


## Research Article

# Order- $\alpha_{CQ}$ Divergence Measures and Aggregation Operators Based on Complex q-Rung Orthopair Normal Fuzzy Sets and Their Application to Multi-Attribute Decision-Making

Zeeshan Ali<sup>1</sup>, Tahir Mahmood<sup>1, </sup>, Abdu Gumaei<sup>2,\*, </sup>

<sup>1</sup>Department of Mathematics and Statistics, International Islamic University, Islamabad, Pakistan

<sup>2</sup>STC's Artificial Intelligence Chair, Department of Information Systems, College of Computer and Information Sciences, King Saud University, Riyadh, Saudi Arabia

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## ABSTRACT

Complex q-rung orthopair fuzzy set (CQROFS) contains the grade of supporting and the grade of supporting against in the form of polar coordinates belonging to unit disc in a complex plane and is a proficient technique to address awkward information, although the normal fuzzy number (NFN) examines normal distribution information in anthropogenic action and a realistic environment. Based on the advantages of both notions, in this manuscript, we explored the novel concept of a complex q-rung orthopair normal fuzzy set (CQRONFS) as an imperative technique to evaluate unreliable and complicated information. Some operational laws based on CQRONFSs are also explored. Additionally, some distance measures, called complex q-rung orthopair normal fuzzy generalized distance measure (CQRONFGDM), complex q-rung orthopair normal fuzzy symmetric distance measure (CQRONFSDM), two types of complex q-rung orthopair normal fuzzy order- divergence measures (CQRONFODMs), and their special cases are discussed. Moreover, weighted averaging, weighted geometric, generalized weighted averaging, and generalized weighted geometric operators based on CQRONFSs are also presented. In last, we solved a numerical example of a multi-attribute decision-making (MADM) problem is shown to justify the proficiency of the presented operators. The advantages, comparative and sensitive analyses are used to express the efficiency and flexibility of the explored approach.

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## 1. INTRODUCTION

In our everyday life, most people are regularly confronted with MADM issues, which may include various other options and numerous assessment components. Because of the multifaceted nature of human social exercises and the vulnerability of indigenous habitats, the way to manage such dubious data has gotten the key to taking care of the MADM issues. Zadeh [1] explored a supporting-based fuzzy set (FS), which viably portrayed the fuzzy data and dubious condition, and hence the advantage to suggest a superior choice. Additionally, Atanassov [2] modified the theory of FS to intuitionistic FS (IFS) containing three components, that is, participation degree, nonenrollment degree, and hesitation degree. IFS has been broadly considered, and various scholars have extended into different kinds of notions [3–6]. Additionally, Yager [7] situated the modified version of IFS is called Pythagorean FS (PFS) and meet the conditions that the square sum of its supporting grade and supporting against grade is not exceeded from unit interval. Several scholars have utilized it in different fields [8–12]. Further, Yager [13] situated the modified version of PFS is called q-rung orthopair FS (QROFS) and meet the conditions that the q-power sum of its supporting grade and supporting against grade is not exceeded from unit interval. Several scholars have utilized it in different fields [14–18].

From the above winning investigations, it has been breaking down that the above examination has been led under the uncertainties or their expansions which can just arrangement with the vulnerability that exists in the data. None of these models can speak to the incomplete numbness of the information and its variances at a given period. Nonetheless, in complex informational indexes, for example, information from the clinical research, database for biometric and facial acknowledgment, and so on. Vulnerability and dubiousness in the information happen simultaneously with changes to the stage (periodicity) of the information. To deal with the periodicity of the information into the decision-making problems, Ramot *et al.* [19] presented the idea of the complex fuzzy set (CFS), a modified version of the FS, displayed by complex-valued supporting grade with a co-domain unit circle in an unpredictable plane. Additionally, Alkouri and Salleh [20] modified the theory of CFS to complex IFS (CIFS) containing three components, that is, complex-valued participation degree, complex-valued nonenrollment degree, and complex-valued hesitation degree. CIFS has been broadly considered and various scholars have extended it into different kinds of notions [21–23]. Additionally, Ullah *et al.* [24] situated the modified version of CIFS is called complex PFS (CPFS) and

\*Corresponding author: Email: [agumaei@ksu.edu.sa](mailto:agumaei@ksu.edu.sa)

meet the conditions that the square sum of its real part (also for the imaginary part) of the supporting grade and real part (also for the imaginary part) of the supporting against grade is not exceeded form unit interval. Several scholars have utilized it in different fields [25]. Further, Liu *et al.* [26,27] situated the modified version of CPFS is called complex QROFS (CQROFS) and meet the conditions that the q-power sum of its real part (also for the imaginary part) of the supporting grade and real part (also for the imaginary part) of the supporting against grade is not exceeded form unit interval. Several scholars have utilized it in different fields [28].

The point of this exploration is to introduce a novel decision-making technique to take care of the MADM issues utilizing CQRONFSs which is a mixture of CQROFSs and normal fuzzy numbers (NFNs) [29] with powerful averaging and geometric operators. The CQRONFSs is a speculation of the QRONFS thinking about the supporting grade and supporting against are complex-valued and are expressed in polar coordinates with NFNs. The amplitude term gives the degree of belongingness of an item in a CQRONFS and the stage terms are commonly identified with periodicity. These stage terms recognize the CQRONFS and customary QRONFS theories. Uncertainties hypothesis manages just each measurement in turn which brings about data misfortune in certain examples. Be that as it may, all things considered, we run over complex characteristic marvels where it gets basic to add the second measurement to the declaration of participation and non-enrollment grades. By presenting this subsequent measurement, the total data can be expert projected in one set, and henceforth, loss of data can stay away from. To delineate the essentialness of the stage term, we give a model. “Consider XYZ organization chooses to set up biometric-based participation gadgets (BBPGs) in the entirety of its workplaces spread everywhere throughout the nation. For this, the organization counsels a specialist who gives the data regarding (i) demonstrates of BBPGs and (ii) creation dates of BBPGs. The organization needs to choose the most ideal model of BBPGs with its creation date all the while. Here, the issue is two-dimensional, which cannot be demonstrated at the same time utilizing customary QRONFS theories. The most ideal approach to speak to the entirety of the data gave by the master is by utilizing CQRONFS theories. The amplitude terms in CQRONFS might be utilized to give the organization’s choice concerning the model of BBPGs, and the stage terms might be utilized to speak to the organization’s judgment concerning creation date of BBPGs.”

When a decision-maker gives  $0.0834e^{i2\pi(0.1005)}$  for truth grade,  $0.1487e^{i2\pi(0.304)}$  for falsity grade, and  $(0.6845, 0.1718)$  for a NFN, then the existing notions are cannot be able to cope with it. In the writing, there are a couple of techniques qualified for dealing with the MADM issues by utilizing the idea of CQRONFS. Because the explored idea of CQRONFS is more powerful and more general than existing notions, whose detail is discussed below:

1. If we choose the value of the imaginary part will be zero in CQRONFS, then the CQRONFS is converted for q-rung orthopair normal FS.
2. If we choose the value of the imaginary part will be zero in CQRONFS for  $q = 2$ , then the CQRONFS is converted for the Pythagorean normal FS.
3. If we choose the value of the imaginary part will be zero in CQRONFS for  $q = 1$ , then the CQRONFS is converted for an intuitionistic normal FS.
4. If we choose the value of the  $q = 2$  in CQRONFS, then the CQRONFS is converted for a complex Pythagorean normal FS.
5. If we choose the value of the  $q = 1$  in CQRONFS, then the CQRONFS is converted for a complex intuitionistic normal FS.
6. If we choose the value of normal FS will be zero in CQRONFS, then the CQRONFS is converted for CQROFS.
7. If we choose the value of the normal FS will be zero in CQRONFS for  $q = 2$ , then the CQRONFS is converted for a complex PFS.
8. If we choose the value of the normal FS will be zero in CQRONFS for  $q = 1$ , then the CQRONFS is converted for a complex IFS. The geometrical expression of the unit disc is discussed in the form of Figure 1.

In this way, inspired by the attributes of the CQRONFS model and the significance of data aggregation, this paper centers around investigating the basic qualities of CQRONFSs and their aggregation operators (AOs) for dealing with the multidimensional complex informational collections. The primary accomplishments of this examination are:

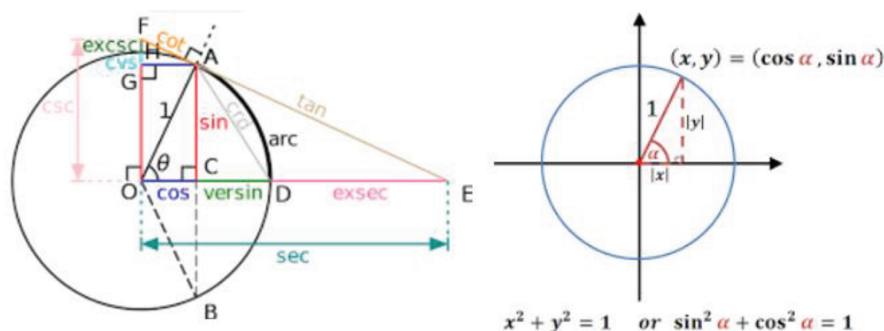


Figure 1 | Expressions of the unit disc in complex plane in unit disc.

1. To handle the uncertainties in a more precise environment using CQRONFSs and their fundamental properties are explored.
2. To explore some new Order- $\alpha_{CQ}$  divergence measures to combine the preferences. We have explored the TOPSIS Method (see Section 6), and the AO and distance measures are part of the TOPSIS method, so this is the reason, why we have explored two different concepts in one work.
3. To explore some new AOs to combine the preferences.
4. To explore an efficient algorithm based on explored measures and operators to solve MADM problems.
5. To illustrate the approach with a numerical example for evaluating the proficiency and reliability of the presented approaches.

To achieve these objectives, we provide more flexibility to the decision-maker to provide their preferences in terms of CQRONFSs to achieve the first objective. The second objective is done by proposing some new Order- $\alpha_{CQ}$  divergence measures based on CQRONFSs. The third objective is done by proposing some new AOs, namely, weighted averaging, weighted geometric, generalized weighted averaging, and generalized weighted geometric operators based on CQRONFSs are also presented to aggregate the different CQRONFSs. Various relations and properties are also studied in it. The fourth objective is completed by developing an efficient algorithm based on explored measures and operators to solve MADM problems for finding to rank the alternatives by using the proposed approaches. Finally, the feasibility and the comparative analysis have been done to fulfill the fourth objective with several existing studies.

The remaining text is outlined as follows: in Section 2, we review some notions like NFN, CQROFS, and their operational laws. In Section 3, we explored the novel concept of CQRONFS is an important technique to evaluate unreliable and complicated information. Some operational laws based on CQRONFSs are also explored. In Section 4, some distance measures are called CQRONFGDM, CQROFNFSMDM, two types of CQRONFODMs, and their special cases are discussed. In Section 5, some weighted averaging, weighted geometric, generalized weighted averaging, and generalized weighted geometric operators based on CQRONFSs are also presented. In Section 6, we solve a numerical example on multi-attribute decision-making (MADM) problem is shown to justify the proficiency of the presented operators. The advantages, comparative and sensitive analysis are used to express the efficiency and flexibility of the explored approach. The conclusion of this article is discussed in Section 7.

## 2. PRELIMINARIES

The purpose of this communication is to review some notions like NFN, CQROFS, and their operational laws. Throughout, this manuscript, the symbol  $\mathcal{X}_{UNI}$  denotes the universal set.

**Definition 1.** [29] A NFN  $\mathcal{N}_{NF} = (\Psi, \Omega)$  on a finite universal set  $\mathcal{X}_{UNI}$  based on truth function of fuzzy number is stated by

$$\mathcal{N}_{NF}(x) = e^{-\left(\frac{x-\Psi}{\Omega}\right)^2} \quad (1)$$

where  $\mathcal{R}_{RN}$  denotes the real number set with  $\Omega, \gamma_{SC} > 0$ .

**Definition 2.** [29] For any two NFNs  $\mathcal{N}_{NF-1} = (\Psi_1, \Omega_1)$  and  $\mathcal{N}_{NF-2} = (\Psi_2, \Omega_2)$ , then

$$\gamma_{SC} \mathcal{N}_{NF-1} = (\gamma_{SC} \Psi_1, \gamma_{SC} \Omega_1) \quad (2)$$

$$\mathcal{N}_{NF-1} + \mathcal{N}_{NF-2} = (\Psi_1 + \Psi_2, \Omega_1 + \Omega_2) \quad (3)$$

The distance measure based on NFNs is stated by

$$\mathcal{d}_{DM}(\mathcal{N}_{NF-1}, \mathcal{N}_{NF-2}) = (\Psi_1 - \Psi_2)^2 + \frac{1}{2}(\Omega_1 - \Omega_2)^2 \quad (4)$$

**Definition 3.** [26,27] The CQROFS  $\mathcal{N}_{CQ}$  is stated by

$$\mathcal{N}_{CQ} = \left\{ \left( \phi'_{\mathcal{N}_{CQ}}(x_i), \psi'_{\mathcal{N}_{CQ}}(x_i) \right) : x \in \mathcal{X}_{UNI} \right\} \quad (5)$$

where  $\phi'_{\mathcal{N}_{CQ}}(x_i) = \phi_{\mathcal{N}_{CQ}}(x_i) e^{i2\pi\eta_{\phi_{\mathcal{N}_{CQ}}}(x_i)}$  denotes the complex-valued truth degree and  $\psi'_{\mathcal{N}_{CQ}}(x_i) = \psi_{\mathcal{N}_{CQ}}(x_i) e^{i2\pi\eta_{\psi_{\mathcal{N}_{CQ}}}(x_i)}$  denotes the complex-valued falsity degree, which holds the following conditions:  $0 \leq \phi_{\mathcal{N}_{CQ}}^{q_{SC}}(x_i) + \psi_{\mathcal{N}_{CQ}}^{q_{SC}}(x_i) \leq 1$  and  $0 \leq \eta_{\phi_{\mathcal{N}_{CQ}}}^{q_{SC}}(x_i) + \eta_{\psi_{\mathcal{N}_{CQ}}}^{q_{SC}}(x_i) \leq 1$ .

Further,  $\mu_{\mathcal{N}_{CQ}}(x_i) = \left(1 - \phi_{\mathcal{N}_{CQ}}^{q_{SC}}(x_i) - \psi_{\mathcal{N}_{CQ}}^{q_{SC}}(x_i)\right)^{\frac{1}{q_{SC}}} e^{i2\pi \left(1 - \eta_{\phi, \mathcal{N}_{CQ}}^{q_{SC}}(x_i) - \eta_{\psi, \mathcal{N}_{CQ}}^{q_{SC}}(x_i)\right)^{\frac{1}{q_{SC}}}}$  represents the hesitancy degree. The complex q-rung orthopair fuzzy number (CQROFNs) is followed as  $\mathcal{N}_{CQ} = \left(\phi_{\mathcal{N}_{CQ}} e^{i2\pi \eta_{\phi, \mathcal{N}_{CQ}}}, \psi_{\mathcal{N}_{CQ}} e^{i2\pi \eta_{\psi, \mathcal{N}_{CQ}}}\right)$ .

