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The Nonsplit Resolving Domination Polynomial of a Graph

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ABSTRACT

Metric representation of a vertex v in a graph G with an ordered subset $R = \{a_1, a_2, \ldots, a_k\}$ of vertices of G is the k-vector $r(v|R) = \left(d(v, a_1), d(v, a_2), \ldots, d(v, a_k)\right)$, where $d(v, a)$ is the distance between v and a in G. The set $R$is called a Resolving set of G, if any two distinct vertices of G have distinct representation with respect to R. The cardinality of a minimum resolving in G is called a dimension of G, and is denoted by $dim(G)$. In a graph $G = (V, E)$, A subset $D \subseteq V$ is a nonsplit resolving dominating set of G if it is a resolving, and nonsplit dominating set of G. The minimum cardinality of a nonsplit resolving dominating set of $G$is known as a nonsplit resolving domination number of $G,$ and is represented by $\gamma_{nsr}(G)$. In network reliability domination polynomial has found its application [20], a resolving set has diverse applications which includes verification of network and its discovery, mastermind game, robot navigation, problems of pattern recognition, image processing, optimization and combinatorial search [19]. Here, we are introducing nonsplit resolving domination polynomial of G. Some properties of the nonsplit Resolving domination polynomial of $G$are studied and nonsplit resolving domination polynomials of some well-known families of graphs are calculated.

Keywords: *Dimension of a graph, Graph polynomial, Resolving domination polynomial, Resolving dominating set.*

1. INTRODUCTION

To analyse the mathematical models, graphs are widely used. At present time the study related to connected domination set has become a very hot topic of research in the field of computer science [1-2]. Connected domination sets can be considered as the virtual backbone of wireless networks. The application of dominating sets and domination number is studied by many research in graph theory to name a few H.L. Abbott, T.V.Wimer, Sampathkumar ,Arumugam, H.B.Walikar, B.Zelinka and many more [21].

An isolated vertex is a vertex with degree zero and a pendant vertex is a vertex with degree one. A pendant edge in a graph Gis an edge incidence with a pendant vertex. The complement of a graph Gis a graph G, with vertex set V (G) in which anytwo vertices are adjacent if and only if they are not adjacent in G. A graph $\overline{K_n}$is the empty (totally disconnected), if no two vertices in it are adjacent. If a graph G consists of disconnected components $H_1$and H_2, then we write $G = H_1 \cup H_2$. If Gconsists of $p \geq 2$ disjoint copies of a graph H, then we write $G = pH$. The corona $G1 \circ G_2$of two graphs $G_1$and $G_2$is the graph obtained by taking one copy of G_1(which has n_1 vertices) and $n_1$copies of $G_2$and then joining the i^{th} vertex of $G_1$to every vertex in the i^{th}copy of G_2. A bipartite graph G is a graph whose vertex set can be partitioned into two subsets $V_1$and V_2such that every edge of Gjoins a vertex of $V_1$with a vertex of V_2. If every vertex of V_1 is joined with every vertex of V_2, then G is said to be complete bipartite graph and denoted by $K_{r,s}$,where $|V_1| =$ rand $|V_2| =$ s. In particular, a complete bipartite graph $K_{1,n-1}$is called a star. The join $G = G_1 + G_2$of graphs $G_1$and $G_2$with disjoint vertex sets $V_1$and $V_2$and edge sets $E_1$and $E_2$is the graph union $G_1 \cup G_2$together with all the edges joining V_1 and V_2 . Graph theory notations and terminologies are not described here [3-8].

Consider nth order connected graph G and $R = \{a_1, a_2, \ldots, a_k\}$ be a subset of vertices of G. The k-vector $r(v/R) = (d(v, w_1), d(v, w_2), \ldots\ldots, d(v, w_k))$ is called the representation (or the code) of a vertex v with respect to a subset R, where $d(v, u)$ is the distance between the vertices v and u in a graph G. The subset R is called a resolving set of G if $r(u|R) \neq r(v|R)$ for every distinct pair of vertices u, v of G. A Resolving set of minimum cardinality is called a minimum resolving set (or a basis) of a graph G and its dimension $dim(G)$ is the *cardinality of a basis* of G. The concepts of resolving set and minimum resolving set has appeared in [16], and later in [17], Slater introduced these terminologies and he has used locating set for Resolving set. He has used the word location number for the cardinality of a minimum resolving set in a graph G. Harary and Melter [9] also studied these concepts. Metric dimension has appeared in various applications of graph theory as a parameter, as diverse as, pharmaceutical chemistry [4], robot navigation [11], combinatorial optimization [12], and sonar and coast guard Loran [16], to name a few. Considerable bibliography is in the paper by Hernando et al. [10], contains.

1.1 Related works

In 2003, the new concept resolving domination in graphs was introduced by Robert and et al. [15]. Resolving dominating set is the one which is both Resolving and dominating set of graph G.Naji and Soner in [13] studied Connected resolving domination of graphs and Subramanian and Arasappan studied the secure Resolving domination in [18]. Recently the new concept of nonsplit Resolving domination of graphs was introduced by Pushpa and Dhananjayamurthy [14]. A connected dominating set is a set of nodes of a network such as WSN (Wireless sensor network), which has small sensing nodes which are capable of computations and wireless communication to monitor geo-fencing of gas and oil pipelines, air pollution, also in health monitoring machines it is used extensively.

