

## Fuzzifying Geospatial Data to Identify Critical Traffic Areas

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### Abstract

This manuscript proposes a framework to design an artifact that combines traffic data of different sources, addresses their low-penetration rate and imprecision, and enables their analysis. The implemented artifact uses probe data of en-route operations of delivery vehicles and Traffic Message Channel-based records. Both datasets are fuzzified and a type-2 fuzzy logic system is then implemented, to determine the traffic criticality of geographical zones. The output of the system is displayed on a map to serve as an analysis tool. With the practical implementation, it is shown that such insights can be obtained, without large amounts of precise information. However, comprehensive evaluation methods are to be developed to verify the validity of the results.

**Keywords:** Type-2 fuzzy sets, traffic analysis, spatio-temporal data, smart logistics, geohash.

decision-making in transportation, especially in the logistics and delivery domain. Advanced travel information system research seeks to determine where and when traffic congestion occurs intending to reduce the time spent on the roads [20, 21].

In the last few years, many studies have been directed to forecast vehicle speeds, traffic flows, transportation cost, among other traffic variables. Such initiatives are based on one-time series methods, simulations, non-parametric approaches, regression, and neural networks. All of them have in common the requirement of large amounts of precise information, and at times, constant tracking of the vehicles circulating on the roads [9, 13].

One source of data to perform traffic analysis is the recorded by global positioning systems (GPS) devices fitted on vehicles. This data is known as floating-car data and, when the records correspond to the movement of vehicles circulating for specific duties (e.g., taxis and logistic trucks), it is known as probe data [2, 13, 3]. Significant insights such a route circulation patterns can be derived from probe data, as shown by the authors of the present article in earlier efforts [16, 17, 18]. However, there are some limitations of working with probe data, which include neglecting the dynamic changes over time of traffic conditions, incomplete and uncertain data, and how traffic is perceived by people.

Transportation and traffic parameters are defined in uncertain, imprecise, ambiguous, and subjective terms [1, 19]. Thus, methods that enable capturing and processing such characteristics are needed. The concept of type-2 fuzzy sets, proposed by Zadeh in 1975 [22] and further studied by Mendel [11], is a promising approach to deal with the ambiguity and incompleteness of traffic-related data due to its capacity of handling randomness and uncertainties in measurements [9, 10]. This research project proposes a fuzzy-data-driven framework to identify areas that

## 1 Introduction

There is no universal definition of traffic congestion. However, according to Aftabuzzaman [1], the definitions found in literature can be categorized into three groups: (i) demand capacity related, meaning that traffic congestion occurs when travel demand exceeds the existing road capacity; (ii) delay-travel time related, that implies travel time or delay more than the normally incurred under free-flow travel conditions; and (iii) cost-related, that refers to the incremental costs resulting from interference among road users. Moreover, systematic road network-wide assessment is an important indicator for

present traffic anomalies, using probe data from delivery vehicles with a low penetration rate and recorded traffic messages from a traffic information center. The day-to-day data is analyzed on a segment-by-segment basis, contrasted, and classified through type-2 fuzzy set methods. It is intended to address the complexities involved with generalizing traffic conditions from low sampling probe data derived from business operations and the imprecision and ambiguity that dealing with transportation parameters entails. Furthermore, this effort aims at providing an analysis tool for industrial practitioners.

This manuscript is structured as follows: Section 2 introduces the theories on which this research work is grounded as well as some related initiatives. Then, the framework developed in this study is described in Section 3. Section 4 presents the results obtained from an implementation. Finally, Section 5 closes the curtains of this work with a summary and concluding remarks.

## 2 Theoretical Background

This section describes the theories applied to the development of this research effort. Precedent works that attempted to achieve similar goals are also discussed.