**Definition 4.** [26,27] For any two CQROFNs  $\mathcal{N}_{CQ-1} = \left(\phi_{\mathcal{N}_{CQ-1}} e^{i2\pi \eta_{\phi, \mathcal{N}_{CQ-1}}}, \psi_{\mathcal{N}_{CQ-1}} e^{i2\pi \eta_{\psi, \mathcal{N}_{CQ-1}}}\right)$  and  $\mathcal{N}_{CQ-2} = \left(\phi_{\mathcal{N}_{CQ-2}} e^{i2\pi \eta_{\phi, \mathcal{N}_{CQ-2}}}, \psi_{\mathcal{N}_{CQ-2}} e^{i2\pi \eta_{\psi, \mathcal{N}_{CQ-2}}}\right)$ , then

$$\mathcal{N}_{CQ-1} \oplus_{CQ} \mathcal{N}_{CQ-2} = \left( \left( \phi_{\mathcal{N}_{CQ-1}}^{q_{SC}} + \phi_{\mathcal{N}_{CQ-2}}^{q_{SC}} - \phi_{\mathcal{N}_{CQ-1}}^{q_{SC}} \phi_{\mathcal{N}_{CQ-2}}^{q_{SC}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( \eta_{\phi, \mathcal{N}_{CQ-1}}^{q_{SC}} + \eta_{\phi, \mathcal{N}_{CQ-2}}^{q_{SC}} - \eta_{\phi, \mathcal{N}_{CQ-1}}^{q_{SC}} \eta_{\phi, \mathcal{N}_{CQ-2}}^{q_{SC}} \right)^{\frac{1}{q_{SC}}}}, \left( \psi_{\mathcal{N}_{CQ-1}} \psi_{\mathcal{N}_{CQ-2}} \right) e^{i2\pi \left( \eta_{\psi, \mathcal{N}_{CQ-1}} \eta_{\psi, \mathcal{N}_{CQ-2}} \right)} \right) \quad (6)$$

$$\mathcal{N}_{CQ-1} \otimes_{CQ} \mathcal{N}_{CQ-2} = \left( \left( \phi_{\mathcal{N}_{CQ-1}} \phi_{\mathcal{N}_{CQ-2}} \right) e^{i2\pi \left( \eta_{\phi, \mathcal{N}_{CQ-1}} \eta_{\phi, \mathcal{N}_{CQ-2}} \right)}, \left( \psi_{\mathcal{N}_{CQ-1}}^{q_{SC}} + \psi_{\mathcal{N}_{CQ-2}}^{q_{SC}} - \psi_{\mathcal{N}_{CQ-1}}^{q_{SC}} \psi_{\mathcal{N}_{CQ-2}}^{q_{SC}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( \eta_{\psi, \mathcal{N}_{CQ-1}}^{q_{SC}} + \eta_{\psi, \mathcal{N}_{CQ-2}}^{q_{SC}} - \eta_{\psi, \mathcal{N}_{CQ-1}}^{q_{SC}} \eta_{\psi, \mathcal{N}_{CQ-2}}^{q_{SC}} \right)^{\frac{1}{q_{SC}}}} \right) \quad (7)$$

$$\gamma_{SC} \mathcal{N}_{CQ-1} = \left( \left( 1 - \left( 1 - \phi_{\mathcal{N}_{CQ-1}}^{q_{SC}} \right)^{\gamma_{SC}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( 1 - \left( 1 - \eta_{\phi, \mathcal{N}_{CQ-1}}^{q_{SC}} \right)^{\gamma_{SC}} \right)^{\frac{1}{q_{SC}}}}, \left( \psi_{\mathcal{N}_{CQ-1}}^{\gamma_{SC}} \right) e^{i2\pi \left( \eta_{\psi, \mathcal{N}_{CQ-1}} \right)^{\gamma_{SC}}} \right) \quad (8)$$

$$\mathcal{N}_{CQ-1}^{\gamma_{SC}} = \left( \left( \phi_{\mathcal{N}_{CQ-1}}^{\gamma_{SC}} \right) e^{i2\pi \left( \eta_{\phi, \mathcal{N}_{CQ-1}} \right)^{\gamma_{SC}}}, \left( 1 - \left( 1 - \psi_{\mathcal{N}_{CQ-1}}^{q_{SC}} \right)^{\gamma_{SC}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( 1 - \left( 1 - \eta_{\psi, \mathcal{N}_{CQ-1}}^{q_{SC}} \right)^{\gamma_{SC}} \right)^{\frac{1}{q_{SC}}}} \right) \quad (9)$$

**Definition 5.** [26,27] For any CQROFN  $\mathcal{N}_{CQ-1} = \left(\phi_{\mathcal{N}_{CQ-1}} e^{i2\pi \eta_{\phi, \mathcal{N}_{CQ-1}}}, \psi_{\mathcal{N}_{CQ-1}} e^{i2\pi \eta_{\psi, \mathcal{N}_{CQ-1}}}\right)$ , then the score function  $\mathcal{S}_{SF}$  and accuracy function  $\mathcal{H}_{AF}$  are stated by

$$\mathcal{S}_{SF}(\mathcal{N}_{CQ-1}) = \frac{\phi_{\mathcal{N}_{CQ-1}}^{q_{SC}} - \psi_{\mathcal{N}_{CQ-1}}^{q_{SC}} + \eta_{\phi, \mathcal{N}_{CQ-1}}^{q_{SC}} - \eta_{\psi, \mathcal{N}_{CQ-1}}^{q_{SC}}}{2} \quad (10)$$

$$\mathcal{H}_{AF}(\mathcal{N}_{CQ-1}) = \frac{\phi_{\mathcal{N}_{CQ-1}}^{q_{SC}} + \psi_{\mathcal{N}_{CQ-1}}^{q_{SC}} + \eta_{\phi, \mathcal{N}_{CQ-1}}^{q_{SC}} + \eta_{\psi, \mathcal{N}_{CQ-1}}^{q_{SC}}}{2} \quad (11)$$

For finding the relationships between any two CQROFNs, we use the following inequalities:

1. If  $\mathcal{S}_{SF}(\mathcal{N}_{CQ-1}) > \mathcal{S}_{SF}(\mathcal{N}_{CQ-2}) \Rightarrow \mathcal{N}_{CQ-1} > \mathcal{N}_{CQ-2}$ ;
2. If  $\mathcal{S}_{SF}(\mathcal{N}_{CQ-1}) = \mathcal{S}_{SF}(\mathcal{N}_{CQ-2}) \Rightarrow$ 
  - 1) If  $\mathcal{H}_{AF}(\mathcal{N}_{CQ-1}) > \mathcal{H}_{AF}(\mathcal{N}_{CQ-2}) \Rightarrow \mathcal{N}_{CQ-1} > \mathcal{N}_{CQ-2}$ ;
  - 2) If  $\mathcal{H}_{AF}(\mathcal{N}_{CQ-1}) = \mathcal{H}_{AF}(\mathcal{N}_{CQ-2}) \Rightarrow \mathcal{N}_{CQ-1} = \mathcal{N}_{CQ-2}$ .

### 3. COMPLEX Q-RUNG ORTHOPAIR NF N

The purpose of this communication is to present the notion of CQRONFN, which is the mixture of CQROFS and NFN to cope with uncertain and awkward information in realistic decision theory. Some basic operational laws for CQRONFN are also explored.

**Definition 6.** The CQRONFN  $\mathcal{N}_{CQN}$  is stated by

$$\mathcal{N}_{CQN} = \left\{ \left( \left( \Psi_{\mathcal{N}_{CQN}}, \Omega_{\mathcal{N}_{CQN}} \right), \left( \phi'_{\mathcal{N}_{CQN}}(x_i), \psi'_{\mathcal{N}_{CQN}}(x_i) \right) \right) : x \in \mathcal{X}_{UNI} \right\} \quad (12)$$

$$\text{where } \phi'_{\mathcal{N}_{CQN}}(x_i) = e^{-\left(\frac{x-\Psi}{\Omega}\right)^2} \left( \phi_{\mathcal{N}_{CQN}}(x_i) e^{i2\pi\eta_{\phi_{\mathcal{N}_{CQN}}}(x_i)} \right) = e^{-\left(\frac{x-\Psi}{\Omega}\right)^2} * \phi_{\mathcal{N}_{CQN}}(x_i) e^{i2\pi\eta_{\phi_{\mathcal{N}_{CQN}}}(x_i)} = \phi_{RP}(x_i) e^{i2\pi\eta_{\phi_{IP}}(x_i)}$$

$$\text{denotes the complex-valued truth degree and } \psi'_{\mathcal{N}_{CQN}}(x_i) = e^{-\left(\frac{x-\Psi}{\Omega}\right)^2} \left( \left( 1 - \left( 1 - \psi_{\mathcal{N}_{CQN}}(x_i) \right) \right) e^{i2\pi \left( 1 - \left( 1 - \eta_{\psi_{\mathcal{N}_{CQN}}}(x_i) \right) \right)} \right) = e^{-\left(\frac{x-\Psi}{\Omega}\right)^2} *$$

$$\left( 1 - \left( 1 - \psi_{\mathcal{N}_{CQN}}(x_i) \right) \right) e^{i2\pi \left( 1 - \left( 1 - \eta_{\psi_{\mathcal{N}_{CQN}}}(x_i) \right) \right)} = \psi_{RP}(x_i) e^{i2\pi\eta_{\psi_{IP}}(x_i)} \text{ denotes the complex-valued falsity degree, which holds the following conditions: } 0 \leq \phi_{RP}^{q_{SC}}(x_i) + \psi_{RP}^{q_{SC}}(x_i) \leq 1 \text{ and } 0 \leq \eta_{\phi_{IP}}^{q_{SC}}(x_i) + \eta_{\psi_{IP}}^{q_{SC}}(x_i) \leq 1. \text{ The complex q-rung orthopair NFN (CQRONFNs) is followed as } \mathcal{N}_{CQN} = \left( \left( \Psi_{\mathcal{N}_{CQN}}, \Omega_{\mathcal{N}_{CQN}} \right), \left( \phi_{RP} e^{i2\pi\eta_{\phi_{IP}}}, \psi_{RP} e^{i2\pi\eta_{\psi_{IP}}} \right) \right).$$

**Definition 7.** For any two CQRONFNs  $\mathcal{N}_{CQN-1} = \left( \left( \Psi_{\mathcal{N}_{CQN-1}}, \Omega_{\mathcal{N}_{CQN-1}} \right), \left( \phi_{RP-1} e^{i2\pi\eta_{\phi_{IP-1}}}, \psi_{RP-1} e^{i2\pi\eta_{\psi_{IP-1}}} \right) \right)$  and  $\mathcal{N}_{CQN-2} = \left( \left( \Psi_{\mathcal{N}_{CQN-2}}, \Omega_{\mathcal{N}_{CQN-2}} \right), \left( \phi_{RP-2} e^{i2\pi\eta_{\phi_{IP-2}}}, \psi_{RP-2} e^{i2\pi\eta_{\psi_{IP-2}}} \right) \right)$ , then

$$\mathcal{N}_{CQN-1} \oplus_{CQN} \mathcal{N}_{CQN-2} = \left( \left( \Psi_{\mathcal{N}_{CQN-1}} + \Psi_{\mathcal{N}_{CQN-2}}, \Omega_{\mathcal{N}_{CQN-1}} + \Omega_{\mathcal{N}_{CQN-2}} \right), \left( \left( \phi_{RP-1}^{q_{SC}} + \phi_{RP-2}^{q_{SC}} - \phi_{RP-1}^{q_{SC}} \phi_{RP-2}^{q_{SC}} \right) \frac{1}{q_{SC}} e^{i2\pi \left( \eta_{\phi_{IP-1}}^{q_{SC}} + \eta_{\phi_{IP-2}}^{q_{SC}} - \eta_{\phi_{IP-1}}^{q_{SC}} \eta_{\phi_{IP-2}}^{q_{SC}} \right) \frac{1}{q_{SC}}}, \left( \psi_{RP-1} \psi_{RP-2} \right) e^{i2\pi \left( \eta_{\psi_{IP-1}} \eta_{\psi_{IP-2}} \right)} \right) \right) \quad (13)$$

$$\mathcal{N}_{CQN-1} \otimes_{CQN} \mathcal{N}_{CQN-2} = \left( \left( \Psi_{\mathcal{N}_{CQN-1}} * \Psi_{\mathcal{N}_{CQN-2}}, \Omega_{\mathcal{N}_{CQN-1}} * \Omega_{\mathcal{N}_{CQN-2}} \right), \left( \left( \phi_{RP-1} \phi_{RP-2} \right) e^{i2\pi \left( \eta_{\phi_{IP-1}} \eta_{\phi_{IP-2}} \right)}, \left( \psi_{RP-1}^{q_{SC}} + \psi_{RP-2}^{q_{SC}} - \psi_{RP-1}^{q_{SC}} \psi_{RP-2}^{q_{SC}} \right) \frac{1}{q_{SC}} e^{i2\pi \left( \eta_{\psi_{IP-1}}^{q_{SC}} + \eta_{\psi_{IP-2}}^{q_{SC}} - \eta_{\psi_{IP-1}}^{q_{SC}} \eta_{\psi_{IP-2}}^{q_{SC}} \right) \frac{1}{q_{SC}}} \right) \right) \quad (14)$$

$$\gamma_{SC} \mathcal{N}_{CQN-1} = \left( \left( \gamma_{SC} \Psi_{\mathcal{N}_{CQN-1}}, \gamma_{SC} \Omega_{\mathcal{N}_{CQN-1}} \right), \left( \left( 1 - \left( 1 - \phi_{RP-1}^{q_{SC}} \right)^{\gamma_{SC}} \right) \frac{1}{q_{SC}} e^{i2\pi \left( 1 - \left( 1 - \eta_{\phi_{IP-1}}^{q_{SC}} \right)^{\gamma_{SC}} \right) \frac{1}{q_{SC}}}, \left( \psi_{RP-1}^{\gamma_{SC}} \right) e^{i2\pi \left( \eta_{\psi_{IP-1}} \right)^{\gamma_{SC}}} \right) \right) \quad (15)$$

$$\mathcal{N}_{CQN-1}^{\gamma_{SC}} = \left( \left( \Psi_{\mathcal{N}_{CQN-1}}^{\gamma_{SC}}, \Omega_{\mathcal{N}_{CQN-1}}^{\gamma_{SC}} \right), \left( \left( \phi_{RP-1}^{\gamma_{SC}} \right) e^{i2\pi \left( \eta_{\phi_{IP-1}} \right)^{\gamma_{SC}}}, \left( 1 - \left( 1 - \psi_{RP-1}^{q_{SC}} \right)^{\gamma_{SC}} \right) \frac{1}{q_{SC}} e^{i2\pi \left( 1 - \left( 1 - \eta_{\psi_{IP-1}}^{q_{SC}} \right)^{\gamma_{SC}} \right) \frac{1}{q_{SC}}} \right) \right) \quad (16)$$

**Definition 8.** For any CQRONFN  $\mathcal{N}_{CQN-1} = \left( \left( \Psi_{\mathcal{N}_{CQN-1}}, \Omega_{\mathcal{N}_{CQN-1}} \right), \left( \phi_{RP-1} e^{i2\pi\eta_{\phi_{IP-1}}}, \psi_{RP-1} e^{i2\pi\eta_{\psi_{IP-1}}} \right) \right)$ , then the score function and accuracy function are stated by

$$\mathcal{S}_{SF}(\mathcal{N}_{CQN-1}) = \frac{\left( \frac{\Psi_{\mathcal{N}_{CQN-1}} + \Omega_{\mathcal{N}_{CQN-1}}}{2} \right) + \left( \phi_{RP-1}^{q_{SC}} - \psi_{RP-1}^{q_{SC}} + \eta_{\phi_{IP-1}}^{q_{SC}} - \eta_{\psi_{IP-1}}^{q_{SC}} \right)}{3} \quad (17)$$

$$\mathcal{H}_{AF}(\mathcal{N}_{CQN-1}) = \frac{\left( \frac{\Psi_{\mathcal{N}_{CQN-1}} + \Omega_{\mathcal{N}_{CQN-1}}}{2} \right) + \left( \phi_{RP-1}^{q_{SC}} - \psi_{RP-1}^{q_{SC}} + \eta_{\phi_{IP-1}}^{q_{SC}} - \eta_{\psi_{IP-1}}^{q_{SC}} \right)}{3} \quad (18)$$

For finding the relationships between any two CQRONFNs, we use the following inequalities:

4. If  $\mathcal{S}_{SF}(\mathcal{N}_{CQN-1}) > \mathcal{S}_{SF}(\mathcal{N}_{CQN-2}) \Rightarrow \mathcal{N}_{CQN-1} > \mathcal{N}_{CQN-2}$ ;
5. If  $\mathcal{S}_{SF}(\mathcal{N}_{CQN-1}) = \mathcal{S}_{SF}(\mathcal{N}_{CQN-2}) \Rightarrow$ 
  - 3) If  $\mathcal{H}_{AF}(\mathcal{N}_{CQN-1}) > \mathcal{H}_{AF}(\mathcal{N}_{CQN-2}) \Rightarrow \mathcal{N}_{CQN-1} > \mathcal{N}_{CQN-2}$ ;
  - 4) If  $\mathcal{H}_{AF}(\mathcal{N}_{CQN-1}) = \mathcal{H}_{AF}(\mathcal{N}_{CQN-2}) \Rightarrow \mathcal{N}_{CQN-1} = \mathcal{N}_{CQN-2}$ .