One of the branches of algebraic graph theory is graph polynomials and there are many graph polynomials that have been introduced and studied widely. Farrell [26-28] proposed the most general approach to graph polynomials. We refer the interested readers to [1,5, 6, 7], for more information on this topic. Alikani et al. [28], in (2009), introduced domination polynomial in graphs to count the number of dominating sets in a graph of different size. In a polynomial

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

Where a_n is called the leading coefficient of $P(x)$, the polynomial $P(x)$ is called monic for $a_n = 1$.

Motivated by domination polynomial of graphs, here we introduce a nonsplit resolving domination polynomial of a graph and study some properties of the nonsplit resolving domination polynomial. For graphs like paths P_n, cycles C_n ,complete graphs K_n ,complete bipartite graph $K_{r,s}$, star graph $K_{1,n-1}$, bi-star graph $B_{r,s}$ and friendship graph F_n, for join and corona product of graphs, the nonsplit resolving domination polynomial will be found.

Some fundamental results which will be required for many of our arguments in this paper are as follows:

Lemma 1.1. [14] For a positive integer number $n \geq 2$,
$\gamma_{nsr}(K_n) = n - 1$.
$\gamma_{nsr}(K_{1,n-1}) = n - 1$.
$\gamma_{nsr}(P_n) = \begin{cases} 1, & n = 2; \\ n-2, & otherwise. \end{cases}$
$\gamma_{nsr}(C_n) = n - 2, n \geq 3$.
$\gamma_{nsr}(K_{r,s}) = r + s - 2, r \geq s \geq 2$.
$\gamma_{nsr}(F_{1,n}) = \frac{n}{2}$.
$\gamma_{nsr}(B_{r,s}) = n - 4$,
where $B_{r,s}$, for $r \geq s \geq 1$is a bistar graph formed from the two star $K_{1,r}$ and $K_{1,s}$ by joined the central vertices by an edge [22-25].

2. PROPERTIES OF NONSPLIT RESOLVING DOMINATION POLYNOMIAL OF GRAPHS

In this section, we investigate nonsplit resolving domination polynomial of a graph and we study some its properties.

Definition 2.1. *Let G be a graph of order n. The nonsplit resolving domination polynomial of G is the polynomial $P(G, x) = \sum_{i=\gamma_{nsr}}^{n} r(G, i)\, x^i$*

Where $r(G, i)$ is the number of nonsplit resolving dominating sets of G of size i.

To illustrate the concept of nonsplit domination polynomial of a graph.

Consider the graph G_1 shown in figure1.

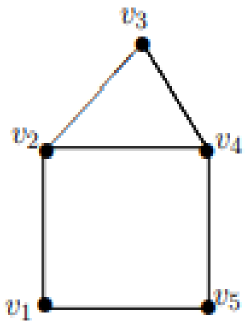

Figure 1 A graph G_1 with $\gamma_{nsr}(G_1) = 2$

It is clear, by easy chick, that $\gamma_{nsr}(G_1) = 2$ and there are only two nonsplit resolving dominating sets of G_1 of size two namely $\{v_1, v_3\}$ and $\{v_3, v_5\}$. For more clarity, the subsets $\{v_1, v_2\}$ and $\{v_4, v_5\}$ are nonsplit dominating sets but it is not a resolving set, and also, the subsets $\{v_1, v_4\}$, $\{v_2, v_5\}$ and $\{v_2, v_4\}$ are resolving sets of G_1 but are not nonsplit sets. There are three nonsplit resolving dominating sets of size three are : $\{v_1, v_2, v_3\}, \{v_1, v_3, v_5\}$ *and* $\{v_3, v_4, v_5\}$. There are five nonsplit resolving dominating sets of size four which they are: $\{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_3, v_5\}, \{v_1, v_2, v_4, v_5\}, \{v_1, v_3, v_4, v_5\}$ *and* $\{v_2, v_3, v_4, v_5\}$. Also there is only one nonsplit resolving dominating set of size five which is $V(G_1)$. Here $r(G_1, 1) = 0, r(G_1, 2) = 2,$

$r(G_1, 3) = 3, r(G_1, 4) = 5$ *and* $r(G_1, 5) = 1$.

Therefore, $P(G_1, x) = x^5 + 5x^4 + 3x^3 + 2x^2$.

In the following result, we present some properties of the coefficients of the nonsplit resolving domination polynomial of a graph.

Proposition 2.2. *Let G be a graph of order* $n \geq 1$. *Then*

Since the only nonsplit resolving dominating set of G with cardinality n is only the set $V(G)$, *so* $r(G, n) = 1$ *and hence* $P(G, x)$ *is a monic polynomial.*

In a graph G without isolated vertices, there are n possible different ways to choose the nonsplit resolving dominating sets of G of size $n - 1$.Therefore,

$r(G, n-1) = n$.

