LA-hyperring and some properties that we will use in next section.

Let H be a non-empty set, then the map $\oplus: H \times H \to P^*(H)$ is called hyperoperation of H, where $P^*(H)$ denotes the set of all non-empty power set of H. A set H with a hyperoperation $\oplus$ is said to be a hypergroupoid, denoted by $(H,\oplus)$. If A and B are two subsets of H, the we denote

$$A \oplus B = \bigcup_{a \in A, b \in B} a \oplus b,$$

$$a \oplus B = \{a\} \oplus B, \qquad A \oplus b = A \oplus \{b\}.$$

[8] A hypergroupoid $(H,\oplus)$ is said to be an LA-hypersemigroup if satisfies left invertive law. If $(H,\oplus)$ satisfies reproduction law, that is $H \oplus a = H = a \oplus H$, for every $a \in H$, then $(H,\oplus)$ is called by LA-hypergroup. An element $e \in H$ is called by a left identity element (resp. pure left identity element) if $a \in a \oplus e$ $(a = a \oplus e)$. In an LA-semihypergroup, the medial law holds, $(a \oplus b) \oplus (c \oplus d) = (a \oplus c) \oplus (b \oplus d)$ for all $a, b, c, d \in H$. An LA-semihypergroup may not contain a left identity element or a pure left identity. In an LA-semihypergroup with pure left identity, the paramedial law $(a \oplus b) \oplus (c \oplus d) = (d \oplus c) \oplus (b \oplus a)$ holds for all $a, b, c, d \in H$.

Definition 2.1 [9] An ordered LA-semihypergroup $(H,\oplus,\leq)$ is a poset $(H,\leq)$ at the same time an LA-semihyepergroup $(H,\oplus)$ such that $a, b, x \in H, a \leq b$ implies $a \oplus x \leq b \oplus x$ and $x \oplus a \leq x \oplus b$ for any $a, b, x \in H$. A non-empty subset A of H is called by LA-subsemihypergroup of an ordered LA-subsemihypergroup $(H,\oplus,\leq)$ if $(A \oplus A] \subseteq (A]$.

If A and B are non-empty subsets of H, then we denote $A \leq B$ if for every $a \in B$ there exist $b \in B$ such that $a \leq b$.

Definition 2.2 [9] A non-empty subset A of an ordered LA-subsemihypergroup $(H,\oplus,\leq)$ is called left (resp. right) hyperideal of $(H,\oplus,\leq)$ if the following conditions hold:

- $H \oplus A \subseteq A$ (resp. $A \oplus H \subseteq A$).
- If $a \in A$ and $b \leq a$, then $b \in A$.

A is called by hyperideal of $(H,\oplus,\leq)$ if it is a left and right hyperideal.

Definition 2.3 [9] An LA-subsemihypergroup B of an ordered LA-semihypergroup $(H,\oplus,\leq)$ is called a bi-hyperideal of $(H,\oplus,\leq)$ if the following conditions hold:

- $(B \oplus H) \oplus B \subseteq B$.
- $If a \in B$ and $b \leq a$, then $b \in B$.

Definition 2.4 [9] A non-empty subset Q of an LA-semihyperring $(H,\oplus,\leq)$ is called by quasi-hyperideal of $(H,\oplus,\leq)$ if the following conditions hold:

- $Q \oplus H \cap H \oplus Q \subseteq Q$.
- $If a \in Q$ and $b \leq a$, then $b \in Q$.

Definition 2.5 [11] A hypergroupoid $(R,\oplus,\otimes,\leq)$ is said to be an LA-hyperring if satisfies the following conditions:

- $(R,\oplus)$ is an LA-hypergroup.
- $(R,\otimes)$ is an LA-hypersemigroup.
- The hyperoperation $\otimes$ is distributive with respect to the hyperoperation $\oplus$.

Definition 2.6 [11] An LA-subhypergroup S of an ordered LA-hyperring $(R,\oplus,\otimes,\leq)$ is said to be a left (resp. right) hyperideal of R if $R \otimes S \subseteq S$ $(S \otimes R \subseteq S)$. If S is a left and right hyperideal, then S is called by hyperideal.

Proposition 2.7 If $(R,\oplus,\otimes)$ be an LA-hyperring with left identity (pure left identity), then every right hyperideal is a left hyperideal.

