Analysis of Junior High School Student’s Mathematical Reasoning Ability in Solving Non-routine Problems on Material of Two-variable Linear Equation Systems

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ABSTRACT

Reasoning ability encourages students to think logically, so it is a very important part in learning process. This qualitative descriptive study aims to determine the mathematical reasoning ability of Junior High School students in solving non-routine problems of the two-variable linear equation systems material. The subjects were 6 students of class VIII.7 SMP Negeri 17 Palembang. Due to Covid-19, the research was conducted online using test and interviews, and analyzed descriptively. The results showed that of the 7 indicators of mathematical reasoning ability used, high-capable subjects fulfill almost all indicators. Indicators are not fulfilled in namely posing an assumption, perform mathematical manipulation, draw conclusions, compile evidence, provide reasons or evidence for the correctness of the solution to problem 1 and find patterns or properties of mathematical phenomena that will be generalized to problem 2. Indicators that are not fulfilled in medium-capable are perform mathematical manipulation, draw conclusions, compile evidence, provide reasons or evidence for the correctness of the solution and find patterns or properties of mathematical phenomena that will be generalized to problem 2. Students with low-capable still not dominant and only permit indicators to present oral, written, picture and diagram math statement and draw conclusions from those statements.

Keywords: Reasoning abilities, problem solving, linear equation system.

1. INTRODUCTION

Students’ mathematical reasoning abilities play a major role in the achievement of mathematics learning [1, 3]. Mathematical reasoning is the ability to understand mathematics in a logical way in order to form a conclusion based on facts and relevant sources [2, 4, 28]. Mathematical reasoning skills help students to conclude and prove a statement, build new ideas and solve mathematical problems [5]. This ability can train students to use their knowledge to think logically and systematically, examine a statement and finally solve a problem [6].

With reasoning students will use the rules, properties and logic of mathematics to understand and understand mathematical concepts and obtain the truth in mathematics and get a correct conclusion [6]. So it can be said that reasoning encourages students to think logically in making conclusions by following existing provisions and statements and making new statements that are proven true so that they can solve problems faced by students.

The world of mathematics requires mathematical reasoning to solve problems because reasoning encourages us to go through a logical thinking process [6, 7]. One of the descriptions of core competencies in the skill aspect is that students are required to demonstrate critical reasoning skills, that is, students do not only have the ability to count but the ability to think logically and critically in solving routine and non-routine problems [8]. NCTM states that improving students’ mathematical reasoning abilities is one of the goals of learning mathematics [24]. So that mathematical reasoning skills are very important to develop because if not, then mathematics will only be material that follows and imitates existing procedures without being meaningful.
Students are said to have mathematical reasoning abilities if students are able to (1) make calculations based on applicable rules, (2) draw general conclusions based on visible mathematical processes, (3) make assumptions, (4) draw conclusions based on similar mathematical processes, and (5) check the validity of an argument [1, 9].

However, the fact that occurs in the field is that the reasoning ability of students in learning mathematics is still very low [9, 10]. The low ability of students' mathematical reasoning becomes a problem that occurs in the learning process characterized by the low ability of students to understand and analyze concepts, use a method and draw conclusions [11, 12].

Most of the students have difficulty when dealing with mathematical problems that involve reasoning because students are still accustomed to memorizing [13]. The description of the mathematics learning process so far is that the teacher only gives questions that emphasize concept understanding, while higher thought processes including reasoning are rarely given [14]. Students have difficulty solving non-routine questions because the majority of learning only adopts routine questions and source books [15, 29, 30]. From this, it can be seen that so far students' reasoning has not been trained, students tend to just memorize and use the formulas that have been given by the teacher so that when students are faced with higher-level questions, students have difficulty solving them.

Previous research has shown students' low reasoning skills when the teacher gives questions on linear equation system material [11, 27]. Two-variable linear equation system is material learned in junior high school and becomes a concept that is difficult to understand by students because students' mathematical reasoning skills in SPLDV material are still low [11, 18]. Students make many mistakes in solving SPLDV problems, for example in changing to a mathematical model and incorrectly performing the solving procedure [19]. In addition, many students still have difficulty identifying variables and determining their solutions because they have not mastered SPLDV well [20, 21].