**Theorem 1.** For any two CQRONFNs  $\mathcal{N}_{CQN-1} = \left( \left( \Psi_{\mathcal{N}_{CQN-1}}, \Omega_{\mathcal{N}_{CQN-1}} \right), \left( \phi_{RP-1} e^{i2\pi\eta_{\phi_{IP-1}}}, \psi_{RP-1} e^{i2\pi\eta_{\psi_{IP-1}}} \right) \right)$  and  $\mathcal{N}_{CQN-2} = \left( \left( \Psi_{\mathcal{N}_{CQN-2}}, \Omega_{\mathcal{N}_{CQN-2}} \right), \left( \phi_{RP-2} e^{i2\pi\eta_{\phi_{IP-2}}}, \psi_{RP-2} e^{i2\pi\eta_{\psi_{IP-2}}} \right) \right)$ , then

$$\mathcal{N}_{CQN-1} \oplus_{CQN} \mathcal{N}_{CQN-2} = \mathcal{N}_{CQN-2} \oplus_{CQN} \mathcal{N}_{CQN-1} \quad (19)$$

$$(\mathcal{N}_{CQN-1} \oplus_{CQN} \mathcal{N}_{CQN-2}) \oplus_{CQN} \mathcal{N}_{CQN-3} = \mathcal{N}_{CQN-1} \oplus_{CQN} (\mathcal{N}_{CQN-2} \oplus_{CQN} \mathcal{N}_{CQN-3}) \quad (20)$$

$$\mathcal{N}_{CQN-1} \otimes_{CQN} \mathcal{N}_{CQN-2} = \mathcal{N}_{CQN-2} \otimes_{CQN} \mathcal{N}_{CQN-1} \quad (21)$$

$$(\mathcal{N}_{CQN-1} \otimes_{CQN} \mathcal{N}_{CQN-2}) \otimes_{CQN} \mathcal{N}_{CQN-3} = \mathcal{N}_{CQN-1} \otimes_{CQN} (\mathcal{N}_{CQN-2} \otimes_{CQN} \mathcal{N}_{CQN-3}) \quad (22)$$

$$\gamma_{SC}(\mathcal{N}_{CQN-1} \oplus_{CQN} \mathcal{N}_{CQN-2}) = \gamma_{SC} \mathcal{N}_{CQN-1} \oplus_{CQN} \gamma_{SC} \mathcal{N}_{CQN-2} \quad (23)$$

$$(\gamma_{SC-1} + \gamma_{SC-2}) \mathcal{N}_{CQN-1} = \gamma_{SC-1} \mathcal{N}_{CQN-1} + \gamma_{SC-1} \mathcal{N}_{CQN-1} \quad (24)$$

$$(\mathcal{N}_{CQN-1} \otimes_{CQN} \mathcal{N}_{CQN-2})^{\gamma_{SC}} = \mathcal{N}_{CQN-1}^{\gamma_{SC}} \otimes_{CQN} \mathcal{N}_{CQN-2}^{\gamma_{SC}} \quad (25)$$

**Proof.** Straightforward.

## 4. DISTANCE MEASURES BASED ON CQRONFNs

The purpose of this section is to explore some distance measures are called CQRONFGDM, CQROFNFSDM, complex q-rung orthopair normal fuzzy Order- $\alpha_{CQ}$  divergence measures  $\mathcal{D}_{GDM}^{SDM-1}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2})$ , and  $\mathcal{D}_{GDM}^{SDM-2}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2})$  and their special cases are discussed. Further, these all measures are also converted into similarity measures.

**Definition 9.** For any two CQRONFNs  $\mathcal{N}_{CQN-1} = \left( \left( \Psi_{\mathcal{N}_{CQN-1}}, \Omega_{\mathcal{N}_{CQN-1}} \right), \left( \phi_{RP-1} e^{i2\pi\eta_{\phi_{IP-1}}}, \psi_{RP-1} e^{i2\pi\eta_{\psi_{IP-1}}} \right) \right)$  and  $\mathcal{N}_{CQN-2} = \left( \left( \Psi_{\mathcal{N}_{CQN-2}}, \Omega_{\mathcal{N}_{CQN-2}} \right), \left( \phi_{RP-2} e^{i2\pi\eta_{\phi_{IP-2}}}, \psi_{RP-2} e^{i2\pi\eta_{\psi_{IP-2}}} \right) \right)$ , the CQRONFGDM is stated by

$$\mathcal{d}_{GDM}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}) = \frac{1}{2} \left( \left( \frac{1 + \phi_{RP-1}^{q_{SC}} - \psi_{RP-1}^{q_{SC}} + \eta_{\phi_{IP-1}}^{q_{SC}} - \eta_{\psi_{IP-1}}^{q_{SC}}}{2} * \Psi_{\mathcal{N}_{CQN-1}} \right)^{q_{SC}} - \left( \frac{1 + \phi_{RP-2}^{q_{SC}} - \psi_{RP-2}^{q_{SC}} + \eta_{\phi_{IP-2}}^{q_{SC}} - \eta_{\psi_{IP-2}}^{q_{SC}}}{2} * \Omega_{\mathcal{N}_{CQN-1}} \right)^{q_{SC}} + \frac{1}{2} \left( \Psi_{\mathcal{N}_{CQN-2}} + \Psi_{\mathcal{N}_{CQN-2}} \right) \right)^{\frac{1}{q_{SC}}} \quad (26)$$

Eq. (26), must hold the following conditions:

1.  $\mathcal{d}_{GDM}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}) \geq 0$ ;
2.  $\mathcal{d}_{GDM}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}) = \mathcal{d}_{GDM}(\mathcal{N}_{CQN-2}, \mathcal{N}_{CQN-1})$ ;
3.  $\mathcal{d}_{GDM}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}) = 0 \Leftrightarrow \mathcal{N}_{CQN-1} = \mathcal{N}_{CQN-2}$ .

In the CQRONFS hypothesis, enrollment and nonmembership degrees are intricate esteemed and are spoken to in polar directions. The abundance term comparing to the participation (nonmembership) degree gives the degree of things (not-belongings) of an item in a CQRONFS, and the stage term related to enrollment (nonmembership) degree gives the extra data, by and large, related with periodicity. The stage terms are novel boundaries of the participation and nonmembership degrees, and these are the boundaries that recognize the customary QRONFS and CQRONFS hypothesis. QRONFS hypothesis manages just each measurement in turn, which brings about data misfortune on certain occasions. In any case, in day-to-day life, we run over complex normal marvels where it gets fundamental to add the second measurement to the statement of participation and nonmembership grades. By presenting this subsequent measurement, the total data can be extended in one set, and henceforth, loss of data can be maintained a strategic distance from. To show the hugeness of the stage term, consider an illustration of a specific organization that chooses to put in new information handling and examination programming. For this, the organization counsels a specialist who gives the data concerning (a) alternate choices of programing (b) relating programing variant. The organization needs to choose the most ideal alternative(s) of programing with its most recent form all the while. Here, the issue is two-dimensional, to be specific, to choose the ideal option of programing and its most recent form. This issue cannot be displayed precisely utilizing the conventional QRONFS hypothesis. Along these lines, the most ideal approach to speak to all the data gave by the master is by utilizing the CQRONFS hypothesis. The adequacy terms in CQRONFS might be utilized to give an organization's choice concerning the option of programing and the stage terms might be utilized to speak to the organization's choice regarding programing adaptation. Various researchers have used various sorts of measures in the fields of FS hypothesis and their expansions. However, cutting-edge nobody investigated the veers estimates dependent on proposed thoughts because the proposed thoughts are more summed up than existing thoughts.

**Definition 10.** For any two CQRONFNs  $\mathcal{N}_{CQN-1} = \left( \left( \Psi_{\mathcal{N}_{CQN-1}}, \Omega_{\mathcal{N}_{CQN-1}} \right), \left( \phi_{RP-1} e^{i2\pi\eta_{\phi_{IP-1}}}, \psi_{RP-1} e^{i2\pi\eta_{\psi_{IP-1}}} \right) \right)$  and  $\mathcal{N}_{CQN-2} = \left( \left( \Psi_{\mathcal{N}_{CQN-2}}, \Omega_{\mathcal{N}_{CQN-2}} \right), \left( \phi_{RP-2} e^{i2\pi\eta_{\phi_{IP-2}}}, \psi_{RP-2} e^{i2\pi\eta_{\psi_{IP-2}}} \right) \right)$ , the complex q-rung orthopair normal fuzzy symmetric divergence measure  $\mathcal{A}_{GDM}^{SDM}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2})$  is given by

$$\mathcal{A}_{GDM}^{SDM}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}) = \frac{1}{2} \left( \mathcal{A}_{GDM}^{SDM-1}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}) + \mathcal{A}_{GDM}^{SDM-2}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}) \right) \quad (27)$$

**Definition 11.** For any two CQRONFNs  $\mathcal{N}_{CQN-1} = \left( \left( \Psi_{\mathcal{N}_{CQN-1}}, \Omega_{\mathcal{N}_{CQN-1}} \right), \left( \phi_{RP-1} e^{i2\pi\eta_{\phi_{IP-1}}}, \psi_{RP-1} e^{i2\pi\eta_{\psi_{IP-1}}} \right) \right)$  and  $\mathcal{N}_{CQN-2} = \left( \left( \Psi_{\mathcal{N}_{CQN-2}}, \Omega_{\mathcal{N}_{CQN-2}} \right), \left( \phi_{RP-2} e^{i2\pi\eta_{\phi_{IP-2}}}, \psi_{RP-2} e^{i2\pi\eta_{\psi_{IP-2}}} \right) \right)$  with  $\mathcal{X}_{UNI} = \{x\}$ , the complex q-rung orthopair normal fuzzy Order- $\alpha_{CQ}$  divergence measures  $\mathcal{A}_{GDM}^{SDM-1}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2})$  and  $\mathcal{A}_{GDM}^{SDM-2}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2})$  are given by

$$\begin{aligned} & \mathcal{A}_{GDM}^{SDM-1}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}) \\ &= \frac{1}{\alpha_{CQ}-1} \log_2 \left( \left( \left( \frac{\phi_{RP-1}^{qSC} + \eta_{\phi_{IP-1}}^{qSC}}{2} * \Psi_{\mathcal{N}_{CQN-1}} \right)^{\alpha_{CQ}} \left( \frac{\phi_{RP-1}^{qSC} + \eta_{\phi_{IP-1}}^{qSC} + \phi_{RP-2}^{qSC} + \eta_{\phi_{IP-2}}^{qSC}}{4} * \Omega_{\mathcal{N}_{CQN-1}} \right)^{1-\alpha_{CQ}} \right)^{\alpha_{CQ}} \right. \\ & \quad \left. + \left( \left( \frac{\psi_{RP-1}^{qSC} + \eta_{\psi_{IP-1}}^{qSC}}{2} * \Psi_{\mathcal{N}_{CQN-2}} \right)^{\alpha_{CQ}} \left( \frac{\psi_{RP-1}^{qSC} + \eta_{\psi_{IP-1}}^{qSC} + \psi_{RP-2}^{qSC} + \eta_{\psi_{IP-2}}^{qSC}}{4} * \Omega_{\mathcal{N}_{CQN-2}} \right)^{1-\alpha_{CQ}} \right)^{\alpha_{CQ}} \right) \end{aligned} \quad (28)$$

$$\mathcal{A}_{GDM}^{SDM-2}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}) = \frac{1}{e^{2^{\alpha_{CQ}-1}} - e} \left( e^{\left( \left( \left( \frac{\phi_{RP-1}^{qSC} + \eta_{\phi_{IP-1}}^{qSC}}{2} * \Psi_{\mathcal{N}_{CQN-1}} \right)^{\alpha_{CQ}} \left( \frac{\phi_{RP-1}^{qSC} + \eta_{\phi_{IP-1}}^{qSC} + \phi_{RP-2}^{qSC} + \eta_{\phi_{IP-2}}^{qSC}}{4} * \Omega_{\mathcal{N}_{CQN-1}} \right)^{1-\alpha_{CQ}} \right)^{\alpha_{CQ}} + \left( \left( \frac{\psi_{RP-1}^{qSC} + \eta_{\psi_{IP-1}}^{qSC}}{2} * \Psi_{\mathcal{N}_{CQN-2}} \right)^{\alpha_{CQ}} \left( \frac{\psi_{RP-1}^{qSC} + \eta_{\psi_{IP-1}}^{qSC} + \psi_{RP-2}^{qSC} + \eta_{\psi_{IP-2}}^{qSC}}{4} * \Omega_{\mathcal{N}_{CQN-2}} \right)^{1-\alpha_{CQ}} \right)^{\alpha_{CQ}} \right)} - e \right) \quad (29)$$

where  $\alpha_{CQ} \in (0, 1)$  and  $e$  represents the exponential functions.

Eqs. (27–29) are also satisfied the three conditions of Definition 9. Further, we have discussed some special cases of the explored measures which are discussed below.

$$\mathbb{S}_{GSM}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}) = 1 - \mathcal{A}_{GDM}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}) \quad (30)$$

Is called complex q-rung orthopair normal fuzzy generalized similarity measure. Similarly, we can find more similarity measures from Eqs. (27–29).



## 5. AOs BASED ON CQRONFNs

The purpose of this communication is to present some AOs based on CQRONFNs is called complex q-rung orthopair normal fuzzy weighted averaging (CQRONFWA), complex q-rung orthopair normal fuzzy weighted geometric (CQRONFWG), complex q-rung orthopair normal fuzzy generalized weighted averaging (CQRONFGWA), complex q-rung orthopair normal fuzzy generalized weighted geometric (CQRONFGWG) operators, and their special cases.

**Definition 12.** For any family of CQRONFNs  $\mathcal{N}_{CQN-j} = \left( \left( \Psi_{\mathcal{N}_{CQN-j}}, \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \phi_{RP-j} e^{i2\pi\eta_{\phi_{IP-j}}}, \psi_{RP-j} e^{i2\pi\eta_{\psi_{IP-j}}} \right) \right), j = 1, 2, \dots, n$ , then the CQRONFWA operator is stated by

$$CQRONFWA(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) = \sum_{j=1}^n \omega_{W-j} \mathcal{N}_{CQN-j} \quad (31)$$

where  $\omega_W = \{\omega_{W-1}, \omega_{W-2}, \dots, \omega_{W-n}\}, \omega_{W-j} \in [0, 1]$  with a condition that is  $\sum_{j=1}^n \omega_{W-j} = 1$ .