3. ORDERED LA-HYPERRING

In this section, we introduce the concept of ordered LA-hyperring and give some examples of this hyperstructure. We also prove the elementary properties of ordered LA-hyperring and study some useful contiditons for ordered LA-hyperring to become an ordered hyperring.

Definition 3.1 A hypergroupiod $(R,\oplus,\otimes,\leq)$ is said to be a ordered LA-hyperring if satistfies the following conditions:

- $(R,\oplus,\otimes)$ is an LA-hyperring.
- If $a \leq b$, then $a \oplus c \leq b \oplus c$ and $c \oplus a \leq c \oplus b$.
- If $a \leq b$, then $a \otimes c \leq b \otimes c$ and $c \otimes a \leq c \otimes b$.

Example 3.2 Let $R = \{a, b, c, d\}$ be a set with the hyperoperations $\oplus$ and $\otimes$ are defined as follows:

Table 1. The hyperoperation $\oplus$ of LA-hyperring

$\oplus$	a	b	c	d
a	$\{a\}$	$\{a,b\}$	R	R
b	$\{a,b\}$	$\{a,b\}$	R	$\{c,d\}$
c	R	$\{c,d\}$	$\{c,d\}$	$\{c,d\}$
d	R	R	R	R

Table 2. The hyperoperation $\otimes$ of LA-hyperring

$\otimes$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a,b\}$	R	R
c	$\{a\}$	R	R	$\{d\}$

d	$\{a\}$	R	$\{c,d\}$	R

And the relation order $\leq$ is defined by: $\leq := \{(a,b),(b,d),(c,d),(a,d),(a,a),(b,b),(c,c),(d,d)\}$

It is easy to verify that $(R,\oplus,\otimes,\leq)$ is an ordered LA-hyperring and we can see that $(R,\oplus)$ and $(R,\otimes)$ are non-associative, $(c \oplus b) \oplus d = R \neq \{c,d\} = c \oplus (b \oplus d)$ and $(c \otimes c) \otimes d = R \neq \{d\} = c \otimes (c \otimes d)$.

Definition 3.3 Let $(R,\oplus,\otimes,\leq)$ be an ordered LA-hyperring and $A \subseteq R$. Define a subset $(A]$ as follows
$(A] = \{t \in R : t \leq a, \text{for some } a \in A\}$

Lemma 3.4 Let $(R,\oplus,\otimes,\leq)$ be an ordered LA-hyperring. Then

- $A \subseteq (A]$ for any $A \subseteq R$.
- If $A \subseteq B$, then $(A] \subseteq (B]$ for any $A, B \subseteq R$.
- $(A] \otimes (B] \subseteq (A \otimes B]$ and $(A] \oplus (B] \subseteq (A \oplus B]$ for any $A, B \subseteq R$.
- $((A] \otimes (B]] = (A \otimes B]$ and $((A] \oplus (B]] = (A \oplus B]$ for any $A, B \subseteq R$.
- If $A \subseteq B$, then $(C \otimes A] \subseteq (C \otimes B]$ and $(A \otimes C] \subseteq (B \otimes C]$.
- If $A \subseteq B$, then $(C \oplus A] \subseteq (C \oplus B]$ and $(A \oplus C] \subseteq (B \oplus C]$.

Proof. The proof is straightforward.

Theorem 3.5 An ordered LA-hyperring $(R,\oplus,\otimes,\leq)$ is an ordered hyperring if and only if:

- $a \oplus b = b \oplus a$.
- $a \otimes (b \otimes c) = (c \otimes b) \otimes a$.

Proof. Let $(R,\oplus,\otimes,\leq)$ be an ordered LA-hyperring. We will show that $(R,\oplus,\otimes,\leq)$ is an ordered hyperring. Let $a,b,c \in R$, then

$$(a \oplus b) \oplus c = (c \oplus b) \oplus a \quad \text{(left invertive law)}$$
$$= a \oplus (b \oplus c)$$

Since $(R,\oplus)$ is a LA-hypergroup, so reproduction axioms holds in R. Thus $(R,\oplus)$ is a hypergroup.

$$(a \otimes b) \otimes c = (c \otimes b) \otimes a \quad \text{(left invertive law)}$$
$$= a \otimes (b \otimes c)$$

Hence, $(R,\oplus,\otimes)$ is a hyperring. Since R is an ordered LA-hyperring, the conditions (R1) and (R2) holds obviously. So $(R,\oplus,\otimes,\leq)$ is an ordered hyperring.