For that, it is necessary to make changes in learning, namely by getting students to work on non-routine questions. Non-routine questions can be used in learning to see the extent of students' reasoning abilities [22]. Problem solving problems or so-called non-routine problems are problems with the solving process that do not use ordinary procedures or problems that contain challenges to solve which require relatively longer than routine problems [23]. Presenting non-routine questions to students means getting them used to applying various mathematical concepts in new situations and when students get used to non-routine questions, they will easily solve routine problems [25, 26]. That is, giving non-routine questions when learning mathematics is very necessary in order to improve students' abilities in learning mathematics and can be used in everyday life.

Given the importance of students’ mathematical reasoning, it is necessary to carry out an in-depth analysis of students' mathematical reasoning abilities. This analysis is in the form of students’ mathematical reasoning abilities in solving problem solving problems or non-routine questions as well as students’ difficulties and mistakes in doing mathematical reasoning.

2. METHOD

This study is a qualitative descriptive research with the focus is the mathematical reasoning ability of Junior High School students in solving non-routine problems with the two-variable linear system (SPLDV) material. Due to pandemic condition, this research was conducted online with video lessons by Youtube, Whatsapp Group and Zoom Meetings. The subjects in this study were 6 students of class VIII.7 SMP Negeri 17 Palembang with high, medium and low ability categories. The data collection techniques used were tests and interviews. The tests were given in the form of 3 problem solving questions of descriptions of the SPLDV material which had been adjusted to the indicators of students 'mathematical reasoning abilities, while interviews were conducted to deepen the analysis of students' reasoning abilities by asking questions that were in accordance with the indicators. Table 1 shows the indicators used in this study.

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Descriptor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present oral, written, picture and diagram math statement</td>
<td>Able to explain the problems from the questions given, be able to determine and identify the information contained in the questions, namely knowing what is known and asked from the questions.</td>
</tr>
<tr>
<td>Possing an assumption</td>
<td>Able to develop arguments and strategies to determine the steps for completion and the elements used.</td>
</tr>
<tr>
<td>Perform mathematical manipulation</td>
<td>Able to use certain methods in solving problems.</td>
</tr>
<tr>
<td>Draw conclusions, compile evidence,</td>
<td>Able to use and show the strategy that has been</td>
</tr>
</tbody>
</table>

Table 1. Indicators of mathematical reasoning.
provide reasons or evidence for the correctness of the solution.

determined in carrying out the steps to solve it appropriately.

Drawing conclusion from the statement

Able to use the knowledge to produce a thought in accordance with the problems.

Checking the validity of an argument

Able to re-examine problems and investigate the truth of the statements.

Find patterns or properties of mathematical phenomena to make generalizations.

Able to find patterns or properties of a problem and can be used to solve new situations so that students are able to draw valid conclusions.

Data analysis was performed using descriptive methods by describing students’ mathematical reasoning abilities that appeared on test results, interviews and researcher notes. The questions used is a non-routine problem that has been adjusted to the indicator of mathematical reasoning ability. The following are questions used in this study:

Table 2. Problem solving two-variable linear equation system material.

<table>
<thead>
<tr>
<th>No.</th>
<th>Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A grandfather named Andi has a granddaughter named Nurul. 3 years ago, Nurul’s age was ( \frac{1}{6} ) of her grandfather’s age. If 21 years from now, Nurul’s age is ( \frac{1}{5} ) of grandfather’s age, then how old will Nurul and Grandpa be in the next 5 years? And is it true that they are now 100 years old?</td>
</tr>
<tr>
<td>2</td>
<td>Ani and Ina go to a bookstore. At the bookstore, everyone who has a member card will be given a discount of IDR 5,000 / book. That day, the bookstore’s birthday and offers each of the same types of books at the same price. Here are some book prices at the shop.</td>
</tr>
</tbody>
</table>

Ani and Ina bought the same type of books with the same quantity, namely textbooks and comics.

a. If Ani is a member of the bookstore and she spent IDR 230,000 while Ina was not a member and spent IDR 260,000, so how many textbooks and comics did they buy?
b. If they get 5% discount from every textbook they buy, how much money will Ani and Ina have to pay each?