### 2.1 Type-2 Fuzzy Sets

Unlike type-1 fuzzy sets, the membership grades of a type-2 fuzzy set (T2 FS) are fuzzy sets themselves instead of crisp values. They were introduced as an extension of traditional fuzzy sets by Zadeh as an alternative to deal with uncertain membership criteria and parameters [22]. Mendel [11] refers to type-2 fuzzy sets as "fuzzy-fuzzy sets that are convenient when the membership functions for a fuzzy set are hard to determine, being one example of this the modeling of words". Formally, a type-2 fuzzy set is defined as follows [22, 10]:

**Definition 1.** A type-2 fuzzy set  $\tilde{A}$ , is characterized by its membership function  $\mu_{\tilde{A}}(x, u)$ , where  $x \in X$  and  $u \in J_x \subseteq [0, 1]$ , that is to say

$$\tilde{A} = ((x, u), \mu_{\tilde{A}}(x, u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0, 1],$$

where  $0 \leq \mu_{\tilde{A}}(x, u) \leq 1$

Type 2-fuzzy sets can also be understood as regions and not as a curve or discrete points as in the case of type-1 fuzzy sets. This characteristic enables to handle in a better way randomness and uncertainty in the data.

However, computations with type-2 fuzzy sets are complex. Thus, most practical implementations and

data modeling are done through one special kind of type-2 fuzzy set called *interval type-2 fuzzy set* (IT2 FS). Interval type-2 fuzzy sets assume a uniform variation of the weighting of the parameters over the whole set, in this way, computations are less intense, and still, the capabilities to process randomness in data is kept [11, 10]. The following definition of IT2 FS [10] is adopted in the present work.

**Definition 2.** Membership grades of every element in type-2 fuzzy sets are type-1 fuzzy sets. If secondary membership grades are equal to 1, then the set is known as an interval type-2 fuzzy set.

Figure 1 presents an example of an IT2 FS, note the so-called *footprint of uncertainty* (FOU) which corresponds to the uncertainty on the left and right end-points. The uniformity observed on the FOU is due to the fact that the figure depicts an IT2 FS.

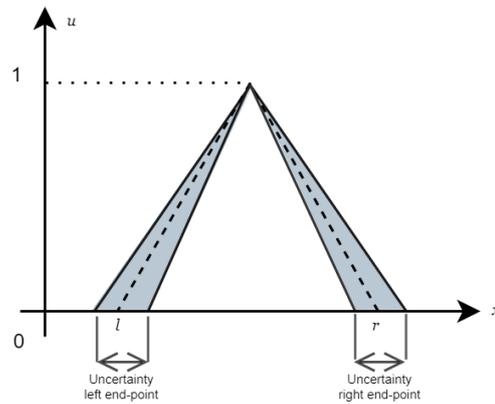


Figure 1: Triangular membership functions depicting a type-2 fuzzy set.

Moreover, according to Pedrycz [14], two considerations need to be taken into account to decide when to use type-2 fuzzy sets. Firstly, the need to apply them must be clear and their use should be straightforward; secondly, there is a sound membership definition procedure that grants the definition of the fuzzy sets. Pedrycz also proposes an example that illustrates how to deal with the aforementioned considerations; in the case of having several datasets coming from different regions that present the influence of locality, it is very likely that they have some variability. Through type-2 fuzzy sets, this variability can be better captured and the result of the aggregation of the datasets can be more faithful to reality. These last statements support the authors' decision of choosing IT2 FS in the development of this work, provided that databases of different nature describing uncertain traffic-related data are used.

## 2.2 Type-2 Fuzzy Sets and Traffic Models

Several preceding initiatives that applied high-order fuzzy sets in traffic-related topics are described in this section.

Authors Li *et. al* [9] built a type-2 fuzzy logic forecasting model for short-term traffic prediction. Historical and real-time data were combined to generate a traffic forecast that performed as well as other approaches reported on the literature. T2 FS performed well handling and including uncertainties derived from the traffic analysis. The main limitations of this work included the usage of interval fuzzy sets for the fuzzy engine and the simple average for the defuzzification process.

Another related initiative is one of Nagarajan *et. al* [12]. The authors proposed a new perspective on traffic control management through the use of triangular IT2 FS and interval neutrosophic sets. The researchers concluded that the application of T2 FS enables the definition of rules that accept uncertainties completely, adaptiveness, and novelty. On the other hand, the computational complexity increases given that the membership functions describing the sets are fuzzy themselves.