Definition 3.6 Let $(R,\oplus,\otimes,\leq)$ is an ordered LA-hyperring, then:

- R is called with left identity (resp. pure left identity) if there is an element $e \in R$ such that $a \in a \otimes e$ ($\{a\} = a \otimes e$) for all $a \in R$.
- A non-empty subset S of R is said to be an LA-subsemihyperring if $(S,\oplus,\otimes,\leq)$ itself is an ordered LA-hyperring.
- An element $a \in R$ is called by an idempotent (resp. weakly idempotent) element of R if $a \otimes a = a$ (resp. $a \otimes a = \{a\}$) .

Example 3.7 Let $A = \{a,b,c\}$ with the hyperoperations $\oplus$ and $\otimes$ are defined as follows:

Table 3. The hyperoperation $\oplus$ of A.

$\oplus$	**a**	**b**	**c**
a	S	S	S
b	$\{a,b\}$	$\{b,c\}$	$\{b,c\}$
c	$\{a,c\}$	$\{b,c\}$	$\{b,c\}$

Table 4. The hyperoperation $\oplus$ of A.

$\otimes$	**a**	**b**	**c**
a	$\{a\}$	$\{b\}$	$\{c\}$
b	$\{c\}$	$\{b,c\}$	$\{c\}$
c	$\{b\}$	$\{b\}$	$\{b,c\}$

And the order relation is defined by $\leq := \{(a,b),(a,c),(a,a),(b,b),(c,c)\}$.

It is easy to verify that $(A,\oplus,\otimes,\leq)$ is an ordered LA-hyperring and a is a pure identity element of A.

Theorem 3.8 A pure left element of an ordered LA-hyperring is unique.

Proof. Let $(R,\oplus,\otimes,\leq)$ be an ordered LA-hyperring and e be a pure left identity element. Assume that a pure left identity is not unique, then there is an element $e' \in R$ such that $\{e'\} = e' \otimes a$, for all $a \in R$.

$$\{e\} = e \otimes e'$$
$$= (e \otimes e) \otimes e' \quad (e \text{ is a pure left identity})$$
$$= (e' \otimes e) \otimes e \quad \text{(invertive law)}$$
$$= e' \otimes e = \{e'\}$$

It contradicts with the assumption that e is not unique. Hence a pure left identity is unique.

4. HYPERIDEAL OF ORDERED LA-HYPERRING

In this section, we study hyperideal, bi-hyperideal, and quasi-hyperideal of ordered LA-hyperring. Also we investigate some elementary properties of some type hyperideal of ordered LA-hyperring.

Definisi 4.1 A non-empty set I of an ordered LA-hyperring $(R,\oplus,\otimes,\leq)$ is called by left (resp. right) hyperideal of $(R,\oplus,\otimes,\leq)$ if satisfies the following condition.

- $(I,\oplus)$ is LA-subhypergroup of $(R,\oplus)$.
- $R\otimes I\subseteq I$ $(I\otimes R\subseteq I)$.
- If $a\in I$ and $b\leq a$, then $b\in I$, for any $b\in R$.

Example 4.2 Let $(R,\oplus,\otimes,\leq)$ be an ordered LA-hyperring with the hyperoperations $\oplus$ and $\otimes$ are defined as follows:

Table 5. The hyperoperation $\oplus$ of R.

$\oplus$	a	b	c
a	R	R	R
b	$\{a,b\}$	$\{b,c\}$	$\{b,c\}$
c	$\{a,c\}$	$\{b,c\}$	$\{b,c\}$

Table 6. The hyperoperation $\otimes$ of R.

$\otimes$	a	b	c
a	R	$\{b,c\}$	$\{b,c\}$
b	$\{b,c\}$	$\{b,c\}$	$\{c\}$
c	$\{b,c\}$	$\{b\}$	$\{b,c\}$

And the relation order is defined by $\leq:=\{(a,a),(b,b),(c,c),(b,a),(c,a)\}$.

It is easy to verify that $I=\{b,c\}$ is a hyperideal of $(R,\oplus,\otimes,\leq)$.

Teorema 4.3 The intersection of any two hyperideals of ordered LA-hyperring $(R,\oplus,\otimes,\leq)$ is a hyperideal of R.