3. Result and Discussion

The results of the analysis were obtained from test questions and interviews. Students work on 3 essay questions for 45 minutes and are supervised by researcher via Zoom Meetings. After the researcher gets the test results, the researcher checks their work steps and pays attention to the indicators that appear. Furthermore, researcher conducted interviews to deepen the analysis on one by one the subjects via Zoom Meetings. The results of the analysis of test and interview questions can be seen in Table 3 below.
Table 3. The results of the analysis of test questions and interview.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Question Number</th>
<th>Indicator 1</th>
<th>Indicator 2</th>
<th>Indicator 3</th>
<th>Indicator 4</th>
<th>Indicator 5</th>
<th>Indicator 6</th>
</tr>
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</tr>
<tr>
<td>RSP</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>2</td>
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<td>✓</td>
<td>✓</td>
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</tr>
<tr>
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<td>3</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Information:
✓ : Appear
✗ : Not Appear

3.1. First Indicator

The first indicator is to present mathematical statements orally, in writing, pictures and diagrams. All subjects were able to fulfill this indicator in all three questions. This indicator expects the subject to understand the problem by writing down the information available and explaining the meaning of the information orally.

![Figure 1](image1.png) Subject RO’s answer to indicator 1 question no. 1.

![Figure 2](image2.png) Subject CPI’s answer to indicator 1 question no. 2.

![Figure 3](image3.png) Subject AS’s answer to indicator 1 question no. 3.

Figures 1, 2 and 3 show the answers of the subjects on indicator 1 in writing. However, there were also other subjects who had not written the information but were able to state the information verbally during the interview. When interviewing, all subjects are able to state what information is available and explain the purpose of the information.

3.2. Second Indicator

The second indicator is proposing an assumption. This indicator expects students to be able to formulate arguments and strategies to determine the steps for completion and the elements used. To answer the questions given, this indicator requires students to be able to form an equation model of the problem and determine the method used to solve it. Most of the subjects met this indicator but there was 1 subject who still could not make his guesses correctly.

![Figure 4](image4.png) RSP’s answer to indicator 2 question no. 1.

![Figure 5](image5.png) CPI’s answer to indicator 2 question no. 2.
Figure 6 RO’s answer to indicator 2 question no. 3.

Figure 4 shows that RSP subject has not been able to formulate a strategy to solve the problem. Problem 1 asks students to make mathematical models in the form of fractions. To facilitate the counting process, almost all students change the form into whole numbers. The following are the results of interviews with RSP.

\begin{align*}
R &: “here you answer \frac{1}{6} = 6, what does that meaning?” \\
RSP &: “I mean that \frac{1}{6} if used as a regular number is equal to 6” \\
R &: “why is that?” \\
RSP &: “as I remember learning like that, Miss”
\end{align*}

From the interview, it can be seen that RSP has not yet understood the concept of the material and has not even understood number operations. Even though this concept is the basis for understanding the SPLDV material.

Figures 5 and 6 shows that the subject fulfills the 2nd indicator by making mathematical models. They are able to assume the variables and determine the equations of a given situation.

3.3. Third Indicator

The third indicator is perform mathematical manipulation. This indicator requires students to use certain methods in solving problems. Researchers taught 3 SPLDV completion methods and students can use one of the three methods or their own method and determine the most effective method. However, on this indicator some subjects are still not fulfilled. Even though there are those who can do it well, there are still many subjects who find it difficult to work on these questions.

Figure 7 shows the DKJ’s answer, which is the only subject who answered correctly and met the 3rd indicator in question number 1. DKJ used a method other than the 3 methods taught. He made 2 mathematical models in the form of the x equation to produce a value for y then substituted it for the value x.

