Advances in Social Science, Education and Humanities Research, volume 550
Proceedings of the 1st International Conference on Mathematics and Mathematics Education (ICMMED 2020)

The Continuing Value and Relevance of Mathematics Appreciation Courses

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ABSTRACT
In today's information and data-driven society, it is vital for an informed citizen to have: (i) working competency in quantitative reasoning, and (ii) and general awareness of math – its true nature, role in society, strengths and limitations, and connections with other disciplines. Mathematics appreciation courses seek to address these two needs. In this paper I will share my experiences teaching a course in the general education program of our university, called Mathematics, Culture and Society. The course focuses primarily on the second goal as it lays the groundwork for students to acquire needed mathematical and quantitative skills with confidence. The course provides a view of mathematics as a dynamic human endeavor that is regarded as both a science and an art. It aims to give students a feeling for the mathematical way of thinking and why mathematicians see it as an intellectually exciting discipline. Remote teaching has led to some adjustments in course implementation, but also introduced some pleasant surprises. The covid-19 pandemic may have disrupted the global landscape of academia, but whether we teach traditionally or in this new normal, I believe such a course will go a long way in improving not only mathematics skills, but also mindsets and attitudes.

Keywords: *Mathematics appreciation, Liberal education, General education, Remote learning.*

1. INTRODUCTION

For many years now, many colleges and universities especially in the United States have been offering what are commonly called *"mathematics appreciation courses"*. These courses are often lodged in a general education program – the part of a liberal education curriculum that is shared by all students. By liberal education, we adopt the definition of the Association of American Colleges and Universities that describes it as an approach to college learning that empowers students to diversity, complexity and change [12]. Liberal education provides students with broad knowledge of the wider world (e.g. science, culture and society) as well as in-depth study in a specific area of interest.

The Institute of Mathematics at the University of the Philippines Diliman, where I am a faculty member, offers mathematics programs up to the doctoral level. Our yearly intake of mathematics majors is around 120 students. We teach all the mathematics courses for all the different faculties. Our students come from a huge science and engineering student body but also from many other non-STEM programs. For many of these students, mathematics is a scary subject that affects performance, even their degree program and career choices.

My research is in abstract algebra, specifically in group theory and algebraic combinatorics. I have no formal training in mathematics education although I try to keep up with some of the literature. I do know that mathematicians will benefit if they reach out to math educators, and conversely. So what I will share are results of nearly four decades of teaching experience and knowledge gained throughout the years from studies and advice of scholars in mathematics and mathematics education.

2. STUDENT ATTITUDES AND MISCONCEPTIONS

Many students enter the university with negative attitudes and misconceptions. We are familiar with these and other perceptions:

- Math is only for nerds and gifted students
- Math is a dead subject
- I am a left-brain person, more inclined to creative disciplines

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- Math is all about rules and techniques for solving equations.

Some of these are formed even at very young ages, often a product of bad teaching and reinforced by even our own parents. Which is a shame, because I believe everyone has an innate talent and appreciation for mathematics. Babies and toddlers enjoy learning numbers and shapes. But somewhere along, maybe when they start learning things like fractions, they start getting disenchanted and frustrated, with some even hating math. I believe that this is partly because many fail to see the true nature of math – a living, growing human activity, one of humankind's greatest intellectual achievements. To me the nature of math can be summed up through two mantras: *Math has beauty and power. Math is everywhere.*

3. MATH APPRECIATION COURSES

Improving mathematical skills of students will be more difficult if these negative attitudes and misconceptions persist. That is why it makes sense to design courses that may hopefully address some of these misconceptions. I would like these courses to reveal mathematics' beauty and power, so that students can value it and use it in their lives. So that even those who have been disadvantaged by poor teaching or by a poor environment for learning can see mathematics' true nature, what mathematicians do, and why they continue with passion learning, gaining more skills and continually improving their craft. Such a course is possible through mathematics appreciation courses.