**Theorem 2.** For any family of CQRONFNs  $\mathcal{N}_{CQN-j} = \left( \left( \Psi_{\mathcal{N}_{CQN-j}}, \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \phi_{RP-j} e^{i2\pi\eta_{\phi_{IP-j}}}, \psi_{RP-j} e^{i2\pi\eta_{\psi_{IP-j}}} \right) \right), j = 1, 2, \dots, n$ , then by using the Eq. (31), we get

$$CQRONFWA(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) = \left( \left( \sum_{j=1}^n \omega_{W-j} \Psi_{\mathcal{N}_{CQN-j}}, \sum_{j=1}^n \omega_{W-j} \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \left( 1 - \prod_{j=1}^n \left( 1 - \phi_{RP-j}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( 1 - \prod_{j=1}^n \left( 1 - \eta_{\phi_{IP-j}}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}}}, \prod_{j=1}^n \psi_{RP-j} e^{i2\pi \prod_{j=1}^n \eta_{\psi_{IP-j}}} \right) \right) \quad (32)$$

**Proof.** By using the mathematical induction, we have proven the Eq. (32), if  $n = 2$ , then we have

$$\begin{aligned} CQRONFWA(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}) &= \omega_{W-1} \mathcal{N}_{CQN-1} \oplus_{CQN} \omega_{W-2} \mathcal{N}_{CQN-2} \\ &= \left( \left( \left( \omega_{W-1} \Psi_{\mathcal{N}_{CQN-1}}, \omega_{W-1} \Omega_{\mathcal{N}_{CQN-1}} \right), \left( \left( 1 - \prod_{j=1}^n \left( 1 - \phi_{RP-j}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( 1 - \prod_{j=1}^n \left( 1 - \eta_{\phi_{IP-j}}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}}}, \prod_{j=1}^n \psi_{RP-j} e^{i2\pi \prod_{j=1}^n \eta_{\psi_{IP-j}}} \right) \right) \right. \\ &\quad \left. \oplus_{CQN} \left( \left( \omega_{W-2} \Psi_{\mathcal{N}_{CQN-2}}, \omega_{W-2} \Omega_{\mathcal{N}_{CQN-2}} \right), \left( \left( 1 - \prod_{j=1}^n \left( 1 - \phi_{RP-j}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( 1 - \prod_{j=1}^n \left( 1 - \eta_{\phi_{IP-j}}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}}}, \prod_{j=1}^n \psi_{RP-j} e^{i2\pi \prod_{j=1}^n \eta_{\psi_{IP-j}}} \right) \right) \right) \\ &= \left( \left( \left( \omega_{W-1} \Psi_{\mathcal{N}_{CQN-1}} + \omega_{W-2} \Psi_{\mathcal{N}_{CQN-2}}, \omega_{W-1} \Omega_{\mathcal{N}_{CQN-1}} + \omega_{W-2} \Omega_{\mathcal{N}_{CQN-2}} \right), \left( \left( 1 - \prod_{j=1}^n \left( 1 - \phi_{RP-j}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( 1 - \prod_{j=1}^n \left( 1 - \eta_{\phi_{IP-j}}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}}}, \prod_{j=1}^n \psi_{RP-j} e^{i2\pi \prod_{j=1}^n \eta_{\psi_{IP-j}}} \right) \right) \right) \end{aligned}$$



$$= \left( \left( \sum_{j=1}^2 \omega_{W-j} \Psi_{\mathcal{N}_{CQN-j}}, \sum_{j=1}^2 \omega_{W-j} \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \left( 1 - \prod_{j=1}^2 \left( 1 - \phi_{RP-j}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( 1 - \prod_{j=1}^2 \left( 1 - \eta_{\phi_{IP-j}}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}}}, \prod_{j=1}^2 \psi_{RP-j} e^{i2\pi \prod_{j=1}^2 \eta_{\psi_{IP-j}}} \right) \right)$$

Further, we choose for  $n = k$ , then we have

$$CQRONFWA(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-k}) = \left( \left( \sum_{j=1}^k \omega_{W-j} \Psi_{\mathcal{N}_{CQN-j}}, \sum_{j=1}^k \omega_{W-j} \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \left( 1 - \prod_{j=1}^k \left( 1 - \phi_{RP-j}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( 1 - \prod_{j=1}^k \left( 1 - \eta_{\phi_{IP-j}}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}}}, \prod_{j=1}^k \psi_{RP-j} e^{i2\pi \prod_{j=1}^k \eta_{\psi_{IP-j}}} \right) \right)$$

We have proved that for  $n = k + 1$ , such that

$$\begin{aligned} & CQRONFWA(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-k+1}) \\ &= \left( \left( \sum_{j=1}^k \omega_{W-j} \Psi_{\mathcal{N}_{CQN-j}} + \omega_{W-k+1} \Psi_{\mathcal{N}_{CQN-k+1}}, \sum_{j=1}^k \omega_{W-j} \Omega_{\mathcal{N}_{CQN-j}} + \omega_{W-k+1} \Omega_{\mathcal{N}_{CQN-k+1}} \right), \right. \\ & \quad \left. \left( \left( 1 - \prod_{j=1}^k \left( 1 - \phi_{RP-j}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} + \left( \left( 1 - \phi_{RP-k+1}^{q_{SC}} \right)^{\omega_{W-k+1}} \right)^{\frac{1}{q_{SC}}} - \left( 1 - \prod_{j=1}^{k+1} \left( 1 - \phi_{RP-j}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( \left( 1 - \prod_{j=1}^k \left( 1 - \eta_{\phi_{IP-j}}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} + \left( \left( 1 - \eta_{\phi_{IP-k+1}}^{q_{SC}} \right)^{\omega_{W-k+1}} \right)^{\frac{1}{q_{SC}}} - \left( 1 - \prod_{j=1}^{k+1} \left( 1 - \eta_{\phi_{IP-j}}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} \right)^{\frac{1}{q_{SC}}}}, \right. \\ & \quad \left. \prod_{j=1}^k \psi_{RP-j} * \psi_{RP-k+1} e^{i2\pi \prod_{j=1}^k \eta_{\psi_{IP-j}} * \eta_{\psi_{IP-k+1}}} \right) \\ &= \left( \left( \sum_{j=1}^n \omega_{W-j} \Psi_{\mathcal{N}_{CQN-j}}, \sum_{j=1}^n \omega_{W-j} \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \left( 1 - \prod_{j=1}^n \left( 1 - \phi_{RP-j}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( 1 - \prod_{j=1}^n \left( 1 - \eta_{\phi_{IP-j}}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}}}, \prod_{j=1}^n \psi_{RP-j} e^{i2\pi \prod_{j=1}^n \eta_{\psi_{IP-j}}} \right) \right) \\ &= CQRONFWA(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) \end{aligned}$$

Hence the result is completed.

Further, we have discussed some properties based on CQRONFNs are called idempotent, boundedness, and monotonicity.

**Theorem 3.** For any family of CQRONFNs  $\mathcal{N}_{CQN-j} = \left( \left( \Psi_{\mathcal{N}_{CQN-j}}, \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \phi_{RP-j} e^{i2\pi \eta_{\phi_{IP-j}}}, \psi_{RP-j} e^{i2\pi \eta_{\psi_{IP-j}}} \right) \right)$ ,  $j = 1, 2, \dots, n$ , if  $\mathcal{N}_{CQN-j} = \mathcal{N}_{CQN}$ , then

$$CQRONFWA(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) = \mathcal{N}_{CQN} \quad (33)$$

**Proof.** By hypothesis, it's clear that  $\mathcal{N}_{CQN-j} = \mathcal{N}_{CQN}$ , then we get

$$\begin{aligned}
 & CQRONFWA(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) \\
 &= \left( \left( \sum_{j=1}^n \omega_{W-j} \Psi_{\mathcal{N}_{CQN-j}}, \sum_{j=1}^n \omega_{W-j} \Omega_{\mathcal{N}_{CQN-j}} \right), \right. \\
 & \quad \left. \left( \left( 1 - \prod_{j=1}^n (1 - \phi_{RP-j}^{q_{sc}})^{\omega_{W-j}} \right)^{\frac{1}{q_{sc}}} e^{i2\pi \left( 1 - \prod_{j=1}^n (1 - \eta_{\phi_{IP-j}}^{q_{sc}})^{\omega_{W-j}} \right)^{\frac{1}{q_{sc}}}}, \prod_{j=1}^n \psi_{RP-j} e^{i2\pi \prod_{j=1}^n \eta_{\psi_{IP-j}}} \right) \right) \\
 &= \left( \left( \sum_{j=1}^n \omega_{W-j} \Psi_{\mathcal{N}_{CQN}}, \sum_{j=1}^n \omega_{W-j} \Omega_{\mathcal{N}_{CQN}} \right), \right. \\
 & \quad \left. \left( \left( 1 - \prod_{j=1}^n (1 - \phi_{RP}^{q_{sc}})^{\omega_{W-j}} \right)^{\frac{1}{q_{sc}}} e^{i2\pi \left( 1 - \prod_{j=1}^n (1 - \eta_{\phi_{IP}}^{q_{sc}})^{\omega_{W-j}} \right)^{\frac{1}{q_{sc}}}}, \prod_{j=1}^n \psi_{RP} e^{i2\pi \prod_{j=1}^n \eta_{\psi_{IP}}} \right) \right) \\
 &= \left( \left( \Psi_{\mathcal{N}_{CQN}}, \Omega_{\mathcal{N}_{CQN}} \right), \right. \\
 & \quad \left. \left( \left( 1 - (1 - \phi_{RP}^{q_{sc}})^{\frac{1}{q_{sc}}} e^{i2\pi (1 - (1 - \eta_{\phi_{IP}}^{q_{sc}})^{\frac{1}{q_{sc}}})} \right)^{\frac{1}{q_{sc}}}, \psi_{RP} e^{i2\pi \eta_{\psi_{IP}}} \right) \right), \sum_{j=1}^n \omega_{W-j} = 1 \\
 &= \mathcal{N}_{CQN}.
 \end{aligned}$$

**Theorem 4.** For any family of CQRONFNs  $\mathcal{N}_{CQN-j} = \left( \left( \Psi_{\mathcal{N}_{CQN-j}}, \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \phi_{RP-j} e^{i2\pi \eta_{\phi_{IP-j}}}, \psi_{RP-j} e^{i2\pi \eta_{\psi_{IP-j}}} \right) \right)$ ,  $j = 1, 2, \dots, n$ , if  $\mathcal{N}_{CQN-j}^- = \left( \left( \min \Psi_{\mathcal{N}_{CQN-j}}, \min \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \min \phi_{RP-j} e^{i2\pi \min \eta_{\phi_{IP-j}}}, \max \psi_{RP-j} e^{i2\pi \max \eta_{\psi_{IP-j}}} \right) \right)$  and  $\mathcal{N}_{CQN-j}^+ = \left( \left( \max \Psi_{\mathcal{N}_{CQN-j}}, \max \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \max \phi_{RP-j} e^{i2\pi \max \eta_{\phi_{IP-j}}}, \min \psi_{RP-j} e^{i2\pi \min \eta_{\psi_{IP-j}}} \right) \right)$ , then

$$\mathcal{N}_{CQN-j}^- \leq CQRONFWA(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) \leq \mathcal{N}_{CQN-j}^+ \quad (34)$$

**Proof.** From the above analysis it is clear that  $\sum_{j=1}^n \omega_{W-j} \Psi_{\mathcal{N}_{CQN-j}}^- \leq \sum_{j=1}^n \omega_{W-j} \Psi_{\mathcal{N}_{CQN-j}}^+$  and  $\sum_{j=1}^n \omega_{W-j} \Omega_{\mathcal{N}_{CQN-j}}^- \leq \sum_{j=1}^n \omega_{W-j} \Omega_{\mathcal{N}_{CQN-j}}^+$ . Further, we can prove that for complex-valued supporting grade and also for complex-valued supporting against, we have

**Case 1:** We have considered the real part of the supporting grade, such that

$$\begin{aligned}
 & \left( 1 - \left( \prod_{j=1}^n \left( 1 - \min_{1 \leq j \leq n} \phi_{RP-j}^{q_{sc}} \right)^{\omega_{W-j}} \right) \right)^{\frac{1}{q_{sc}}} \leq \left( 1 - \left( \prod_{j=1}^n \left( 1 - \phi_{RP-j}^{q_{sc}} \right)^{\omega_{W-j}} \right) \right)^{\frac{1}{q_{sc}}} \leq \left( 1 - \left( \prod_{j=1}^n \left( 1 - \max_{1 \leq j \leq n} \phi_{RP-j}^{q_{sc}} \right)^{\omega_{W-j}} \right) \right)^{\frac{1}{q_{sc}}} \\
 & \Rightarrow \left( 1 - \left( \left( 1 - \min_{1 \leq j \leq n} \phi_{RP-j}^{q_{sc}} \right)^{\sum_{j=1}^n \omega_{W-j}} \right) \right)^{\frac{1}{q_{sc}}} \leq \left( 1 - \left( \prod_{j=1}^n \left( 1 - \phi_{RP-j}^{q_{sc}} \right)^{\omega_{W-j}} \right) \right)^{\frac{1}{q_{sc}}} \leq \left( 1 - \left( \left( 1 - \max_{1 \leq j \leq n} \phi_{RP-j}^{q_{sc}} \right)^{\sum_{j=1}^n \omega_{W-j}} \right) \right)^{\frac{1}{q_{sc}}}
 \end{aligned}$$

Because  $\sum_{j=1}^n \omega_{W-j} = 1$ , so

$$\Rightarrow \min_{1 \leq j \leq n} \phi_{RP-j} \leq \left( 1 - \left( \prod_{j=1}^n \left( 1 - \phi_{RP-j}^{q_{sc}} \right)^{\omega_{W-j}} \right) \right)^{\frac{1}{q_{sc}}} \leq \max_{1 \leq j \leq n} \phi_{RP-j}$$

Similarly, we can find for the imaginary part of the complex-valued supporting grade, we have

$$\Rightarrow \min_{1 \leq j \leq n} \eta_{\phi_{IP-j}} \leq \left( 1 - \left( \prod_{j=1}^n \left( 1 - \eta_{\phi_{IP-j}}^{q_{sc}} \right)^{\omega_{W-j}} \right) \right)^{\frac{1}{q_{sc}}} \leq \max_{1 \leq j \leq n} \eta_{\phi_{IP-j}}$$

**Case 2:** We have considered the real part of the supporting against the grade, such that

$$\begin{aligned} \prod_{j=1}^n \min_{1 \leq j \leq n} \psi_{RP-j}^{\omega_{W-j}} &\leq \prod_{j=1}^n \psi_{RP-j}^{\omega_{W-j}} \leq \prod_{j=1}^n \max_{1 \leq j \leq n} \psi_{RP-j}^{\omega_{W-j}} \\ \Rightarrow \min_{1 \leq j \leq n} \psi_{RP-j} \sum_{j=1}^n \omega_{W-j} &\leq \prod_{j=1}^n \psi_{RP-j}^{\omega_{W-j}} \leq \max_{1 \leq j \leq n} \psi_{RP-j} \sum_{j=1}^n \omega_{W-j} \end{aligned}$$

And because  $\sum_{j=1}^n \omega_{W-j} = 1$  so

$$\min_{1 \leq j \leq n} \psi_{RP-j} \leq \prod_{j=1}^n \psi_{RP-j}^{\omega_{W-j}} \leq \max_{1 \leq j \leq n} \psi_{RP-j}$$

Then combined the above two cases, which is discussed for real parts, we have

$$\min_{1 \leq j \leq n} \phi_{RP-j} - \max_{1 \leq j \leq n} \psi_{RP-j} \leq \left( \left( 1 - \left( \prod_{j=1}^n (1 - \phi_{RP-j}^{q_{SC}})^{\omega_{W-j}} \right) \right)^{\frac{1}{q_{SC}}} - \prod_{j=1}^n \psi_{RP-j}^{\omega_{W-j}} \right) \leq \max_{1 \leq j \leq n} \phi_{RP-j} - \min_{1 \leq j \leq n} \psi_{RP-j}$$

By the Eqs. (10) and (11), we get

$$\mathcal{S}_{SF}(\mathcal{N}_{CQN-j}^-) \leq \mathcal{S}_{SF}(\mathcal{N}_{CQN-j}) \leq \mathcal{S}_{SF}(\mathcal{N}_{CQN-j}^+)$$

Similarly, we will find imaginary parts. So based on cases (1) and (2) and Eq. (10), we get

$$\mathcal{N}_{CQN-j}^- \leq \text{CQRONFWA}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) \leq \mathcal{N}_{CQN-j}^+$$