In the work of Li *et.al* [10], a long-term forecasting scheme based on IT2 FS was developed. The central limit theorem was applied to convert traffic flow point data into confidence intervals, to obtain the membership functions for the T2 FS. The method developed by the researchers handles the uncertainty and randomness of traffic flow while diminishing the effects of noise from the data. It was found that the use of upper and lower limits to forecasting the results enables a higher prediction of traffic flow with high precision and stable errors.

Contrary to the prior efforts, this manuscript proposes a framework to fuzzify probe data and traffic-related incidents to identify different types of critical traffic areas, while addressing the issues of working with low-penetration data and imprecision in traffic measurement. Moreover, this work seeks to develop an analysis tool that does not require large amounts of data and that eases the interpretation of data coming from two very different data sources.

## 3 Framework and Artifact Design

The design science research for information systems guidelines were adopted to conduct this research work. The reason that led to choose this research methodology is the fact that its employment entails the development of artifacts that enable the extension

of existing knowledge [7]. Moreover, this project was executed in collaboration with two industrial partners based in Switzerland from the postal service and traffic control domains, and with a transdisciplinary approach (i.e., incorporating practical experiences into the solution process [5]).

The framework to identify critical traffic areas is built upon four main components: i) data cleaner; ii) fuzzifier; iii) fuzzy inference engine; and, iv) visualizer. It takes inspiration from the Interval Type-2 Fuzzy Logic System (IT2 FLS) proposed by Mendel [11] and the practical implementation completed by Li *et. al* [9]. Figure 2 depicts these components and their main operations.

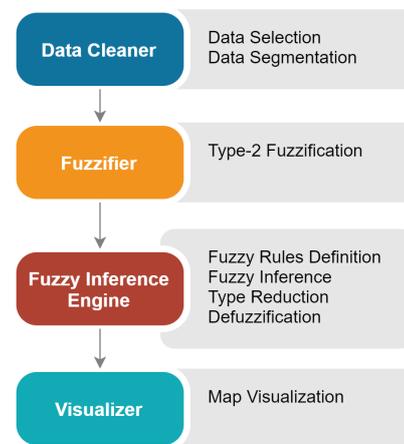


Figure 2: Framework to identify critical traffic areas based on type-2 fuzzy sets.

### 3.1 Data Cleaner

Two different data sources were used in the development of the case study:

1. *DB1 - Probe data*: GPS data of six months of operations of logistic vehicles of a postal company based in Switzerland were used. Its records contained information on the location of the vehicles during their delivery operations, among them mileage, speed, events (e.g., parked, motor on, and motor off), and postal code. Only data points of the area of Bern (Switzerland) were considered. The operations performed on the dataset adhere to the ones completed on a previous effort of the researchers [17] as well as the following:
  - (a) Duplicates, inconsistent and invalid records were removed.
  - (b) Selection of records timestamped from 6 A.M. to 10 A.M.

- (c) Segmentation through level-7 geohashes<sup>1</sup> to perform an average speed deduction on a segment fashion. Moreover, the geohashes are later used for visualization.
2. *DB2 - Traffic Message Channel-based records:* Traffic messages delivered through the Traffic Message Channel (TMC) technology [4] and processed by the Swiss national competence center for traffic, during the years 2018 to 2020, constituted the second data source. These messages are generated by the traffic monitoring responsible (i.e., policemen and municipalities) and they record incidents varying from traffic anomalies, road accidents, to road works and events that might cause traffic delays. The following steps were followed when selecting the data for this case study:
- Messages containing on their description these words (translated from German) were selected: *traffic, stagnant, heavy traffic, lost time, waiting time, traffic suspended until further notice, and traffic obstruction.*
  - Duplicates, inconsistent and invalid records were removed. Records timestamped from 6 A.M. to 10 A.M. were selected.
  - Given that the records contained only the start and end locations where the traffic incident takes place, the locations in between the start and end were augmented onto the database, as these locations are also affected by the traffic anomaly. To this end, the official Swiss TMC location code table provided by our partner was used.
  - The duration in minutes of the traffic incidents was deduced according to the rules explained during workshops accomplished during the development of this project.
  - GPS coordinates of the locations registering anomalies were converted to level-7 geohashes, to be used later for aggregation and visualization purposes.