Figure 8 shows that RSP has not been able to perform mathematical manipulation. He answered correctly that the number of textbooks was 2 and the comics was 4, but he did not write down the steps to solve them. The following is an excerpt from the interview with RSP.

\begin{align*}
R &: “this question is asked for comic, why are you looking for the price of a novel?” \\
RSP &: “oh yes, I wrote it wrong Miss. \\
R &: “okay, no problem. Where do you get 2 and 4 from?” \\
RSP &: “by trial and error, Miss. It is known that the price that Ani issued was IDR 230,000. then try it, for example, how many textbooks, how many comics, if you add up the result is IDR
\end{align*}
230,000. so I get the textbooks 2 and comics 4.

R : ‘how about Ina?’

RSP : “same, just try it. But there is no discount so the price is normal”

It can be seen that RSP actually understand the meaning of the questions but has not been able to use certain methods to solve these problems. He did not even pay attention to the information in the questions that Ani and Ina buy the same number of books so that if you get Ani’s number then you do not need to look for Ina’s. This is because RSP still does not understand the material given.

Figure 9 shows that the DKJ has been able to use the method to solve these problems. But, DKJ is not precise when performing subtraction operations in equation 1. In the \( y + (x - 14) = 72 - 24 \) part it should produce \( y + x = 62 \), but it adds up like this: \( y + (x - 14) = 72 - 24 - y + x = 72 - (24 + 14) \rightarrow y + x = 72 - 38 \) so that \( y + x = 34 \) results in the wrong equation. DKJ is an active subject during learning and does not seem to have any difficulties. In the interview, DKJ admits that he is wrong in the concept of addition and realizes that the answer is wrong until the end. This means that DKJ understands the concepts and procedures of the work, but is not thorough in doing it.

### 3.4. Fourth Indicator

The fourth indicator is drawing conclusions, compiling evidence, providing reasons or evidence for the correctness of the solution. This indicator expects students to be able to use and show a predetermined strategy in carrying out the completion steps appropriately. This means that students must understand every step of their work. On this indicator, half of the subjects have fulfilled it. This indicator is seen at the time of the interview by asking questions such as “how could it be like this?” and some subjects can explain their answer but some are still confused about their own answers which indicates they still do not understand their answers.

### 3.5. Fifth Indicator

The fifth indicator is drawing conclusions from the statement. This indicator expects student to be able to make conclusions or use their knowledge to produce a thought in accordance with the problem. This means that students do not just work on the problem but also understand the direction and purpose of the question. On this indicator, all subjects meet in every question.

### 3.6 Sixth Indicator

The 6th indicator is checking the correctness of an argument. This indicator expects students to be able to re-examine the problem and investigate the truth of the statements they have made. Those who fulfill this indicator are subjects with high ability and moderate ability. Low ability subjects were still confused when asked whether they were sure of the answer. This indicator is shown through interviews with researchers because none of them wrote down the steps to check the truth in their answers. The following is an excerpt from the interview with the subject.

R : “In question number 2, you answered 2 for textbooks and 4 for comics, right?”

AS : “Yes, Miss”

R : “How can you be sure that this answer is correct?”

AS : “2 and 4 are included in the equation with the variables, Miss. Which is 70,000x + 30,000y = 260,000”

It appears that AS has been able to account for the answer and sure that the answer is correct by substituting the value obtained for each existing equation.

### 3.7 Seventh Indicator

The 7th indicator is finding patterns or properties of mathematical symptoms to make generalizations. This indicator expects students to be able to find patterns or properties of a problem and can be used to solve new situations so that students are able to draw valid conclusions. In this study, researchers provide other situations in each question so that when students have...
obtained a solution, they can use the solution to solve other existing situations. Half of the subjects met this indicator and half were unable.

Figure 13 Subject DKJ’s answer to indicator 7 question no. 1.

Figure 14 Subject RO’s answer to indicator 7 question no. 1.