**Theorem 5.** For any family of CQRONFNs  $\mathcal{N}_{CQN-j} = \left( \left( \Psi_{\mathcal{N}_{CQN-j}}, \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \phi_{RP-j} e^{i2\pi\eta_{\phi_{IP-j}}}, \psi_{RP-j} e^{i2\pi\eta_{\psi_{IP-j}}} \right) \right), j = 1, 2, \dots, n$ , and  $\mathcal{N}_{CQN-*j} = \left( \left( \Psi_{\mathcal{N}_{CQN-*j}}, \Omega_{\mathcal{N}_{CQN-*j}} \right), \left( \phi_{RP-*j} e^{i2\pi\eta_{\phi_{IP-*j}}}, \psi_{RP-*j} e^{i2\pi\eta_{\psi_{IP-*j}}} \right) \right)$  if  $\Psi_{\mathcal{N}_{CQN-j}} \leq \Psi_{\mathcal{N}_{CQN-*j}}, \phi_{RP-j} \leq \phi_{RP-*j}, \eta_{\phi_{IP-j}} \leq \eta_{\phi_{IP-*j}}, \psi_{RP-j} \geq \psi_{RP-*j}$  and  $\eta_{\psi_{IP-j}} \geq \eta_{\psi_{IP-*j}}$ , then

$$\text{CQRONFWA}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) \leq \text{CQRONFWA}(\mathcal{N}_{CQN-*1}, \mathcal{N}_{CQN-*2}, \dots, \mathcal{N}_{CQN-*n}) \quad (35)$$

**Proof.** Consider that  $\text{CQRONFWA}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) = (\check{X}, \check{Y})$  and  $\text{CQRONFWA}(\mathcal{N}_{CQN-*1}, \mathcal{N}_{CQN-*2}, \dots, \mathcal{N}_{CQN-*n}) = (\check{X}', \check{Y}')$ , then for proving we consider the supporting grade, whose real part follows:  $\check{X} \leq \check{X}'$  and  $\check{Y} \geq \check{Y}'$ . If  $\Psi_{\mathcal{N}_{CQN-j}} \leq \Psi_{\mathcal{N}_{CQN-*j}}, \phi_{RP-j} \leq \phi_{RP-*j}, \eta_{\phi_{IP-j}} \leq \eta_{\phi_{IP-*j}}, \psi_{RP-j} \geq \psi_{RP-*j}$  and  $\eta_{\psi_{IP-j}} \geq \eta_{\psi_{IP-*j}}$ , then we obtain

$$\sum_{j=1}^n \omega_{W-j} \Psi_{\mathcal{N}_{CQN-j}} \leq \sum_{j=1}^n \omega_{W-j} \Psi_{\mathcal{N}_{CQN-*j}}$$

And the real part of the supporting grade is following as

$$\left( 1 - \prod_{j=1}^n (1 - \phi_{RP-j}^{q_{SC}})^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} \leq \left( 1 - \prod_{j=1}^n (1 - \phi_{RP-*j}^{q_{SC}})^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}}$$

Similarly, for the imaginary part of the supporting grade, we have

$$\left( 1 - \prod_{j=1}^n (1 - \eta_{\phi_{IP-j}}^{q_{SC}})^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} \leq \left( 1 - \prod_{j=1}^n (1 - \eta_{\phi_{IP-*j}}^{q_{SC}})^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}}$$

And the real part of the supporting against grade is following as

$$\prod_{j=1}^n \psi_{RP-j} \geq \prod_{j=1}^n \psi_{RP-*j}$$

Similarly, for the imaginary part of the supporting against the grade, we have

$$\prod_{j=1}^n \eta_{\psi_{IP-j}} \geq \prod_{j=1}^n \eta_{\psi_{IP-*j}}$$

By combing these all, we get

$$\begin{aligned} & \left( \left( \sum_{j=1}^n \omega_{W-j} \Psi_{\mathcal{N}_{CQN-j}}, \sum_{j=1}^n \omega_{W-j} \Omega_{\mathcal{N}_{CQN-j}} \right), \right. \\ & \left. \left( \left( 1 - \prod_{j=1}^n \left( 1 - \phi_{RP-j}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( 1 - \prod_{j=1}^n \left( 1 - \eta_{\phi_{IP-j}}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}}}, \prod_{j=1}^n \psi_{RP-j} e^{i2\pi \prod_{j=1}^n \eta_{\psi_{IP-j}}} \right) \right) \\ & \leq \left( \left( \sum_{j=1}^n \omega_{W-j} \Psi_{\mathcal{N}_{CQN-*j}}, \sum_{j=1}^n \omega_{W-j} \Omega_{\mathcal{N}_{CQN-*j}} \right), \right. \\ & \left. \left( \left( 1 - \prod_{j=1}^n \left( 1 - \phi_{RP-*j}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( 1 - \prod_{j=1}^n \left( 1 - \eta_{\phi_{IP-*j}}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}}}, \prod_{j=1}^n \psi_{RP-*j} e^{i2\pi \prod_{j=1}^n \eta_{\psi_{IP-*j}}} \right) \right) \end{aligned}$$

Hence

$$CQRONFWA(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) \leq CQRONFWA(\mathcal{N}_{CQN-*1}, \mathcal{N}_{CQN-*2}, \dots, \mathcal{N}_{CQN-*n}).$$

**Definition 13.** For any family of CQRONFNs  $\mathcal{N}_{CQN-j} = \left( \left( \Psi_{\mathcal{N}_{CQN-j}}, \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \phi_{RP-j} e^{i2\pi \eta_{\phi_{IP-j}}}, \psi_{RP-j} e^{i2\pi \eta_{\psi_{IP-j}}} \right) \right), j = 1, 2, \dots, n$ , then the CQRONFWG operator is stated by

$$CQRONFWG(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) = \sum_{j=1}^n \mathcal{N}_{CQN-j}^{\omega_{W-j}} \quad (36)$$

where  $\omega_W = \{\omega_{W-1}, \omega_{W-2}, \dots, \omega_{W-n}\}$ ,  $\omega_{W-j} \in [0, 1]$  with a condition that is  $\sum_{j=1}^n \omega_{W-j} = 1$ .

**Theorem 6.** For any family of CQRONFNs  $\mathcal{N}_{CQN-j} = \left( \left( \Psi_{\mathcal{N}_{CQN-j}}, \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \phi_{RP-j} e^{i2\pi \eta_{\phi_{IP-j}}}, \psi_{RP-j} e^{i2\pi \eta_{\psi_{IP-j}}} \right) \right), j = 1, 2, \dots, n$ , then by using the Eq. (36), we get

$$\begin{aligned} & CQRONFWG(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) \\ & = \left( \left( \sum_{j=1}^n \Psi_{\mathcal{N}_{CQN-j}}^{\omega_{W-j}}, \sum_{j=1}^n \Omega_{\mathcal{N}_{CQN-j}}^{\omega_{W-j}} \right), \right. \\ & \left. \left( \prod_{j=1}^n \phi_{RP-j} e^{i2\pi \prod_{j=1}^n \eta_{\phi_{IP-j}}}, \left( 1 - \prod_{j=1}^n \left( 1 - \psi_{RP-j}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( 1 - \prod_{j=1}^n \left( 1 - \eta_{\psi_{IP-j}}^{q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{q_{SC}}}} \right) \right) \end{aligned} \quad (37)$$

**Proof.** Straightforward.

**Definition 14.** For any family of CQRONFNs  $\mathcal{N}_{CQN-j} = \left( \left( \Psi_{\mathcal{N}_{CQN-j}}, \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \phi_{RP-j} e^{i2\pi \eta_{\phi_{IP-j}}}, \psi_{RP-j} e^{i2\pi \eta_{\psi_{IP-j}}} \right) \right), j = 1, 2, \dots, n$ , then the CQRONFGWA operator is stated by

$$CQRONFGWA(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) = \left( \sum_{j=1}^n \omega_{W-j} \mathcal{N}_{CQN-j}^{\gamma_{SC}} \right)^{\frac{1}{\gamma_{SC}}} \quad (38)$$

where  $\omega_W = \{\omega_{W-1}, \omega_{W-2}, \dots, \omega_{W-n}\}$ ,  $\omega_{W-j} \in [0, 1]$  with a condition that is  $\sum_{j=1}^n \omega_{W-j} = 1$  and  $\gamma_{SC} \in (-\infty, 0) \cup (0, \infty)$ .

**Theorem 7.** For any family of CQRONFNs  $\mathcal{N}_{CQN-j} = \left( \left( \Psi_{\mathcal{N}_{CQN-j}}, \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \phi_{RP-j} e^{i2\pi\eta_{\phi_{IP-j}}}, \psi_{RP-j} e^{i2\pi\eta_{\psi_{IP-j}}} \right) \right)$ ,  $j = 1, 2, \dots, n$ , then by using the Eq. (38), we get

$$\begin{aligned} & CQRONFGWA(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) \\ &= \left( \left( \left( \sum_{j=1}^n \omega_{W-j} \Psi_{\mathcal{N}_{CQN-j}}^{\gamma_{SC}} \right)^{\frac{1}{\gamma_{SC}}}, \left( \sum_{j=1}^n \omega_{W-j} \Omega_{\mathcal{N}_{CQN-j}}^{\gamma_{SC}} \right)^{\frac{1}{\gamma_{SC}}} \right), \right. \\ & \quad \left( 1 - \prod_{j=1}^n \left( 1 - \phi_{RP-j}^{\gamma_{SC} q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{\gamma_{SC} q_{SC}}} e^{i2\pi \left( 1 - \prod_{j=1}^n \left( 1 - \eta_{\phi_{IP-j}}^{\gamma_{SC} q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{\gamma_{SC} q_{SC}}}}, \right. \\ & \quad \left. \left( \left( 1 - \left( 1 - \left( \prod_{j=1}^n \left( 1 - \left( 1 - \psi_{RP-j}^{q_{SC} \gamma_{SC}} \right)^{1/q_{SC}} \right)^{\omega_{W-j}} \right)^{q_{SC}} \right)^{\frac{1}{\gamma_{SC}}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( 1 - \left( 1 - \left( \prod_{j=1}^n \left( 1 - \left( 1 - \eta_{\psi_{IP-j}}^{q_{SC} \gamma_{SC}} \right)^{1/q_{SC}} \right)^{\omega_{W-j}} \right)^{q_{SC}} \right)^{\frac{1}{\gamma_{SC}}} \right)^{\frac{1}{q_{SC}}}} \right) \right) \end{aligned} \quad (39)$$

**Proof.** Straightforward.

Theorems 3–5 are the same for Definitions 13 and 14.

**Definition 15.** For any family of CQRONFNs  $\mathcal{N}_{CQN-j} = \left( \left( \Psi_{\mathcal{N}_{CQN-j}}, \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \phi_{RP-j} e^{i2\pi\eta_{\phi_{IP-j}}}, \psi_{RP-j} e^{i2\pi\eta_{\psi_{IP-j}}} \right) \right)$ ,  $j = 1, 2, \dots, n$ , then the CQRONFGWG operator is stated by

$$CQRONFGWG(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) = \frac{1}{\gamma_{SC}} \left( \sum_{j=1}^n (\gamma_{SC} \mathcal{N}_{CQN-j})^{\omega_{W-j}} \right) \quad (40)$$

where  $\omega_W = \{\omega_{W-1}, \omega_{W-2}, \dots, \omega_{W-n}\}$ ,  $\omega_{W-j} \in [0, 1]$  with a condition that is  $\sum_{j=1}^n \omega_{W-j} = 1$  and  $\gamma_{SC} \in (-\infty, 0) \cup (0, \infty)$ .

**Theorem.** For any family of CQRONFNs  $\mathcal{N}_{CQN-j} = \left( \left( \Psi_{\mathcal{N}_{CQN-j}}, \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \phi_{RP-j} e^{i2\pi\eta_{\phi_{IP-j}}}, \psi_{RP-j} e^{i2\pi\eta_{\psi_{IP-j}}} \right) \right)$ ,  $j = 1, 2, \dots, n$ , then by using the Eq. (40), we get

$$\begin{aligned} & CQRONFGWG(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-2}, \dots, \mathcal{N}_{CQN-n}) \\ &= \left( \left( \left( \sum_{j=1}^n \omega_{W-j} \Psi_{\mathcal{N}_{CQN-j}}^{\gamma_{SC}} \right)^{\frac{1}{\gamma_{SC}}}, \left( \sum_{j=1}^n \omega_{W-j} \Omega_{\mathcal{N}_{CQN-j}}^{\gamma_{SC}} \right)^{\frac{1}{\gamma_{SC}}} \right), \right. \\ & \quad \left( 1 - \left( 1 - \left( \prod_{j=1}^n \left( 1 - \left( 1 - \phi_{RP-j}^{q_{SC} \gamma_{SC}} \right)^{1/q_{SC}} \right)^{\omega_{W-j}} \right)^{q_{SC}} \right)^{\frac{1}{\gamma_{SC}}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left( 1 - \left( 1 - \left( \prod_{j=1}^n \left( 1 - \left( 1 - \eta_{\phi_{IP-j}}^{\gamma_{SC} q_{SC}} \right)^{1/q_{SC}} \right)^{\omega_{W-j}} \right)^{q_{SC}} \right)^{\frac{1}{\gamma_{SC}}} \right)^{\frac{1}{q_{SC}}}}, \right. \\ & \quad \left. \left( 1 - \prod_{j=1}^n \left( 1 - \psi_{RP-j}^{\gamma_{SC} q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{\gamma_{SC} q_{SC}}} e^{i2\pi \left( 1 - \prod_{j=1}^n \left( 1 - \eta_{\psi_{IP-j}}^{\gamma_{SC} q_{SC}} \right)^{\omega_{W-j}} \right)^{\frac{1}{\gamma_{SC} q_{SC}}}} \right) \end{aligned} \quad (41)$$

**Proof.** Straightforward.

## 6. MADM METHOD BASED ON CQRONFNs

In this study, we present the proficiency and reliability of the explored approach, we develop a MADM technique based on CQRONFNs. For solving these issues, we choose the family of alternatives and their attributes concerning weight vector, whose representations are followed as  $\mathcal{A}_{AL} = \{\mathcal{A}_{AL-1}, \mathcal{A}_{AL-2}, \dots, \mathcal{A}_{AL-n}\}$ ,  $\mathcal{E}_{AT} = \{\mathcal{E}_{AT-1}, \mathcal{E}_{AT-2}, \dots, \mathcal{E}_{AT-m}\}$  and  $\omega_W = (\omega_{W-1}, \omega_{W-2}, \dots, \omega_{W-n})^T$ . Further, we choose the CQRONFNs as alternatives  $\mathcal{A}_{AL-j}$  and their attributes  $\mathcal{E}_{AT-j}$  is followed as  $\mathcal{N}_{CQN-j} = \left( \left( \Psi_{\mathcal{N}_{CQN-j}}, \Omega_{\mathcal{N}_{CQN-j}} \right), \left( \phi_{RP-j} e^{i2\pi\eta_{\phi_{IP-j}}}, \psi_{RP-j} e^{i2\pi\eta_{\psi_{IP-j}}} \right) \right)$ , then the steps of the MADM technique is summarized as follows:

### TOPSIS Method

**Step 1:** By using Eq. (42), we construct the decision matrix, whose entry in the form of CQRONFNs.

$$\mathcal{D}_{DM} = (\mathcal{N}_{CQN-jk})_{n \times m} \quad (42)$$

**Step 2:** By using Eq. (43), we normalize the decision matrix, which is given in step 1, if needed, we have

$$\mathcal{D}_{DM} = \begin{cases} \left( \left( \Psi_{\mathcal{N}_{CQN-jk}}, \Omega_{\mathcal{N}_{CQN-jk}} \right), \left( \phi_{RP-jk} e^{i2\pi\eta_{\phi_{IP-jk}}}, \psi_{RP-jk} e^{i2\pi\eta_{\psi_{IP-jk}}} \right) \right) & \text{for benefit types} \\ \left( \left( \Psi_{\mathcal{N}_{CQN-jk}}, \Omega_{\mathcal{N}_{CQN-jk}} \right), \left( \psi_{RP-jk} e^{i2\pi\eta_{\psi_{IP-jk}}}, \phi_{RP-jk} e^{i2\pi\eta_{\phi_{IP-jk}}} \right) \right) & \text{for cost types} \end{cases} \quad (43)$$