Moreover, the information recorded on the *DB1* consisted of geospatial information located mostly on the streets of the city of Bern; the data register on *DB2* contains information of traffic events that happened on the main highways surrounding Bern and the main axes that go through the city.

<sup>1</sup><https://www.movable-type.co.uk/scripts/geohash.html>

### 3.2 Fuzzifier

Once the data sources have been cleaned, it is necessary to select the variables of interest. From the *DB1*, the deduced speed *ss* at the different segments was selected; from the *DB2*, the time of traffic anomalies *tt* recorded was the clear variable to be observed. The input variables are crisp values, thus, the fuzzifier is in charge of taking those crisp values to fuzzy ones. To that effect, the parametric principle of justifiable granularity to build type-2 membership functions was applied [15]. By implementing this principle, we ensure that the membership functions are built in a way that they are experimentally justifiable and also exhibit sound semantics.

Moreover, given the nature of the data and as per the guidelines of the principle of justifiable granularity, the datasets were split using clustering [15, 14], and their centroids were used as prototypes of each set; later, the principle is applied to the individual subsets. The resulting membership functions used in this work are Gaussian with fixed standard deviation  $\sigma$  and uncertain mean  $m$ , considering that the mean traffic time over the time, as well as the mean speed with what vehicle circulates, is uncertain and we wanted to capture that variability when adjusting the membership functions. The following expression defines the membership functions for our two variables.

$$A(x; m, \sigma) = e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} \quad m \in [m_1, m_2] \quad (1)$$

being the interval  $m_1, m_2$  defined over the historical data. As type-2 fuzzy sets are defined, the upper and lower membership functions need to be defined as well. The upper membership function is given by:

$$\bar{\mu}_{\tilde{A}}(x) = \begin{cases} A(x; m_1, \sigma) & x \leq m_1 \\ 1 & m_1 \leq x \leq m_2 \\ A(x; m_2, \sigma) & x \geq m_2 \end{cases} \quad (2)$$

And the lower membership function is defined by:

$$\underline{\mu}_{\tilde{A}}(x) = \begin{cases} A(x; m_2, \sigma) & x \leq \frac{m_1+m_2}{2} \\ A(x; m_1, \sigma) & x \geq \frac{m_1+m_2}{2} \end{cases} \quad (3)$$

### 3.3 Fuzzy Inference Engine

As defined by Mendel, rules are the core of a fuzzy logic system [11], and in this case, they were defined from the available data.

The fuzzy rules handle the fuzzy values input coming from the definitions of the type-2 fuzzy sets. They are declared as a set of IF-THEN clauses, with the IF-part

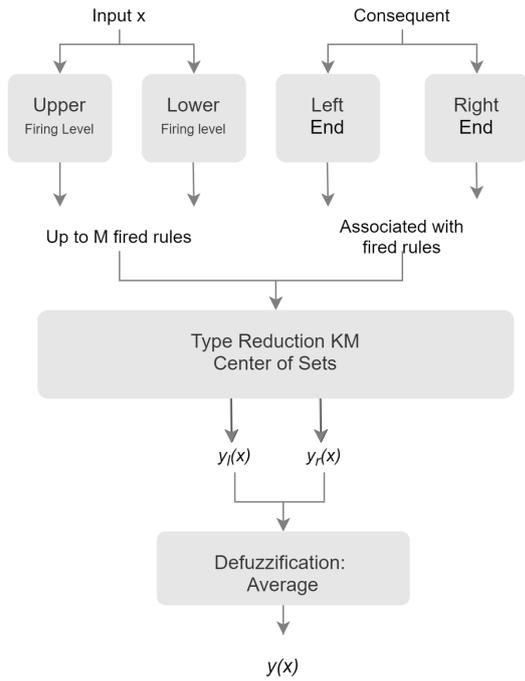


Figure 3: Computations of the Fuzzy Inference Engine, adapted from [11].