Math appreciation courses have been around for many years now. In the 1950s, the mathematics educator Herbert Gross, concerned that his basic math students were not planning on entering a STEM-oriented career and that they would soon forget the math they were expected to learn in college, undertook the task of developing a type of course that would show how the development of mathematics mirrored the development of civilization [9]. He felt that such a course would not only give students a better understanding and appreciation of mathematics, but that it would also dramatically reduce their fear of mathematics. This eventually let to the classic textbook: *"Mathematics, the Chronicle of Human Endeavor"* [6]. Herbert Gross is considered a pioneer in distance learning. In the 1960s to the 70s, he videotaped his lectures in many courses, particularly in Calculus. You can find his lectures as part of MIT's OpenCourseWare. Also many of them are considered internet "cult classics". Some are available on YouTube. He also founded the Association of Mathematics Teachers at Two Year Colleges.

In 1952, Dr. Howard Fehr, professor of math education at Columbia University Teachers College, prolific author and internationally known educator, wrote that "appreciation of mathematics" – which was, even then, included in most national reports as one of the desired goals of mathematics education was seldom defined or explicitly described – except that somehow appreciation involves aesthetic and the emotions [5]. Fehr tried to explain what *"mathematics appreciation"* meant to him. As an example, he said that geometrical designs, pictures or cut-outs made from rulers, compasses or other instruments were all nice and gave satisfaction and emotional pleasure. They can aid as a means of motivation to, but were not in themselves an appreciation of, mathematics. He differentiated the type of appreciation one derived from seeing something explained by or is an outcome of applied mathematics but without knowing the underlying mathematics. He referred to this as appreciation of tone, of color, of design, and not of mathematics and that it constituted an entirely different form of mental structure than the kind of appreciation derived from actual knowledge of and search for mathematics. He wrote that the appreciation of mathematics comes when we see it and use it as a means of explaining our environment, explaining a design, and so on. It is also a search for or extension of abstract logical structures as satisfying mental creations.

I do not wish to dwell on these different notions of appreciation. I too enjoy cut-outs, pictures or designs, but never only for show and tell. Instead, these are often used for introduction and examination of the mathematical concepts involved. I have seen that many students appreciate learning about the aesthetic and creative aspects involved with mathematical objects and ideas. To me, simply stated, appreciation involves not only hearing about math, but actually doing math and getting actively engaged in the process.

In 1977, the Committee on the Undergraduate Programs in Mathematics (CUPM) of the Mathematical Association of America (MAA) established a panel to consider the content of those college and university courses that treat mathematics appreciation for students in the arts and humanities. Many universities have some of these types of courses, often lodged in a general education program. This is the component of the undergraduate program taken by all or most students regardless of their majors. The CUPM panel came up with a well-thought and comprehensive report, approved by the MAA, that discusses the philosophy of such courses, things to stress in a math appreciation course, things to avoid, approaches to course organization, as well the matter of topic selection [10]. Many of their recommendations are appropriate to this day. The panel also decided that they were satisfied with use of the term *"Mathematics Appreciation"*, as opposed to other names such as *"Mathematics for Poets"*.

There are typically two types of general education courses, not quite mutually exclusive. One type focuses on quantitative literacy. But a second type focuses more on the nature of mathematics, its development, role in

society that will provide students with a broad awareness of math and its uses. I wish to focus on this latter type.

These are some frequent features of mathematics appreciation courses [11]:

- Explores historical achievements and current topics
- Examines a wide spectrum of topics showing something of the breadth of mathematics
- Includes some significant applications
- Requires creativity and exploration above memorization of facts and formulas
- Encourages critical thinking and develops problem solving skills
- Examines various strategies used in mathematics, including experimentation and proof.

There are mathematicians and math educators who have voiced concern about these college-level courses for non-majors. Paul Sally Jr., mathematics professor at the University of Chicago, remarked in an invited address before a 2003 joint meeting of the American Mathematical Society and the Mathematical Association of America, that *"the trend toward teaching college students about mathematics rather than to actually do mathematics has led to growing numbers of college graduates who are numerically illiterate"* [8]. And that *"the notion that one has to 'interest' students in mathematics in order to make them do it has gone much too far, to the point that mathematics in many cases has just disappeared entirely from the course"*. The criticism is valid. But to me, the problem has more to do with the implementation of the course. The recommendations of the CUPM clearly advise against imprecise ideas and mere talk about mathematics.

4. MATH APPRECIATION COURSE AT THE UNIVERSITY OF PHILIPPINES

In 2013, the Philippines embarked on its largest reform program in education. A law approved the establishment of the K-12 program, making Kindergarten and the addition of two years in senior high school (Years 11 and 12) mandatory for all. By 2018, the first batch of students who all went through K-12 entered college. This meant that curricular programs in college all had to undergo massive revision.