**Step 3:** By using Eq. (41), we aggregate the values, which are normalized in step 2.

**Step 4:** By using Eqs. (44) and (45), we evaluate the positive and negative ideas, such that

$$\mathcal{N}_{CQN-j1}^+ = \left\{ \begin{aligned} & \left( \left( \max \Psi_{\mathcal{N}_{CQN-j1}}, \max \Omega_{\mathcal{N}_{CQN-j1}} \right), \left( \max \phi_{RP-j1} e^{i2\pi\max\eta_{\phi_{IP-j1}}}, \min \psi_{RP-j1} e^{i2\pi\min\eta_{\psi_{IP-j1}}} \right) \right), \\ & \left( \left( \max \Psi_{\mathcal{N}_{CQN-j2}}, \max \Omega_{\mathcal{N}_{CQN-j2}} \right), \left( \max \phi_{RP-j2} e^{i2\pi\max\eta_{\phi_{IP-j2}}}, \min \psi_{RP-j2} e^{i2\pi\min\eta_{\psi_{IP-j2}}} \right) \right), \dots, \\ & \left( \left( \max \Psi_{\mathcal{N}_{CQN-jm}}, \max \Omega_{\mathcal{N}_{CQN-jm}} \right), \left( \max \phi_{RP-jm} e^{i2\pi\max\eta_{\phi_{IP-jm}}}, \min \psi_{RP-jm} e^{i2\pi\min\eta_{\psi_{IP-jm}}} \right) \right) \end{aligned} \right\} \quad (44)$$

$$\mathcal{N}_{CQN-j1}^- = \left\{ \begin{aligned} & \left( \left( \min \Psi_{\mathcal{N}_{CQN-j1}}, \min \Omega_{\mathcal{N}_{CQN-j1}} \right), \left( \min \phi_{RP-j1} e^{i2\pi\min\eta_{\phi_{IP-j1}}}, \max \psi_{RP-j1} e^{i2\pi\max\eta_{\psi_{IP-j1}}} \right) \right), \\ & \left( \left( \min \Psi_{\mathcal{N}_{CQN-j2}}, \min \Omega_{\mathcal{N}_{CQN-j2}} \right), \left( \min \phi_{RP-j2} e^{i2\pi\min\eta_{\phi_{IP-j2}}}, \max \psi_{RP-j2} e^{i2\pi\max\eta_{\psi_{IP-j2}}} \right) \right), \dots, \\ & \left( \left( \min \Psi_{\mathcal{N}_{CQN-jm}}, \min \Omega_{\mathcal{N}_{CQN-jm}} \right), \left( \min \phi_{RP-jm} e^{i2\pi\min\eta_{\phi_{IP-jm}}}, \max \psi_{RP-jm} e^{i2\pi\max\eta_{\psi_{IP-jm}}} \right) \right) \end{aligned} \right\} \quad (45)$$

**Step 5:** By using the Eq. (28), we examine the Order- $\alpha_{CQ}$  divergence measures, by using positive and negative ideals, we have

$$\mathcal{A}_{GDM}^{SDM-1}(\mathcal{N}_{CQN-j}, \mathcal{N}_{CQN-j1}^+) \text{ and } \mathcal{A}_{GDM}^{SDM-1}(\mathcal{N}_{CQN-j}, \mathcal{N}_{CQN-j1}^-).$$

**Step 6:** By using Eq. (46), we examine the overall distance measure based on the  $\mathcal{A}_{GDM}^{SDM-1}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-j1}^+)$  and  $\mathcal{A}_{GDM}^{SDM-1}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-j1}^-)$ , we have

$$D_{OD-j} = \frac{\mathcal{A}_{GDM}^{SDM-1}(\mathcal{N}_{CQN-j}, \mathcal{N}_{CQN-j1}^+)}{\mathcal{A}_{GDM}^{SDM-1}(\mathcal{N}_{CQN-j}, \mathcal{N}_{CQN-j1}^+) + \mathcal{A}_{GDM}^{SDM-1}(\mathcal{N}_{CQN-j}, \mathcal{N}_{CQN-j1}^-)} \quad (46)$$

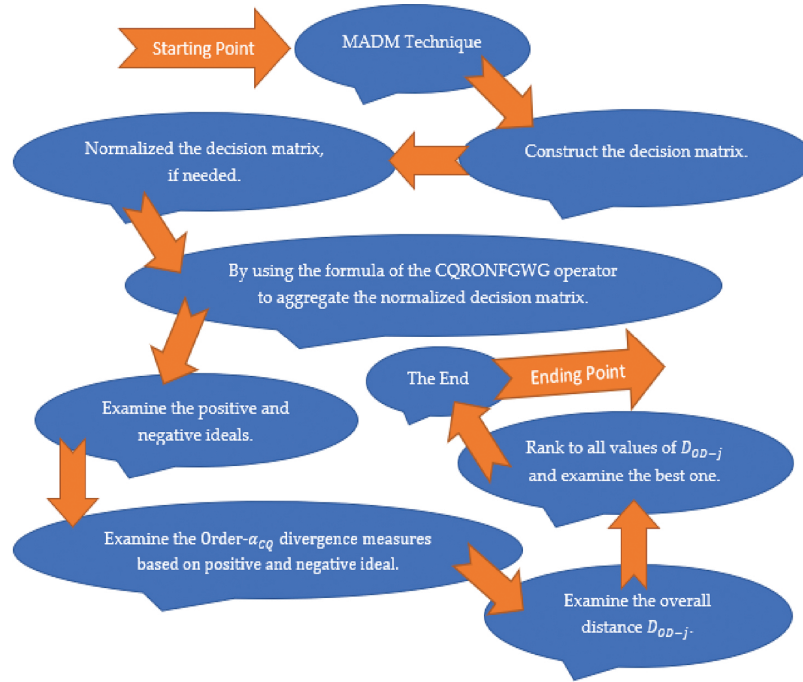
**Step 7:** Rank all alternatives, which we get in step 6, and examine the best alternative from the family of alternatives.

**Step 8:** The end. The graphical representation of the explored algorithm is summarized in the form of Figure 2.

### Example 1.

With the improvement of internet business stages, web-based shopping has become a typical utilization propensity for buyers. A customer plans to purchase a cell phone on a web-based business stage. The mobile phones are considered as alternatives, whose representations are followed as  $\mathcal{A}_{AL} = \{\mathcal{A}_{AL-1}, \mathcal{A}_{AL-2}, \mathcal{A}_{AL-3}, \mathcal{A}_{AL-4}, \mathcal{A}_{AL-5}\}$  and their attributes whose representation with details is followed as

- $\mathcal{C}_{AT-1}$ : Creditability of merchant;
- $\mathcal{C}_{AT-2}$ : Online satisfaction rate;
- $\mathcal{C}_{AT-3}$ : The preference of mobile phone system;
- $\mathcal{C}_{AT-4}$ : Price preference.



**Figure 2** | Geometrical interpretation of the explored algorithm.

**Table 1** | Original decision matrix, whose every entry in the form of complex q-rung orthopair normal fuzzy numbers.

Symbols	$\mathcal{C}_{AT-1}$	$\mathcal{C}_{AT-2}$	$\mathcal{C}_{AT-3}$	$\mathcal{C}_{AT-4}$
$\mathcal{A}_{AL-1}$	$\begin{pmatrix} (0.6, 0.2), \\ 0.4e^{i2\pi(0.5)}, \\ 0.2e^{i2\pi(0.3)} \end{pmatrix}$	$\begin{pmatrix} (0.7, 0.2), \\ 0.5e^{i2\pi(0.6)}, \\ 0.1e^{i2\pi(0.2)} \end{pmatrix}$	$\begin{pmatrix} (0.8, 0.1), \\ 0.3e^{i2\pi(0.4)}, \\ 0.1e^{i2\pi(0.3)} \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.2), \\ 0.5e^{i2\pi(0.3)}, \\ 0.2e^{i2\pi(0.4)} \end{pmatrix}$
$\mathcal{A}_{AL-2}$	$\begin{pmatrix} (0.61, 0.21), \\ 0.41e^{i2\pi(0.51)}, \\ 0.21e^{i2\pi(0.31)} \end{pmatrix}$	$\begin{pmatrix} (0.71, 0.21), \\ 0.51e^{i2\pi(0.61)}, \\ 0.11e^{i2\pi(0.21)} \end{pmatrix}$	$\begin{pmatrix} (0.81, 0.11), \\ 0.31e^{i2\pi(0.41)}, \\ 0.11e^{i2\pi(0.31)} \end{pmatrix}$	$\begin{pmatrix} (0.51, 0.21), \\ 0.51e^{i2\pi(0.31)}, \\ 0.21e^{i2\pi(0.41)} \end{pmatrix}$
$\mathcal{A}_{AL-3}$	$\begin{pmatrix} (0.62, 0.22), \\ 0.42e^{i2\pi(0.52)}, \\ 0.22e^{i2\pi(0.32)} \end{pmatrix}$	$\begin{pmatrix} (0.72, 0.22), \\ 0.52e^{i2\pi(0.62)}, \\ 0.12e^{i2\pi(0.22)} \end{pmatrix}$	$\begin{pmatrix} (0.82, 0.12), \\ 0.32e^{i2\pi(0.42)}, \\ 0.12e^{i2\pi(0.32)} \end{pmatrix}$	$\begin{pmatrix} (0.52, 0.22), \\ 0.52e^{i2\pi(0.32)}, \\ 0.22e^{i2\pi(0.42)} \end{pmatrix}$
$\mathcal{A}_{AL-4}$	$\begin{pmatrix} (0.63, 0.23), \\ 0.43e^{i2\pi(0.53)}, \\ 0.23e^{i2\pi(0.33)} \end{pmatrix}$	$\begin{pmatrix} (0.73, 0.23), \\ 0.53e^{i2\pi(0.63)}, \\ 0.13e^{i2\pi(0.23)} \end{pmatrix}$	$\begin{pmatrix} (0.83, 0.13), \\ 0.33e^{i2\pi(0.43)}, \\ 0.13e^{i2\pi(0.33)} \end{pmatrix}$	$\begin{pmatrix} (0.53, 0.23), \\ 0.53e^{i2\pi(0.33)}, \\ 0.23e^{i2\pi(0.43)} \end{pmatrix}$
$\mathcal{A}_{AL-5}$	$\begin{pmatrix} (0.64, 0.24), \\ 0.44e^{i2\pi(0.54)}, \\ 0.24e^{i2\pi(0.34)} \end{pmatrix}$	$\begin{pmatrix} (0.74, 0.24), \\ 0.54e^{i2\pi(0.64)}, \\ 0.14e^{i2\pi(0.24)} \end{pmatrix}$	$\begin{pmatrix} (0.84, 0.14), \\ 0.34e^{i2\pi(0.44)}, \\ 0.14e^{i2\pi(0.34)} \end{pmatrix}$	$\begin{pmatrix} (0.54, 0.24), \\ 0.54e^{i2\pi(0.34)}, \\ 0.24e^{i2\pi(0.44)} \end{pmatrix}$

For alternatives  $\mathcal{A}_{AL-j}$  and their attributes  $\mathcal{C}_{AT-j}$ , the weight vector is followed as  $\omega_W = (0.2, 0.25, 0.35, 0.0.2)^T$ . Based on the above analysis, the procedure of the MADM technique is summarized in the following ways:

**Step 1:** By using Eq. (42), we construct the decision matrix, whose entries in the form of CQRONFNs.

**Step 2:** By using Eq. (43), we normalize the decision matrix, which is given in step 1, if needed. The matrix, which is mention in the Table 1 is not needed to normalize it.



**Step 3:** By using Eq. (41), we aggregate the values, which are normalized in step 2, for  $\gamma_{SC} = 2$ .

$$\begin{aligned}\mathcal{N}_{CQN-1} &= ((0.6845, 0.1718), (0.0834e^{i2\pi(0.1005)}, 0.1487e^{i2\pi(0.304)})) \\ \mathcal{N}_{CQN-2} &= ((0.6943, 0.1814), (0.088e^{i2\pi(0.1056)}, 0.1582e^{i2\pi(0.3138)})) \\ \mathcal{N}_{CQN-3} &= ((0.7042, 0.191), (0.0927e^{i2\pi(0.1108)}, 0.1678e^{i2\pi(0.3236)})) \\ \mathcal{N}_{CQN-4} &= ((0.7141, 0.2007), (0.0975e^{i2\pi(0.1162)}, 0.1774e^{i2\pi(0.3335)})) \\ \mathcal{N}_{CQN-5} &= ((0.7239, 0.2105), (0.1024e^{i2\pi(0.1217)}, 0.187e^{i2\pi(0.3433)}))\end{aligned}$$

**Step 4:** By using Eqs. (44) and (45), we evaluate the positive and negative ideas, such that

$$\begin{aligned}\mathcal{N}_{CQN}^+ &= ((0.7239, 0.2105), (0.1024e^{i2\pi(0.1217)}, 0.1487e^{i2\pi(0.304)})) \\ \mathcal{N}_{CQN}^- &= ((0.6845, 0.1718), (0.0834e^{i2\pi(0.1005)}, 0.187e^{i2\pi(0.3433)}))\end{aligned}$$

**Step 5:** By using the Eq. (28), we examine the Order- $\alpha_{CQ}$  divergence measures, by using positive and negative ideals, we have

$$\mathcal{d}_{GDM}^{SDM-1}(\mathcal{N}_{CQN-j}, \mathcal{N}_{CQN-j1}^+) \text{ and } \mathcal{d}_{GDM}^{SDM-1}(\mathcal{N}_{CQN-j}, \mathcal{N}_{CQN-j1}^-).$$

**Step 6:** By using Eq. (46), we examine the overall distance measure based on the  $\mathcal{d}_{GDM}^{SDM-1}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-j1}^+)$  and  $\mathcal{d}_{GDM}^{SDM-1}(\mathcal{N}_{CQN-1}, \mathcal{N}_{CQN-j1}^-)$ , we have

$$\begin{aligned}D_{OD-1} &= \frac{0.482}{0.482 + 0.562} = 0.4616 \\ D_{OD-2} &= 0.4617 \\ D_{OD-3} &= 0.4618 \\ D_{OD-4} &= 0.4619 \\ D_{OD-5} &= 0.4620\end{aligned}$$

**Step 7:** Rank all alternatives, which we get in step 6 and we examine the best alternative from the family of alternatives, such that

$$D_{OD-5} \geq D_{OD-4} \geq D_{OD-3} \geq D_{OD-2} \geq D_{OD-1}$$

The best alternative is  $D_{OD-5}$ .

**Step 8:** The end.

## 6.1. Comparative Analysis

Keeping the advantages of the explored notions is called CQRONFSs and their AOs, we solve some numerical examples to examine the reliability and effectiveness of the presented work. For coping with such kind of issues, we considered different kinds of information and resolved it by using explored and existing operators, whose information's are discussed below.

Additionally, the compassion among presented work and existing works are discussed to observe the proficiency and expertise of the explored approach. The existing technique of intuitionistic normal fuzzy AOs was proposed by Wang and Li [30], and the q-rung orthopair normal fuzzy AOs were presented by Yang *et al.* [31].

From the above analysis, it is clear that the existing approaches [30,31] and explored approach in these manuscripts give the same ranking results, which is in the form of  $\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$ . If, we can check the values, which are obtained in Example 1, which is following as:  $D_{OD-5} \geq D_{OD-4} \geq D_{OD-3} \geq D_{OD-2} \geq D_{OD-1}$  is also the same as the ranking values of Table 2. From the above discussion, we obtained with the help of explored work and existing works, the best alternative is  $\mathcal{A}_{AL-5}$ . The graphical representation of the information, which is discussed in Table 4, is explained with the help of Figure 3. The aggregated values for proposed work and existing works are illustrated in Table 3, for  $\gamma_{SC} = 2$ .

From Table 2 we considered the complex intuitionistic normal fuzzy information and resolved it by using the explored and existing operators [30,31] to examine the proficiency and expertise of the presented approach. Further, to find the reliability of the explored operator, we choose the complex Pythagorean normal fuzzy information and solve it by using the explored and existing operators [30,31].

From Figure 3 there are mentions five kinds of series, which are denoted the graph of alternatives in different colors. From Figure 3, we easily obtained that which one is the best alternative, see the above figure the series five is moved on the top in all series, so series five is the best alternative from the set of alternatives. The information's are discussed in Table 5 for  $\gamma_{SC} = 2$  and the weight vectors are discussed at the beginning of Example 1.