being the antecedent and the THEN-portion the consequent. The  $i$ -th rule of the fuzzy engine has the following format:

$$\text{IF } ss \text{ is } F_e^1 \text{ and } tt \text{ is } F_e^2 \text{ THEN } tc \text{ is } G_e^1$$

where  $F_i^l$  is the antecedent,  $G_e^n$  is the consequent, ( $ss$ ) is the input variable that corresponds to the speed of segment, ( $tt$ ) is the traffic time, and ( $tc$ ) is the output, which describes the traffic criticality, meaning how critical is the traffic in a determined zone.

Moreover, the fuzzy inferential engine calculates a so-called ‘firing level’ for each of the rules defined, based on the input and antecedents of the rules; the firing level is then applied to shape the consequent fuzzy sets. The inferential engine implemented in this works follows an interval type-2 Mamdani fuzzy logic system, which uses minimum and product implication models [11].

Given that the consequent will be mapped into a type-2 set, a *type reduction* to type-1 one fuzzy set and a *defuzzification* process needs to be conducted as well to obtain a crisp output. These two steps need to be executed to later use the output and visualize a score on a map. To this end, the Karnik-Mendel algorithms (KM) with center-of-sets defuzzification are used as they are easy to use and implement, and they converge to the solutions monotonically and

super-exponentially fast [8, 11]. After the type reduction takes place, an interval set  $[y_l(x), y_r(x)]$  is obtained; the final defuzzified value is computed by averaging  $[y_l(x)$  and  $y_r(x)]$ . Figure 3 presents a summary of the computations performed at this stage.

### 3.4 Visualizer

The visualizer allows displaying how critical, in terms of traffic, a zone is (output  $tc$  of the IT2 FLS). For that effect, the historical values of speed and recorded traffic time per geohash segment (see Sec. 3.1) are fed to the IT2 FLS. The output results are then aggregated and visualized on a map.

## 4 Results

The implementation results of the artifact built upon the methods explained in Section 3 are presented next. The Python programming language was used to perform the data cleaning, the package `pyit2fls` [6] was used to fuzzify the data, and the library `Folium` for the visualization .

### 4.1 Data Cleaning

After the data cleaning process, the *DB1* was composed of about 315 014 sampling points, and each record was described in terms of 14 fields among which there were position, timestamps, distance, non-traffic vehicular related information, and events depicting internal vehicular state such as power on/off, ignition, movement and waiting. Once the selection, segmentation, and calculation of the segment speed were done, the final database had 34470 records which contained timestamp information, deduced speed, and the GPS coordinates encoded with Geohash.

Regarding the *DB2*, once the cleaning and selection steps were completed, it was formed by 3 008 records and 15 fields among them timestamps of the start and end of the event, coordinates and names of the start and end locations, duration, and description of the event. Furthermore, after the data augmentation process took place (see Sec. 3.1), the total of records in the database was 5 013. These final records contained additional fields depicting the time of the day and the coordinates where the incident happened encoded with Geohash.

### 4.2 Fuzzification and Inference

With the data selection, it was possible to define the type-2 fuzzy sets for the selected variables. As it was

previously defined, the principle of justifiable granularity was applied over both datasets, using the programming language Python and the library `pyit2fls`.

Figure 4 presents the IT2 FS for the variable Segment Speed  $ss$  obtained from the *DB1*; the values on the horizontal axis correspond to the speed in KM/H which has been scaled (1 : 100) for visualization and processing purposes. Figure 5 shows the resulting IT2 FS for the variable Traffic Time, derived from the *DB2*; the values on the horizontal scale correspond to the duration of a traffic anomaly in minutes, which have scaled (1 : 100) as well, for visualization and processing purposes.