The results showed that of the 7 indicators of mathematical reasoning ability used, the indicator that was fulfilled by all subjects was the 1st indicator, namely present oral, written, picture and diagram and the 5th indicator was drawing conclusions from the statement. While the indicator that appears the least is the 3rd indicator, which is doing mathematical manipulation.

The first indicator expects students to be able to explain the problem from the questions given, be able to determine and identify the information contained in the questions, namely knowing what is known and asked from the questions. So, students not only write down what they know and ask but are also able to explain the meaning of the information so that it shows that students really understand the problem. This indicator appears in all research subjects. Students are able to write down what they know and ask about the questions and during the interview are given in-depth questions such as “do you understand the meaning of this question?” and the researcher asked the subjects to explain in his own language the intent and purpose of the question. This can happen because students are accustomed to being taught to write information when working on math problems. This is supported by [16, 17] that the subject is able to understand the problem by translating the story problem into a simple form because it is implicitly accustomed to being taught by the teacher in the classroom.

Another indicator that appears in all subjects is the fifth indicator, which is to draw conclusions from statements. The subject is able to make conclusions on the problems they are working on. Even though there were some subjects who answered wrongly, they still made conclusions on their thoughts so that this indicator still emerged. This is because students are used to making conclusions when working on story problems. Likewise what was said by [31] in his research that the ability of students to draw conclusions reached a very high category because students were used to using deductive thinking, which means that in solving problems students used their experiences to strengthen reasoning which then became provisions for solving problems.

Meanwhile, the indicator that appears the least is the third indicator, which is doing mathematical manipulation. Not one subject succeeded in bringing up this indicator in the three questions, which means that at least one subject made a miscalculation in one of the questions. The subjects admitted that they had difficulty determining the mathematical model because of the problems they thought were difficult. Students also do not understand the method of elimination due to the provision of material only through instructional videos. Non-routine problems have become new to them so that many are confused about solving it, especially when there is not much time. In accordance with the definition of non-routine problems, namely problems with the solving process that do not use ordinary procedures or a problem that contains challenges in solving which require relatively longer time than routine questions [23, 32].

4. CONCLUSION

Mathematical reasoning is an important aspect of learning because with that good reasoning students are able to solve routine and non-routine problems. Students should be accustomed to working on non-routine problems so that they are accustomed to using various procedures to improve their mathematical reasoning skills so that they are able to solve problems well. Based on the results of data analysis obtained, it can be concluded that of the 7 mathematical reasoning indicators used, students with high abilities fulfill almost all reasoning indicators. Only a few indicators are not fulfilled in namely posing an assumption, perform mathematical manipulation and draw conclusions, compile evidence, provide reasons or evidence for the correctness of the solution to problem 1 and find patterns or properties of mathematical phenomena that will be generalized to problem 2. Medium ability students also meet almost all indicators. Indicators that are not fulfilled are indicators of perform mathematical manipulation on problem 1 and 3, draw conclusions, compile evidence, provide reasons or evidence for the correctness of the solution to problems number 1 and 2 then find patterns or properties of mathematical phenomena that will be
generalized to problem 2. Students with low abilities are still not dominant in indicators of reasoning ability. They only permit indicators to present oral, written, picture and diagram math statement and draw conclusions from those statements. The indicators that appear the most are the first indicator, which is present oral, written, pictures and diagrams and the fifth indicator is drawing conclusions from the statement. While the indicator that appears the least is the third indicator, which is doing mathematical manipulation.

ACKNOWLEDGMENTS

The researcher would like to thank the validator, namely Mr. Jeri Araiku, M.Pd who always helpful in advising each instrument research. Furthermore, the researchers would like to thank SMP Negeri 17, The Principal, Mrs. Dra. Hj. Tetrytanti, M.Si, Deputy Public Relations Mrs. Emi Trisna, S.Pd., M.Si, Mathematics Teacher Mrs. Hj. Een Supriyanti, S.Pd., M.Si who has allowed and facilitated the research process there. And no less important to class VIII.7 students who have been willing to be the subjects of this research.

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