Based on the explored operators are called WA, WG, GWA, GWG operators based on complex Pythagorean normal fuzzy information's, the comparison among presented work and existing works are discussed to observe the proficiency and expertise of the explored approach. The existing technique of intuitionistic normal fuzzy AOs was proposed by Wang and Li [30], and the q-rung orthopair normal fuzzy AOs were presented by Yang *et al.* [31]. The aggregated values for proposed work and existing works are illustrated in Table 6.

From the above analysis, it is clear that the existing approaches [30,31] and explored approach in this manuscripts give the different ranking results, which are in the form of  $\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$  and  $\mathcal{A}_{AL-1} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-3} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-5}$ . From the above discussion, we obtained with the help of explored work and existing works, the best alternatives are  $\mathcal{A}_{AL-5}$  and  $\mathcal{A}_{AL-1}$ . The graphical representation of the information, which is discussed in Table 6, is explained with the help of Figure 4. The positive and negative ideals are discussed in the form of Table 2.

**Table 2** | By using Eq. (28), we examine the distance measures based on positive and negative ideals.

Symbols	$\mathcal{D}_{GDM}^{SDM-1} \left( \mathcal{N}_{CQN-j}, \mathcal{N}_{CQN-j1}^+ \right)$	$\mathcal{D}_{GDM}^{SDM-1} \left( \mathcal{N}_{CQN-j}, \mathcal{N}_{CQN-j1}^- \right)$
$\mathcal{N}_{CQN-1}$	0.482	0.562
$\mathcal{N}_{CQN-2}$	0.476	0.555
$\mathcal{N}_{CQN-3}$	0.469	0.547
$\mathcal{N}_{CQN-4}$	0.462	0.539
$\mathcal{N}_{CQN-5}$	0.455	0.53

**Table 3** | Comparison between explored work with some existing operators.

Methods	Operators	Score Values	Ranking
Wang and Li [30]	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.3523, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.3571,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.362, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.3668,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.3717$	
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.3392, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.3444,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.3495, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.3546,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.3597$	
	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.4213, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.4304,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.4394, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.4485,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.4575$	
	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1814, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1838,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1863, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1888,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.1913$	
Yang <i>et al.</i> [31]	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2498, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2575,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2654, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2734,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.2816$	
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2081, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2128,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2174, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2219,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.2264$	
	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2591, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2669,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2748, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2829,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.2911$	

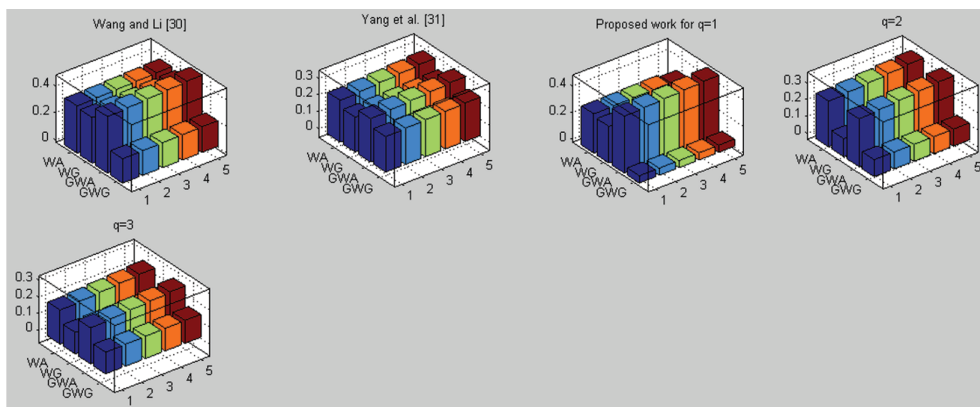
Continued

**Table 3** | Comparison between explored work with some existing operators. (Continued)

Methods	Operators	Score Values	Ranking
Proposed method for q = 1	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2115, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.216,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2204, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2248,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.2291$	
	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2928, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.296,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2992, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.3024,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.3057$	
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2724, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2759,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2794, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2829,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.2864$	
	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.4233, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.4317,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.4399, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.4482,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.4564$	
Proposed method for q = 2	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.0531, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.0532,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.0532, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.0534,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.0537$	
	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2725, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2818,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2911, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.3006,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.3102$	
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1076, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1086,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1094, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1102,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.1109$	
	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2856, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2948,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.3042, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.3137,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.3233$	
Proposed method for q = 3	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1006, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1009,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.101, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.101,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.101$	
	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2031, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2105,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2182, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.226,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.234$	
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1288, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1309,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.133, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.135,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.1368$	
	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2156, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2232,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.231, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.239,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.2471$	
Proposed method for q = 3	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1293, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1312,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1331, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1349,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.1366$	

**Table 4** | Original decision matrix, whose information in the form of complex Pythagorean normal fuzzy numbers.

Symbols	$\mathcal{C}_{AT-1}$	$\mathcal{C}_{AT-2}$	$\mathcal{C}_{AT-3}$	$\mathcal{C}_{AT-4}$
$\mathcal{A}_{AL-1}$	$\begin{pmatrix} (0.6, 0.2), \\ (0.6e^{i2\pi(0.7)}, \\ 0.5e^{i2\pi(0.4)}) \end{pmatrix}$	$\begin{pmatrix} (0.7, 0.2), \\ (0.7e^{i2\pi(0.8)}, \\ 0.4e^{i2\pi(0.3)}) \end{pmatrix}$	$\begin{pmatrix} (0.8, 0.1), \\ (0.9e^{i2\pi(0.7)}, \\ 0.11e^{i2\pi(0.3)}) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.2), \\ (0.5e^{i2\pi(0.6)}, \\ 0.5e^{i2\pi(0.4)}) \end{pmatrix}$
$\mathcal{A}_{AL-2}$	$\begin{pmatrix} (0.61, 0.21), \\ (0.61e^{i2\pi(0.71)}, \\ 0.51e^{i2\pi(0.41)}) \end{pmatrix}$	$\begin{pmatrix} (0.71, 0.21), \\ (0.71e^{i2\pi(0.81)}, \\ 0.41e^{i2\pi(0.31)}) \end{pmatrix}$	$\begin{pmatrix} (0.81, 0.11), \\ (0.92e^{i2\pi(0.72)}, \\ 0.12e^{i2\pi(0.32)}) \end{pmatrix}$	$\begin{pmatrix} (0.51, 0.21), \\ (0.51e^{i2\pi(0.61)}, \\ 0.51e^{i2\pi(0.41)}) \end{pmatrix}$
$\mathcal{A}_{AL-3}$	$\begin{pmatrix} (0.62, 0.22), \\ (0.62e^{i2\pi(0.72)}, \\ 0.52e^{i2\pi(0.42)}) \end{pmatrix}$	$\begin{pmatrix} (0.72, 0.22), \\ (0.72e^{i2\pi(0.82)}, \\ 0.42e^{i2\pi(0.32)}) \end{pmatrix}$	$\begin{pmatrix} (0.82, 0.12), \\ (0.93e^{i2\pi(0.73)}, \\ 0.13e^{i2\pi(0.33)}) \end{pmatrix}$	$\begin{pmatrix} (0.52, 0.22), \\ (0.52e^{i2\pi(0.62)}, \\ 0.52e^{i2\pi(0.42)}) \end{pmatrix}$
$\mathcal{A}_{AL-4}$	$\begin{pmatrix} (0.63, 0.23), \\ (0.63e^{i2\pi(0.73)}, \\ 0.53e^{i2\pi(0.43)}) \end{pmatrix}$	$\begin{pmatrix} (0.73, 0.23), \\ (0.73e^{i2\pi(0.83)}, \\ 0.43e^{i2\pi(0.33)}) \end{pmatrix}$	$\begin{pmatrix} (0.83, 0.13), \\ (0.94e^{i2\pi(0.74)}, \\ 0.14e^{i2\pi(0.34)}) \end{pmatrix}$	$\begin{pmatrix} (0.53, 0.23), \\ (0.53e^{i2\pi(0.63)}, \\ 0.53e^{i2\pi(0.43)}) \end{pmatrix}$
$\mathcal{A}_{AL-5}$	$\begin{pmatrix} (0.64, 0.24), \\ (0.64e^{i2\pi(0.74)}, \\ 0.54e^{i2\pi(0.44)}) \end{pmatrix}$	$\begin{pmatrix} (0.74, 0.24), \\ (0.74e^{i2\pi(0.84)}, \\ 0.44e^{i2\pi(0.34)}) \end{pmatrix}$	$\begin{pmatrix} (0.84, 0.14), \\ (0.95e^{i2\pi(0.75)}, \\ 0.15e^{i2\pi(0.35)}) \end{pmatrix}$	$\begin{pmatrix} (0.54, 0.24), \\ (0.54e^{i2\pi(0.64)}, \\ 0.54e^{i2\pi(0.44)}) \end{pmatrix}$

**Figure 3** | Geometrical representation for the informations of Table 4.

From Figure 4 there are mentions five kinds of series, which are denoted the graph of alternatives in different colors. From Figure 4, we easily obtained that which one is the best alternative, see the above figure the series five is moved on the top in all series, so series five is the best alternative from the set of alternatives. From Tables 1 and 5 we considered the complex intuitionistic normal fuzzy information, complex Pythagorean normal fuzzy information and resolved it by using the explored and existing operators [30,31] to examine the proficiency and expertise of the presented approach. Further, to find the reliability of the explored operator, we choose the complex q-rung orthopair normal fuzzy information and solve it by using the explored and existing operators [30,31]. The information is discussed in Table 7.

Based on the explored operators are called WA, WG, GWA, GWG operators based on complex Pythagorean normal fuzzy information's, the comparison among presented work and existing works are discussed to observe the proficiency and expertise of the explored approach. The existing technique of intuitionistic normal fuzzy AOs was proposed by Wang and Li [30], and the q-rung orthopair normal fuzzy AOs were presented by Yang *et al.* [31]. The aggregated values for proposed work and existing works are illustrated in Table 8.

From the above analysis, it is clear that the existing approaches [30,31] and explored approach in this manuscripts give different ranking results, which are in the form of  $\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$  and  $\mathcal{A}_{AL-1} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-3} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-5}$ . From the above discussion, we obtained with the help of explored work and existing works, the best alternatives are  $\mathcal{A}_{AL-5}$  and  $\mathcal{A}_{AL-1}$ . The graphical representation of the information, which is discussed in Table 8, is explained with the help of Figure 5.

From Figure 5 there are mentions five kinds of series, which are denoted the graph of alternatives in different colors. From Figure 5, we easily obtained that which one is the best alternative, see the above figure the series five is moved on the top in all series, so series five is

**Table 5** Comparison between explored work with some existing operators.

Methods	Operators	Score Values	Ranking
Wang and Li [30]	WA	Failed	Failed
	WG	Failed	Failed
	GWA	Failed	Failed
	GWG	Failed	Failed
Yang et al. [31]	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.4457, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.473,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.4908, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.5097,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.5298$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1772, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1819,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1848, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1876,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.1903$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.472, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.5,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.5176, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.536,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.5557$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1714, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1751,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1772, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1791,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.181$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
Proposed method for q = 1	WA	Failed	Failed
	WG	Failed	Failed
	GWA	Failed	Failed
	GWG	Failed	Failed
Proposed method for q = 2	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.5059, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.5281,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.5433, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.559,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.5754$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1063, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1111,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1141, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1174,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.1209$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.5212, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.5437,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.5589, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.5747,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.591$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.0478, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.0478,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.0469, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.046,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.0452$	$\mathcal{A}_{AL-1} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-5}$
Proposed method for q = 3	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.4198, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.4451,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.4625, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.4808,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.5001$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1049, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1067,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1074, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1081,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.1087$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$

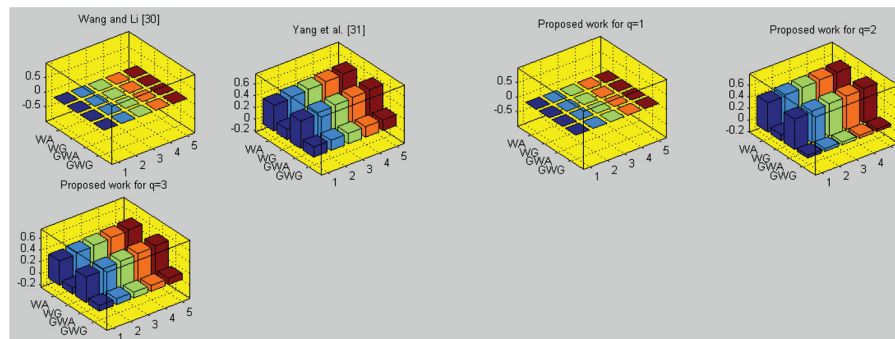
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**Table 5** | Comparison between explored work with some existing operators. (Continued)

Methods	Operators	Score Values	Ranking
Proposed method for q = 3	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.4407, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.4664,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.4836, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.5016,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.5207$	
	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.0991, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1001,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1001, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.11$	

**Table 6** | Original decision matrix, whose information in the form of complex q-rung orthopair normal fuzzy numbers.

Symbols	$\mathcal{C}_{AT-1}$	$\mathcal{C}_{AT-2}$	$\mathcal{C}_{AT-3}$	$\mathcal{C}_{AT-4}$
$\mathcal{A}_{AL-1}$	$\begin{pmatrix} (0.6, 0.2), \\ (0.9e^{i2\pi(0.8)}, \\ 0.8e^{i2\pi(0.7)}) \end{pmatrix}$	$\begin{pmatrix} (0.7, 0.2), \\ (0.8e^{i2\pi(0.9)}, \\ 0.7e^{i2\pi(0.8)}) \end{pmatrix}$	$\begin{pmatrix} (0.8, 0.1), \\ (0.85e^{i2\pi(0.75)}, \\ 0.75e^{i2\pi(0.75)}) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.2), \\ (0.75e^{i2\pi(0.56)}, \\ 0.85e^{i2\pi(0.46)}) \end{pmatrix}$
$\mathcal{A}_{AL-2}$	$\begin{pmatrix} (0.61, 0.21), \\ (0.91e^{i2\pi(0.81)}, \\ 0.81e^{i2\pi(0.71)}) \end{pmatrix}$	$\begin{pmatrix} (0.71, 0.21), \\ (0.81e^{i2\pi(0.91)}, \\ 0.71e^{i2\pi(0.81)}) \end{pmatrix}$	$\begin{pmatrix} (0.81, 0.11), \\ (0.86e^{i2\pi(0.76)}, \\ 0.76e^{i2\pi(0.76)}) \end{pmatrix}$	$\begin{pmatrix} (0.51, 0.21), \\ (0.76e^{i2\pi(0.56)}, \\ 0.86e^{i2\pi(0.46)}) \end{pmatrix}$
$\mathcal{A}_{AL-3}$	$\begin{pmatrix} (0.62, 0.22), \\ (0.92e^{i2\pi(0.82)}, \\ 0.82e^{i2\pi(0.72)}) \end{pmatrix}$	$\begin{pmatrix} (0.72, 0.22), \\ (0.82e^{i2\pi(0.92)}, \\ 0.72e^{i2\pi(0.82)}) \end{pmatrix}$	$\begin{pmatrix} (0.82, 0.12), \\ (0.87e^{i2\pi(0.77)}, \\ 0.77e^{i2\pi(0.77)}) \end{pmatrix}$	$\begin{pmatrix} (0.52, 0.22), \\ (0.77e^{i2\pi(0.57)}, \\ 0.87e^{i2\pi(0.47)}) \end{pmatrix}$
$\mathcal{A}_{AL-4}$	$\begin{pmatrix} (0.63, 0.23), \\ (0.93e^{i2\pi(0.83)}, \\ 0.83e^{i2\pi(0.73)}) \end{pmatrix}$	$\begin{pmatrix} (0.73, 0.23), \\ (0.83e^{i2\pi(0.93)}, \\ 0.73e^{i2\pi(0.83)}) \end{pmatrix}$	$\begin{pmatrix} (0.83, 0.13), \\ (0.88e^{i2\pi(0.78)}, \\ 0.78e^{i2\pi(0.78)}) \end{pmatrix}$	$\begin{pmatrix} (0.53, 0.23), \\ (0.78e^{i2\pi(0.58)}, \\ 0.88e^{i2\pi(0.48)}) \end{pmatrix}$
$\mathcal{A}_{AL-5}$	$\begin{pmatrix} (0.64, 0.24), \\ (0.94e^{i2\pi(0.84)}, \\ 0.84e^{i2\pi(0.74)}) \end{pmatrix}$	$\begin{pmatrix} (0.74, 0.24), \\ (0.84e^{i2\pi(0.94)}, \\ 0.74e^{i2\pi(0.84)}) \end{pmatrix}$	$\begin{pmatrix} (0.84, 0.14), \\ (0.89e^{i2\pi(0.79)}, \\ 0.79e^{i2\pi(0.79)}) \end{pmatrix}$	$\begin{pmatrix} (0.54, 0.24), \\ (0.79e^{i2\pi(0.59)}, \\ 0.89e^{i2\pi(0.49)}) \end{pmatrix}$

**Figure 4** | Geometrical representation for the informations of Table 6.

the best alternative from the set of alternatives. Further, we examine the reliability and proficiency of the explored measures and operators based on CQRONFSS. We illustrate a numerical example which is discussed below is taken from Ref. [31].