Proof. Let I_1 and I_2 be two hyperideals of R. First, we will show that $I_1\cap I_2$ is an LA-subhypergroup of $(R,\oplus)$. Let $x,y\in I_1\cap I_2$, then we have $x\oplus y\subseteq I_1\oplus I_1\subseteq I_1$ and $x\oplus y\subseteq I_2\oplus I_2\subseteq I_2$. Since $x\oplus y\subseteq I_1$, $x\oplus y\subseteq I_2$ and I_1,I_2 are hyperideal of R, then the left invertive law is satisfied in $I_1\cap I_2$. By definition of hyperideal, I_1 and I_2 are LA-subhypergroup of $(R,\oplus)$, then for any $r\in R$ and $x\in I_1\cap I_2$ we get $r\oplus I_1\cap I_2\subseteq r\oplus I_1=I_1$ and $r\oplus I_1\cap I_2\subseteq r\oplus I_2=I_2$. Conversely, $I_1\cap I_2\subseteq I_1=r\oplus I_1$ and $I_1\cap I_2\subseteq I_2=r\oplus I_2$. The same way can used to show that $I_1\cap I_2\oplus r=I_1\cap I_2$. Therefore, $I_1\cap I_2$ is an LA-subhypergroup of $(R,\oplus)$.
Now we will show that $(I_1\cap I_2)$ satisfy (I2) conditions. Consider $(I_1\cap I_2)\otimes R\subseteq I_1\otimes R\subseteq I_1$ and $(I_1\cap I_2)\otimes R\subseteq I_2\otimes R\subseteq I_2$. This implies that $(I_1\cap I_2)\otimes R\subseteq I_1\cap I_2$. The case for right hyperideal can be seen in the similar way. Since I_1 and I_2 are hyperideals of R, then the third condition holds obviously. So $I_1\cap I_1$ is a hyperideal of R.

Theorem 4.4 If $(R,\oplus,\otimes,\leq)$ be an ordered LA-hyperring with a pure left identity e, then for all $a\in R$:

- $(R\otimes a]$ is a left hyperideal of R.
- $(a\otimes R]$ is a right hyperideal of R.

Proof. Let $\{x\},\{y\}\subseteq(R\otimes a]$ where $\{x\}\subseteq r_1\otimes a$ and $\{y\}\subseteq r_2\otimes a$ for some $r_1,r_2\in R$. This implies that $r_1\otimes a\leq s_1\otimes a$ and $r_2\otimes a\leq s_2\otimes a$ for some $s_1,s_2\in R$.

$$\begin{aligned}\{x\}\oplus\{y\}&\subseteq(r_1\otimes a)\oplus(r_2\otimes a)\\&\leq(s_1\otimes a)\oplus(s_2\otimes a)\\&=(s_1\oplus s_2)\otimes a\subseteq R\otimes a\end{aligned}$$

Thus $\{x\}\oplus\{y\}\subseteq(R\otimes a]$. For any $r\in R$, we have

$$\begin{aligned}\{x\}\oplus(R\otimes a]&\subseteq r_1\otimes a\oplus(R\otimes a]\\&\leq r_1\otimes a\oplus r\otimes a\\&=(r_1\oplus r)\otimes a\subseteq R\otimes a\end{aligned}$$

And

$$\begin{aligned}\{x\}\subseteq r_1\otimes a&\leq s_1\otimes a\\&\subseteq R\otimes a\\&=(r\oplus R)\otimes a\\&=(r\otimes a)\oplus(R\otimes a)\end{aligned}$$

Hence $\{x\}\oplus(R\otimes a]=(R\otimes a]$. The similar way can be used to show that $(R\otimes a]\oplus\{x\}=(R\otimes a]$. Therefore, $(R\otimes a]$ is an LA-subhypergroup of R.
Now let $r\in R$, then

$$\begin{aligned}r\otimes\{x\}&\leq r\otimes(r_1\otimes a)\\&\subseteq(e\otimes r)\otimes(r_1\otimes a) &&\text{(pure left identity)}\\&=(a\otimes r_1)\otimes(r\otimes e) &&\text{(paramedial law)}\\&=(a\otimes r_1)\otimes r &&\text{(pure left identity)}\\&=(r\otimes r_1)\otimes a &&\text{(medial law)}\\&\subseteq R\otimes a\end{aligned}$$

So $(R\otimes a]$ is a hyperideal of R.

Theorem 4.5 Let $(R,\oplus,\otimes,\leq)$ be an ordered LA-hyperring with a pure left identity. If I is a hyperideal of R, then $(I\otimes I]$ is also a hyperideal of R.