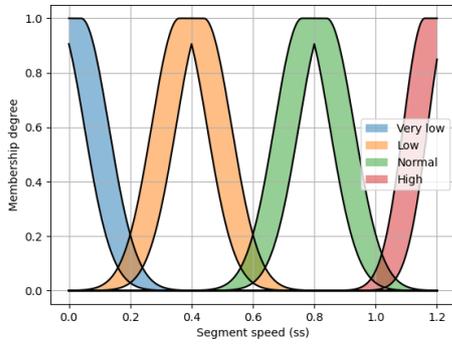


Figure 4: IT2 FS for the variable Segment Speed  $ss$ .

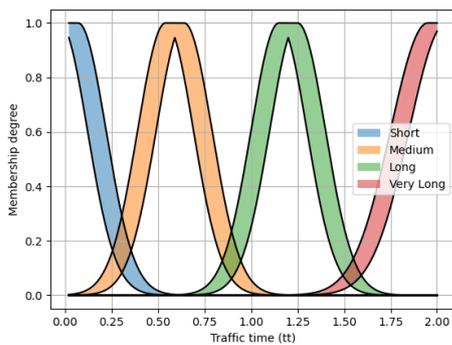


Figure 5: IT2 FS for the variable Traffic Time  $tt$ .

The implementation of the fuzzy system took place afterward. It started with the definition of the fuzzy rules. A total of sixteen rules could be obtained from the combination of the two observed variables. Table 1 shows four of the rules that were used in the implementation of the IT2 FLS. Furthermore, as previously described, the IT2 FLS follows the Mamdani model that was also implemented using the `pyit2fls` package.

Rule	Antecedent		Consequent
	$ss$	$tt$	$tc$
R1	high	short	not critical
R2	normal	medium	low critical
R3	low	long	critical
R4	very low	very long	very critical

Table 1: Sample of the fuzzy if-then rules base of the IT2FS.

The output domain is defined in the interval  $[0, 1]$  for simplicity with Gaussian membership functions with uncertain mean, similar to how the input sets were defined. Figure 6 depicts the output set of the IT2 FLS indicating the traffic criticality  $tc$ .

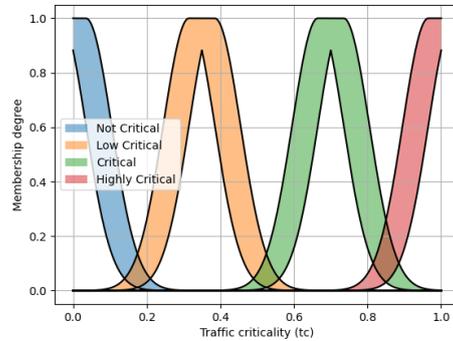


Figure 6: Output type-2 fuzzy set depicting traffic criticality  $tc$ .

As an example of output, consider the value of the segment speed  $ss = 0.45$  meaning "low speed" and the traffic time  $tt = 0.49$  meaning "medium traffic time"; after running the IT2 FLS, the result for the traffic criticality is  $tc = 0.67$  which indicates that the area presenting such characteristics is a "critical one". Figure 7 presents the results of the output  $tc$  for the aforementioned input with the type-2 result and the reduced one.

### 4.3 Visualization

The visualization of the traffic criticality was performed using the crisp output of the IT2 FLS and implemented with Python, using the libraries `Folium` to render the maps and `Libgeohash`<sup>2</sup> to handle the Geohash encoding of the locations. Figure 8 shows an example of the visualization for two areas in the surroundings of Bern. It should be pointed out that the data displayed corresponds to records timestamped in the morning, between 6 A.M. and 10 A.M., as it was explained in Section 3.1.