Example 2.

As economic globalization makes enterprises face a more complex internal and external environment, finding an appropriate partner is an important way to maintain their competitiveness, which is affected by many factors. To select a suitable global partner, an enterprise has selected five candidate enterprises in the global scope. The set of alternative enterprises is  $\mathcal{A}_{AL} = \{\mathcal{A}_{AL-1}, \mathcal{A}_{AL-2}, \mathcal{A}_{AL-3}, \mathcal{A}_{AL-4}, \mathcal{A}_{AL-5}\}$ , and four attributes are considered, namely, R& D capability ( $\mathcal{C}_{AT-1}$ ), business operation level ( $\mathcal{C}_{AT-2}$ ), international cooperation level ( $\mathcal{C}_{AT-3}$ ) and credit level ( $\mathcal{C}_{AT-4}$ ). The set of attributes  $\mathcal{C}_{AT} = \{\mathcal{C}_{AT-1}, \mathcal{C}_{AT-2}, \mathcal{C}_{AT-3}, \mathcal{C}_{AT-4}\}$  is formed, and they are all benefit-oriented indicators. The corresponding weight is  $\omega_W = \{0.3, 0.2, 0.2, 0.3\}^T$ , and the decision information matrix as shown in Table 8 is constructed according to the decision information. Besides, considering the problem of q-RONF information aggregation based on WA, WG, GWA, and GWG operators.

Table 7 | Comparison between explored work with some existing operators.

Methods	Operators	Score Values	Ranking
Wang and Li [30]	WA	Failed	Failed
	WG	Failed	Failed
	GWA	Failed	Failed
	GWG	Failed	Failed
Yang et al. [31]	WA	Failed	Failed
	WG	Failed	Failed
	GWA	Failed	Failed
	GWG	Failed	Failed
Proposed method for q=1	WA	Failed	Failed
	WG	Failed	Failed
	GWA	Failed	Failed
	GWG	Failed	Failed
Proposed method for q = 2	WA	Failed	Failed
	WG	Failed	Failed
	GWA	Failed	Failed
	GWG	Failed	Failed
Proposed method for q = 8	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.289, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.3092,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.3301, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.3529,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.378$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.0627, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.0588,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.0528, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.0458,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.0379$	$\mathcal{A}_{AL-1} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-5}$
	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.3143, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.3358,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.358, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.3822,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.4087$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.0544, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.0494,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.0425, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.0346,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.0256$	$\mathcal{A}_{AL-1} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-5}$

Table 8 | Original decision matrix.

Symbols	$\mathcal{C}_{AT-1}$	$\mathcal{C}_{AT-2}$	$\mathcal{C}_{AT-3}$	$\mathcal{C}_{AT-4}$
$\mathcal{A}_{AL-1}$	((0.8, 0.7), (0.6, 0.7))	((0.4, 0.5), (0.2, 0.4))	((0.8, 0.5), (0.1, 0.7))	((0.4, 0.3), (0.6, 0.2))
$\mathcal{A}_{AL-2}$	((0.6, 0.4), (0.7, 0.9))	((0.5, 0.6), (0.4, 0.3))	((0.4, 0.4), (0.7, 0.2))	((0.8, 0.5), (0.6, 0.4))
$\mathcal{A}_{AL-3}$	((0.9, 0.8), (0.5, 0.4))	((0.7, 0.8), (0.7, 0.2))	((0.4, 0.3), (0.8, 0.8))	((0.6, 0.6), (0.3, 0.5))
$\mathcal{A}_{AL-4}$	((0.7, 0.6), (0.7, 0.8))	((0.8, 0.7), (0.8, 0.6))	((0.5, 0.4), (0.6, 0.9))	((0.6, 0.4), (0.7, 0.3))
$\mathcal{A}_{AL-5}$	((0.5, 0.3), (0.8, 0.4))	((0.7, 0.5), (0.5, 0.5))	((0.5, 0.3), (0.6, 0.6))	((0.8, 0.6), (0.8, 0.7))



Based on the explored operators are called WA, WG, GWA, GWG operators based on complex Pythagorean normal fuzzy information's, the compassion among presented work and existing works are discussed to observe the proficiency and expertise of the explored approach. The existing technique of intuitionistic normal fuzzy AOs was proposed by Wang and Li [30], and the q-rung orthopair normal fuzzy AOs were presented by Yang *et al.* [31]. The aggregated values for proposed work and existing works are illustrated in Table 9.

From the above analysis, the ranking results of the proposed and existing notions are the same which is in the form of  $\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$ . The best alternative is  $\mathcal{A}_{AL-5}$ . Additionally, we choose the intuitionistic normal fuzzy information and examine the best alternative by using the explored and existing operators. The intuitionistic normal fuzzy information is stated in the form of Table 10.

Based on the explored operators are called WA, WG, GWA, GWG operators based on complex Pythagorean normal fuzzy information's, the compassion among presented work and existing works are discussed to observe the proficiency and expertise of the explored approach. The existing technique of intuitionistic normal fuzzy AOs was proposed by Wang and Li [30], and the q-rung orthopair normal fuzzy AOs were presented by Yang *et al.* [31]. The aggregated values for proposed work and existing works are illustrated in Table 11.

From the above analysis, the ranking results of the proposed and existing notions are the same which is in the form of  $\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$ . The best alternative is  $\mathcal{A}_{AL-5}$ . To determine the consistency of the parameters  $\gamma_{SC}$  by using the information of Example 1, which is discussed in the form of Table 12.

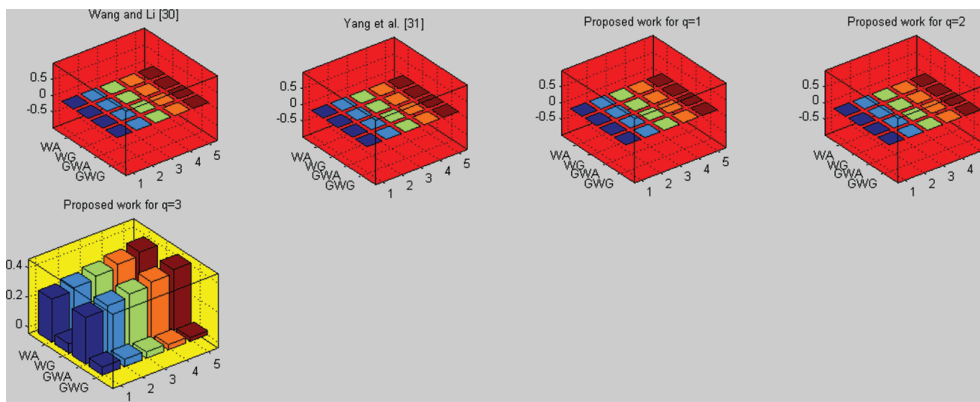


Figure 5 | Geometrical representation for the informations of Table 8.

Table 9 | Comparison between explored work with some existing operators.

Methods	Operators	Score Values	Ranking
Wang and Li [30]	WA	Failed	Failed
	WG	Failed	Failed
	GWA	Failed	Failed
	GWG	Failed	Failed
Yang et al. [31]	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.042, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.132,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.13, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.131,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.186$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-4}$ $\geq \mathcal{A}_{AL-3} \geq \mathcal{A}_{AL-1}$
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.142, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.232,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.23, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.231,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.286$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-4}$ $\geq \mathcal{A}_{AL-3} \geq \mathcal{A}_{AL-1}$
	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.049, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.139,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.19, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.137,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.189$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-4}$ $\geq \mathcal{A}_{AL-3} \geq \mathcal{A}_{AL-1}$
	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.112, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.163,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.133, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.154,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.192$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-4}$ $\geq \mathcal{A}_{AL-3} \geq \mathcal{A}_{AL-1}$

Continued

**Table 9** Comparison between explored work with some existing operators. (Continued)

Methods	Operators	Score Values	Ranking
Proposed method for $q = 1$	WA	Failed	Failed
	WG	Failed	Failed
	GWA	Failed	Failed
	GWG	Failed	Failed
Proposed method for $q = 2$	WA	Failed	Failed
	WG	Failed	Failed
	GWA	Failed	Failed
	GWG	Failed	Failed
Proposed method for $q = 8$	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.042, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.132,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.13, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.131,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.186$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-4}$ $\geq \mathcal{A}_{AL-3} \geq \mathcal{A}_{AL-1}$
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.142, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.232,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.23, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.231,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.286$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-4}$ $\geq \mathcal{A}_{AL-3} \geq \mathcal{A}_{AL-1}$
	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.049, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.139,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.19, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.137,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.189$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-4}$ $\geq \mathcal{A}_{AL-3} \geq \mathcal{A}_{AL-1}$
	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.112, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.163,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.133, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.154,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.192$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-4}$ $\geq \mathcal{A}_{AL-3} \geq \mathcal{A}_{AL-1}$

**Table 10** Original decision matrix.

	$\mathcal{C}_{AT-1}$	$\mathcal{C}_{AT-2}$	$\mathcal{C}_{AT-3}$	$\mathcal{C}_{AT-4}$
$\mathcal{A}_{AL-1}$	((0.2, 0.1), (0.4, 0.3))	((0.12, 0.11), (0.14, 0.13))	((0.22, 0.21), (0.24, 0.23))	((0.32, 0.1), (0.34, 0.3))
$\mathcal{A}_{AL-2}$	((0.3, 0.2), (0.5, 0.4))	((0.13, 0.12), (0.15, 0.14))	((0.23, 0.22), (0.25, 0.24))	((0.33, 0.2), (0.35, 0.4))
$\mathcal{A}_{AL-3}$	((0.4, 0.3), (0.6, 0.3))	((0.14, 0.13), (0.16, 0.13))	((0.24, 0.23), (0.26, 0.23))	((0.34, 0.3), (0.36, 0.3))
$\mathcal{A}_{AL-4}$	((0.5, 0.2), (0.7, 0.2))	((0.15, 0.12), (0.17, 0.12))	((0.25, 0.22), (0.27, 0.22))	((0.35, 0.2), (0.37, 0.2))
$\mathcal{A}_{AL-5}$	((0.6, 0.1), (0.8, 0.1))	((0.16, 0.11), (0.18, 0.11))	((0.26, 0.21), (0.28, 0.21))	((0.36, 0.1), (0.38, 0.1))

**Table 11** Comparison between explored works with some existing operators.

Methods	Operators	Score Values	Ranking
Wang and Li [30]	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2234, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2378,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2435, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2547,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.26457$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2246, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2358,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2416, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2556,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.2678$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2278, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2317,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2419, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2511,$ $\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.2615$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$ $\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$

Continued

**Table 11** | Comparison between explored works with some existing operators. (Continued)

Methods	Operators	Score Values	Ranking
Yang et al. [31]	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2242, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2347,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2455, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2557,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.2611$	
	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1234, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1378,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1435, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1547,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.16457$	
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1246, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1358,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1416, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1556,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.1678$	
	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1278, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1317,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1419, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1511,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.1615$	
	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1242, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1347,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1455, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1557,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.1611$	
	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2234, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2378,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2435, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2547,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.26457$	
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2234, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2378,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2435, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2547,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.26457$	
Proposed method for q = 1	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2234, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2378,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2435, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2547,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.26457$	
	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2234, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2378,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2435, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2547,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.26457$	
Proposed method for q = 2	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.112, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.188,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.197, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.201,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.217$	
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.177, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.193,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.202, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.211,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.228$	
	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.114, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.189,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.199, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.207,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.214$	

Continued

**Table 11** Comparison between explored works with some existing operators. (Continued)

Methods	Operators	Score Values	Ranking
Proposed method for q = 3	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.117, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.192,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.197, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.204,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.218$	
	WA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1234, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1378,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1435, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1547,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.16457$	
	WG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1246, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1358,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1416, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1556,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.1678$	
	GWA	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1278, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1317,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1419, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1511,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.1615$	
	GWG	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1242, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1347,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.1455, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.1557,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
		$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.1611$	

**Table 12** Discussed for different values of the parameter  $\gamma_{SC}$ .

Parameter	Score Values	Ranking Values
$\gamma_{SC} = 2$	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2031, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2105,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
	$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2182, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.226,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.234$	
$\gamma_{SC} = 3$	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2017, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2077,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
	$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2152, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2253,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.2336$	
$\gamma_{SC} = 4$	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2011, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2075,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
	$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2149, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2248,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.2331$	
$\gamma_{SC} = 5$	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.2008, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.2066,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
	$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2145, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2244,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.2329$	
$\gamma_{SC} = 10$	$\mathcal{S}_{SF}(\mathcal{A}_{AL-1}) = 0.1912, \mathcal{S}_{SF}(\mathcal{A}_{AL-2}) = 0.1927,$	$\mathcal{A}_{AL-5} \geq \mathcal{A}_{AL-4} \geq \mathcal{A}_{AL-3}$
	$\mathcal{S}_{SF}(\mathcal{A}_{AL-3}) = 0.2002, \mathcal{S}_{SF}(\mathcal{A}_{AL-4}) = 0.2011,$	$\geq \mathcal{A}_{AL-2} \geq \mathcal{A}_{AL-1}$
	$\mathcal{S}_{SF}(\mathcal{A}_{AL-5}) = 0.2024$	

Therefore, the explored operators based on CQRONFSs are more proficient and more flexible than existing operators, which is discussed in Ref. [30,31]. Hence, the presented approach is extensively powerful and more general than complex intuitionistic normal fuzzy setFSs and complex Pythagorean normal FSs.

## 7. CONCLUSION

One of the most proficient and beneficial theories is called CQROFS, containing the grade of supporting and the grade of supporting against in the form of polar coordinates belonging to unit disc in a complex plane. CQROFS is a proficient technique to address awkward information, although the NFN is examining normal distribution information in anthropogenic action and realistic environment. Based on the

advantages of both notions, in this manuscript, we explored the novel concept of CQRONFS as an important technique to evaluate unreliable and complicated information. Some operational laws based on CQRONFSs are also explored. Additionally, some distance measures are called CQRONFGDM, CQROFNFSDM, two types of CQRONFODMs, and their special cases are discussed. Moreover, weighted averaging, weighted geometric, generalized weighted averaging, and generalized weighted geometric operators based on CQRONFSs are also presented. In last, we solve a numerical example of the MADM problem is shown to justify the proficiency of the presented operators. The advantages, comparative and sensitive analysis are used to express the efficiency and flexibility of the explored approach.

In further research, considering the superiority of new CQRONFSs, we can extend them to some other work based on FSs [32,33], picture FSs [34], PFSs [35], hesitant fuzzy setFSs [36], and so on [37–42].

## CONFLICTS OF INTEREST

The authors declare no conflict of interest.

## AUTHORS' CONTRIBUTIONS

All authors have equally contributed to this manuscript.

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