Proof. First we show that $(A\otimes A]$ is an LA-subhypergroup of $(R,\oplus,\otimes,\leq)$. Let $\{x\},\{y\}\subseteq(I\otimes I]$, which implies that $\{x\}\leq x_1\otimes x_2$ and $\{y\}\leq y_1\otimes y_2$ for some $x_1\otimes x_2,y_1\otimes y_2\subseteq I\otimes I$. Then we have $\{x\}\oplus\{y\}\leq x_1\otimes x_2\oplus y_1\otimes y_2\subseteq I\oplus I$. Next we show that $(I\otimes I]$ satisfy the reproduction law. Let $\{a\}\subseteq z\oplus(I\otimes I]$ then $\{a\}\subseteq\{z\}\oplus\{i\}$ where $\{z\},\{i\}\subseteq(I\otimes I]$. By definition of $(I\otimes I]$, we have $\{z\}\leq z_1\otimes z_2$ and $i\leq i_1\otimes i_2$ for some $z_1,z_2,i_1,i_2\in I$.
Then

$$\begin{aligned}\{a\}&\subseteq\{z\}\oplus\{i\}\\&\leq z_1\otimes z_2\oplus i_1\otimes i_2\\&\subseteq I\oplus I\end{aligned}$$

And

$$\begin{aligned}\{i\}&\leq i_1\otimes i_2\subseteq i_1\otimes I\\&=i_1\otimes(b\oplus I) &&\text{(for all } b\in I)\\&=(i_1\otimes b)\oplus(i_1\otimes I)\end{aligned}$$

$\subseteq (i_1 \otimes b) \oplus (I \otimes I)$

Thus $(I \otimes I]$ is an LA-subhypergroup of $(R,\oplus)$. Let $\{x\} \subseteq R \otimes (I \otimes I]$ where $\{x\} \subseteq r \otimes (i_1 \otimes i_2)$. This implies that $i_1 \otimes i_2 \leq y \otimes z$ for some $y, z \in I$. We have

$\{x\} \subseteq r \otimes (i_1 \otimes i_2)$
$\leq r \otimes (y \otimes z)$
$= (e \otimes r) \otimes (y \otimes z)$ (e is a pure identity)
$= (e \otimes i_1) \otimes (r \otimes i_2)$ (medial law)
$\subseteq (I \otimes I)$

The case for $\{x\} \subseteq (I \otimes I] \otimes R$ can be seen in similar way. Let $\{x\} \subseteq (I \otimes I]$, then $\{x\} \leq x_1 \otimes x_2$ for some $x_1, x_2 \in I$. If $\{y\} \leq \{x\}$, then $\{y\} \leq \{x\} \leq x_1 \otimes x_2$. Hence $\{y\} \subseteq (I \otimes I]$. So $(I \otimes I]$ is a hyperideal of R.

Definition 4.6 A non-empty set B of an ordered LA-hyperring $(R,\oplus,\otimes,\leq)$ is called by bi-hyperideal of $(R,\oplus,\otimes,\leq)$ if satisfies the following condition.

- $(B,\oplus)$ is LA-subhypergroup of $(R,\oplus)$.
- $(B \otimes R) \otimes B \subseteq B$.
- If $a \in B$ and $b \leq a$, then $b \in B$, for any $b \in R$.

Theorem 4.7 Let $(R,\oplus,\otimes,\leq)$ be an ordered LA-hyperring with a pure left identity e. If B_1 and B_2 are bi-hyperideals of R, then $(B_1 \otimes B_2]$ is a bi-hyperideal of R.

Proof. The proof is straightforward.

Theorem 4.8 If $(R,\oplus,\otimes,\leq)$ is an ordered LA-hyperring, then every left (right) hyperideal of R is a bi-hyperideal of R.

Proof. Let I be a left hyperideal of R. we will show that I satisfy the (B2) conditions.
$(I \otimes R) \otimes I = (R \otimes R) \otimes I \subseteq R \otimes I$.
Since I is a left hyperideal of R, we have $R \otimes I \subseteq I$.
The case for a right hyperideal I of R, we get
$(I \otimes R) \otimes I \subseteq I \otimes I \subseteq I$. So, I is a bi-hyperideal of R.

Definition 4.9 A non-empty set Q of an ordered LA-hyperring $(R,\oplus,\otimes,\leq)$ is called by quasi-hyperideal of $(R,\oplus,\otimes,\leq)$ if satisfies the following condition.