<sup>2</sup><https://pypi.org/project/libgeohash/>

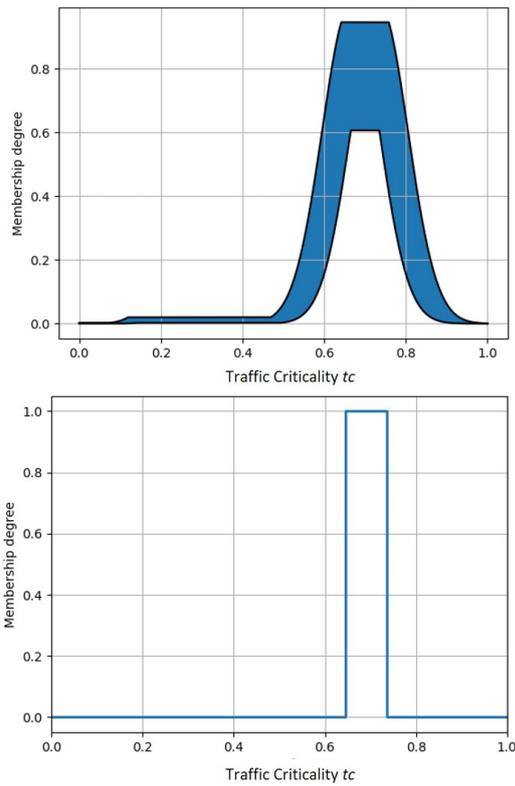


Figure 7: Sample output of the IT2 FLS for  $ss = 0.45$  and  $tt = 0.49$ . The upper graphic denotes the type-2 result and the lower the type-reduced one.

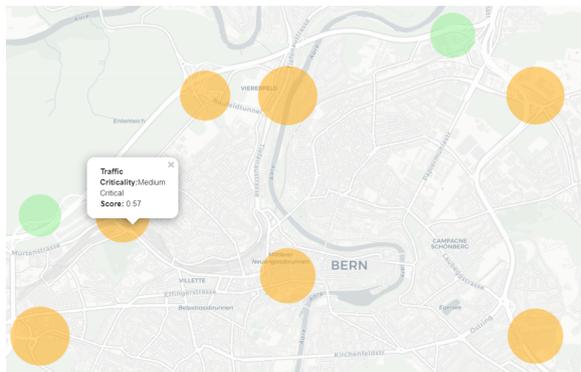


Figure 8: Example of the visualization results.

## 5 Summary and Lessons Learned

This research effort introduces a framework to enable the design of an artifact for the analysis of traffic zones. Powered by type-2 fuzzy sets, this endeavor aims at capturing and handling the uncertainty, incompleteness, and randomness of traffic-related data to represent reality more faithfully.

Four main components are part of the proposed

framework: i) data cleaner, ii) fuzzifier, iii) fuzzy inference engine, and 4) visualizer. The data clear executes processes for selecting data and deducing values of speed from probe data and traffic time duration from TMC-based messages. The fuzzifier applies the principle of parametric justifiable granularity to convert the input crisp data into type-2 fuzzy sets, with Gaussian membership functions with uncertain mean. After the definition of fuzzy rules, the fuzzy inference engine performs an inference, followed by a type reduction and defuzzification to produce an output score. This output represents the traffic criticality of a geographic area. Lastly, the scores obtained from the fuzzy inference engine are aggregated and visualized on a map.

Despite the existing efforts, this research project distinguishes itself for its praxis-oriented nature, transdisciplinarity, and the type of data used in its implementation. The practice knowledge derived from this work is also valuable, it enabled the discovery of potential future collaborations, to get insights that data by separate were not showing, and also to lead to the development of future tools that follow digital ethics and privacy principles. This last statement is claimed since no data resulting from the constant tracking of non-aware users was used and yet, significant results were obtained.

The results of this effort were discussed with the practice partners. As per their knowledge and by comparing the obtained insights to their existing running systems, the results showed consistency and helped to confirm several conjectures they already had and to discover raise awareness of several aspects that were being overlooked. Nevertheless, the authors of this work are aware that a comprehensive validation process must be implemented towards an evaluation of the validity of the results.

Furthermore, this research work contributes with a practical implementation that uses fuzzy sets of higher-order and thus, it also contributes to the assessment of how approximative methods (i.e., fuzzy logic, and computational intelligence) could be applied to develop working solutions that don't need large amounts of precise information (i.e., machine learning solutions) and that can combine data of different nature to give it a new meaning. The results obtained in this effort could be used as a basis to develop further solutions in the field of green logistics, urban planning, and traffic control. Future work will focus on improving the visualization tool so it offers different levels of granularity in time and space as well as the inclusion of real-time information, to conduct prediction tasks.

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