As part of the reorganization of the curriculum, our university revised our general education program. The current format is a hybrid of some mandatory courses plus electives chosen from an approved menu. The objectives of this revised General Education program are: (i) broaden intellectual and cultural horizons; (ii) hone critical and creative thinking; (iii) develop a passion for learning and scholarship; (iv) cultivate a high sense of intellectual and moral integrity; and (v) foster a commitment to nationalism and social justice.

My Institute has offered a general education course in mathematics for close to 60 years now. In the early years, the course was more remedial in nature. There was a focus on skills that were not introduced or mastered in school mathematics. The course has evolved through the years, and its current version – a mathematics appreciation course – was a result of revisions made when the new K-12 program was implemented. The course is an approved course in the revised General Education Program. It is mandatory for most non-STEM students, and an elective course for others. It is now offered in all campuses of the University of the Philippines System. The name of this course is *"Mathematics, Culture and Society"*.

I wish to share with you the features of this course. You will see that it is greatly influenced by the CUPM recommendations, as well as those of organizations like the National Council of Teachers of Mathematics and the American Mathematical Society.

Keith Devlin is a former Stanford professor and prolific author of popular mathematics books. You may be familiar with his course, offered sometimes with titles like "Mathematical Thinking" or "An Overview of Mathematics". Devlin remarked that the name "Mathematics Appreciation" is okay, but that it implies the absence of the experience of *doing mathematics* that he believes is an all-important part of the course [4]. Among his numerous works, the material in his book "The Language of Mathematics" [3] forms the backbone of his course. Our course mirrors Devlin's philosophy.

Let me now briefly describe our math course. The course is officially described as *"appreciation of the beauty and power of mathematics through the examination of its nature, development, utility and relationship with culture and society"*. In their report, the CUPM summarized the ultimate goal of such courses – that is, to instill in the student an appreciation of mathematics. And for this to occur, students must come to understand the historical and contemporary role of mathematics, and to place the discipline properly in the context of other human achievements [10]. We have adopted the same goal for our course.

The course outcomes expected from students are: (i) explain the nature of mathematics as an intellectual and creative discipline; (ii) recognize the importance of mathematics in various human activities; (iii) relate the concepts of mathematics to their field/s of interest; (iv) critically discuss local and global issues and trends in mathematics; (v) produce creative work inspired by mathematical ideas; and (vi) analyze the interplay of mathematics and society.

The course caters to students of diverse interests with different levels of mathematical training. Lessons can be selected from a wide range of topics because of

the richness of math as a field of study and the diversity of the student community. The course adheres to the recommendation of the CUPM panel that no particular selection of topics or teaching strategy should be universally adopted in a mathematics appreciation course. There is latitude in the selection of subtopics to keep the course fresh and to capitalize on the strengths of the faculty and the local context.

The course is divided into five parts. The section titles and suggested hours are as follows: I. Introduction (4.5 hours); II. The Nature of Math: Math as a language, tool, art, and way of thinking and knowing (15 hours); III. Utility and Ubiquity: Math in different disciplines (18 hours); IV. Issues and Trends in Math (7.5 hours), and V. Synthesis: What is mathematics, really (3 hours). The title of the last section is adopted from Reuben Hersch's book [7], whose humanistic philosophy is an influence on the course. Although no textbook is prescribed, an important reference is the classic work, "The Mathematical Experience" by Philip Davis and Hersch [2], a book that introduced me to profound insights on the practice of modern mathematics through a philosophical and historical perspective.

Various subtopics are suggested for Section II. These are: Logic and Reasoning; Philosophical Foundations; Abstraction and Symbols; Axiomatic Systems; Sets; Numbers; Shapes; and Growth. Section III focuses more on applications of math and connections with other disciplines. The fourth section is shorter but is an important part of the course. Here we explore certain local or global issues and trends in mathematics. Typically two topics are chosen from among the following: Ethnomathematics; Math and Gender; Impact of Technology on Math; State of Mathematics Education in the Philippines; and Humanistic Mathematics.