- $(Q,\oplus)$ is LA-subhypergroup of $(Q,\oplus)$.
- $(Q \otimes R) \cap (R \otimes Q) \subseteq Q$.
- *If* $a \in Q$ and $b \leq a$, then $b \in Q$, for any $b \in R$.

Theorem 4.10 Let $(R,\oplus,\otimes,\leq)$ be an ordered LA-hyperring. If Q is a quasi-hyperideal of R, then Q is an LA-subhyperring of R.

Proof. We will show that $Q \otimes Q \subseteq Q$. Let $x, y \in Q$, then
$x \otimes y \subseteq Q \otimes Q \subseteq Q \otimes R$ and
$x \otimes y \subseteq Q \otimes Q \subseteq R \otimes Q$. Hence
$x \otimes y \subseteq (Q \otimes R) \cap (R \otimes Q)$. By definition of quasi-hyperideal of R, we get $x \otimes y \subseteq Q$. So, Q is an LA-subhyperring of R.

Theorem 4.11 If $(R,\oplus,\otimes,\leq)$ is an ordered LA-hyperring with a pure left identity, then every quasi-hyperideal of R is a bi-hyperideal of R.

Proof. Let e be a pure left identity of R and Q be a quasi-hyperideal of R. We will show that Q satisfy the condition $(Q \otimes R) \otimes Q \subseteq Q$. Then

$(Q \otimes R) \otimes Q \subseteq (Q \otimes R) \otimes (Q \otimes e)$
$= (Q \otimes Q) \otimes (R \otimes e)$
$\subseteq Q \otimes R$

and

$(Q \otimes R) \otimes Q \subseteq (R \otimes R) \otimes Q$
$\subseteq R \otimes Q$

Hence, we get $(Q \otimes R) \otimes Q \subseteq Q \otimes R \cap R \otimes Q \subseteq Q$. So, Q is a bi-hyperideal of R.

5. CONCLUSION

An ordered LA-hyperring is a hyperstructure with a partial order relation as a generalization of LA-ring and hyperring. We obtained some elementary properties of ordered LA-hyperring and some useful contiditons for ordered LA-hyperring to become an ordered hyperring. Also, we investigated some properties of hyperideal, bi-hyperideal, and quasi-hyperideal of ordered LA-hyperring.

AUTHORS' CONTRIBUTIONS

All authors have equally contributed to this work.

ACKNOWLEDGMENTS

We would like to thank to all referees for valuable comments. The comments and suggestions have really helped in the development of this paper.

REFERENCES

[1] MARTY and F., "Sur une generalization de la notion de groupe," *8th Congr. Math. Scandinaves*, pp. 45–49, 1934.

[2] P. Corsini and V. Leoreanu, *Applications of Hyperstructure Theory*. Dordrecht: Kluwer Academic, 2003.

[3] M. A. Kazim and N. Naseeruddin, "On almost semigroups," *Aligarh Bull. Math.*, vol. 2, pp. 1–7, 1972.

[4] M. S. Kamran, "Conditions for LA-semigroup to resemble associative structures," Quad-i-Azam University, 1993.

[5] S. M. Yusuf, "On the Left Almost Ring," *Proc. 7th Int. Pure Math. Conf.*, 2006.

[6] T. Shah, "On LA-Rings of Finitely Nonzero Functions," vol. 5, no. 5, pp. 209–222, 2010.

[7] F. Hussain, S. Firdous, and N. Sadiq, "Direct Product of two Near Left Almost Rings," *Int. J. Algebr. Stat.*, vol. 5, no. 2, p. 77, 2016.

[8] K. Hila and J. Dine, "On Hyperideals in Left Almost Semihypergroups," *ISRN Algebr.*, vol. 2011, pp. 1–8, 2011.

[9] N. Yaqoob and M. Gulistan, "Partially ordered left almost semihypergroups," *J. Egypt. Math. Soc.*, vol. 23, no. 2, pp. 231–235, 2015.

[10] N. Yaqoob, P. Corsini, and F. Yousafzai, "On intra-regular left almost semihypergroups with pure left identity," *J. Math.*, vol. 2013, 2013.

[11] I. Rehman, N. Yaqoob, and S. Nawaz, "Hyperideals and hypersystems in LA-hyperrings," *Songklanakarin J. Sci. Technol.*, vol. 39, no. 5, pp. 651–657, 2017.