I enjoy teaching the different topics in the course. Among my favorites are lessons on Euclid and the axiomatic method, leading to the topic of the discovery of non-euclidean geometries. We also discuss concepts such as geometric transformations, curvature and finite geometries. These transition to wonderful sessions on graphs and networks. My students enjoy learning about small world networks and how these are modeled in social media networks, collaboration and friendship graphs. The lesson on symmetry is motivated by examples from nature and from designs and artistic work through the ages, including an exploration of the mathematical ideas behind the weaving patterns and textile designs of Philippine indigenous groups. Ideas abound from the arts and humanities, including those from Lewis Carroll's Alice in Wonderland and literary forms around the world. Contemporary applications such as coding theory, cryptography and bitcoin currency are appreciated. The lesson on voting systems were timed to coincide with the recent US elections and students had an opportunity to examine the difference between the electoral college and our own presidential election method. Due to the pandemic, the lessons on exponential growth and mathematical modeling resonated.

Students appreciate the hours spent on discussing ethnomathematics and culturally relevant education. A consistent activity I require is a reaction paper on the article of Alan Bishop, an Australian mathematics educator, on western mathematics as a weapon of cultural imperialism [1]. A lively discussion is often elicited on the issue of mathematics and gender. History is used as a backdrop for introducing many topics, but facts and dates are not emphasized.

Because the course is handled by various professors, we have allowed a certain degree of flexibility – with the agreement to cover most topics found in the course outline's second level, particularly in Section II, but allowing different emphases. For the applications of math in the third section, there is even greater freedom in the choice of topics, catering to the class profile and the professor's interests and expertise. In most lessons, we have tried to engage students to do mathematics – not your typical equation solving, but through various activities that require higher order thinking skills.

5. REMOTE TEACHING DURING THE COVID-19 PANDEMIC

Before the pandemic, a typical class of mine would have around 100 students. We met twice a week, one and a half hours per meeting, for a total of 48 hours in a 16-week semester. Due to the pandemic, our semester has been shortened to 14 weeks. All instruction is done remotely. There are no face-to-face sessions. My current class has 103 students. I have limited synchronous sessions to at most one session every two weeks.

When we worked out the course revisions a few years back, we formed a committee with writers from the math departments in our different campuses to come up with modules (student learning materials) for each topic. We came up with 12 modules. Each module corresponded to a topic in the course outline. We listed the objectives for each topic and lessons were presented not only through text but also with curated open educational resources. We indicated learning and assessment activities and suggested schedules and timetables. Around the same time, we also started producing videos with the help of our colleagues from the UP Open University and TVUP, the university's digital TV channel. Many of these videos were for our Mathematics, Culture and Society course.

These efforts in writing modules and producing videos gave us a head start when the pandemic started and we had to abruptly end face-face-instruction. They were very useful in the new remote delivery mode as we

had a substantial number of suitable learning materials to continue delivering the course. For this semester, we had more time to prepare course packs for our courses. These packs include the course guide with the course outlines, primary and secondary learning materials, study schedule, course requirements, assessment guide, learning tips and so on. We also produced more video materials. In addition there are so many wonderful professionally created materials on the internet that are freely accessible that have been included in our course materials.

Reactions of my current students toward the course are mostly positive, quite similar to previous years. They find many of the lessons interesting and stimulating. I even found their performance on exams better. Their main concerns were mostly about online connectivity. This semester, I have also asked them to do a learning log. From their reflections, I believe that most of goals of this math appreciation course are being met. I think that in the future, even with the possibility of full face-to-face meetings, such a course will be very amenable to a blended mode of delivery.

6. CONCLUSION

I would sincerely suggest, if you are not teaching a similar course, to try designing and offering your own math appreciation course. It need not correspond exactly to the course I described. But as the CUPM report stressed, *"do not accept anyone else's blueprint"*; adding, *"if you can communicate, in your own way, why you believe that mathematics is beautiful and important, the course will fulfill its purpose"* [10].

With careful handling by competent and caring teachers, the course will play a role in influencing student attitudes toward math, and help break down walls between disciplines. The course will help students how to learn mathematics as they gain broader and critical knowledge of mathematics' nature and development. With positive attitudes gained or reinforced, foundations are built that will be useful for the acquisition and improvement of mathematical skills, even in education's new normal.

Good mathematics appreciation courses continue to have value and relevance. Through these courses, students will have a broader and deeper awareness and appreciation of mathematics – its true nature, its role in society, its strengths and limitations, and its connections with other disciplines – allowing them to see and experience the beauty and power of mathematics.

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