$P(G, x)$ *has no constant term for every* $i < \gamma_{nsr}(G)$. *Hence the nonsplitresolving domination root of* $P(G, x)$ is zero *with multiplicity* $\gamma_{nsr}(G)$.

$P(G, x)$*is strictly increasing function in* $[0, \infty)$.*For any subgraph H of a graph G ,* $\deg(P(G, x)) \geq \deg(P(H, x))$.

Proof. The proof is immediate consequences the definition of the nonsplit resolving domination polynomial of a graph.

Now, we show that, from the nonsplit resolving domination polynomial [29] of a graph G, we can determine the number of isolated vertices in G.

Theorem 2.3. *Let G be a graph of order* $n \geq 2$*with s isolated vertices. If* $P(G, x) = \sum_{i=1}^{n} r(G, i)x^i$ *, is its nonsplit resolving domination polynomial, then* $s = n - r(G, n-1)$.

Proof. Let G be a graph of order $n \geq 2$ and let $S \subseteq V(G)$be the set of all isolated vertices in G, with $|S| = s$. Then for any vertex $v \in V(G) - S$, the set $V(G) - \{v\}$ is a nonsplit resolving dominating set of G. Therefore, $r(G, n-1) = |V(G) - S)| = n - s$, and hence $s = n - r(G, n-1)$.,

From Theorem 5.2 in [14], and by Proposition 2.2, part (3), the following result immediate consequence.

Theorem 2.4. *Let* $\boldsymbol{g}$ *be a graph with n vertices. Then* $\boldsymbol{r(g, 1) \neq 0}$, *if and only if* $\boldsymbol{g \cong k_2 or\ k_1}$.*3. Math and equations*

3. THE NONSPLIT RESOLVING DOMINATION POLYNOMIAL FOR SOME WELL-KNOWN GRAPHS

In this section, we present the explicit formulas of nonsplit resolving domination polynomial for some well-known classes of graphs.

Theorem 3.1. For the complete graph K_n , for $n \geq 2$, $P(K_n, x) = x^{n-1}(x + n)$.

Proof. Let $K_n$be the complete graph with at least two vertices. Then by Lemma 1.1, $\gamma_{nsr}(K_n) = n - 1$ and hence by Proposition 2.2, parts (1) and (2), $r(K_n, n) = 1$ and $r(K_n, n-1) = n$. Therefore

$$P(K_n, x) = \sum_{i=n-1}^{n} r(K_n, i)x^i \quad (1)$$

The nonsplit polynomial representation of a complete graph is given by (1).

Theorem 3.2. *For the trivial graph* $K_1, P(K_1, x) = x$.

Theorem 3.3. *Let* P_n,*for* $n \geq 2$, *be the path. Then*

$$P(P_n, x) = \begin{cases} x(x+2), & if\ n = 2; \\ x^{n-2}\left(x^2 + nx + (n-3)\right). & otherwise. \end{cases}$$

Proof. Let $P_n$be the path with at least two vertices. From Lemma 1.1, we have $\gamma_{nsr}(P_n) = 1$, if $n = 2$ and$\gamma_{nsr}(P_n) = n - 2$, if $n \geq 3$. Hence, we consider the following two cases:

Case 1: If $n = 2$, then $P_2 = K_2$, and hence by Theorem 3.6 $P(P_2, x) = x(x + 2)$. (2)

Case 2: If $n \geq 3$, then by Proposition 2.2, $r(P_n, n) = 1$and $r(P_n, n-1) = n - 1$. Now, let us select nonsplit resolving dominating set $D$of cardinality $n - 2$. Then $V - D$is connected and $V - D \cong P_2, i.e., V - D$is an edge in P_n. Since $P_n$has size $m = n - 1$ and $V(P_n) - e$, for any non-pendant edge $e = uv, u, v \in V(P_n)$ is a nonsplit resolving dominating set of P_n. Then we can choose D by $m - 2 = n - 3$ ways and hence, $r(P_n, n-2) = n - 3$. Therefore,

$$P(P_n, x) = \sum_{i=n=2}^{n} r(P_n, i)x^i$$

$$P(P_n, x) = r(P_n, n-2)x^{n-2} + r(P_n, n-1)x^{n-1} + r(P_n, n)x^n$$

$$P(P_n, x) = (n-3)x^{n-2} + nx^{n-1} + x^n$$

$$P(P_n, x) = x^{n-2}\left(x^2 + nx + (n-3)\right) \quad (3)$$

Theorem 3.4. *Let C_n, for $n \geq 3$, be the cycle graph. Then*

$$P(C_n, x) = \begin{cases} x^2(x+3), & if\ n = 3; \\ x^{n-2}(x^2+nx+n), & otherwise. \end{cases}$$

Proof. Let C_n be the cycle graph of order $n \geq 3$. Then by Lemma 1.1, we have $\gamma_{nsr}(C_n) = 2$, if $n = 3$ and $\gamma_{nsr}(C_n) = n-2$, for every $n \geq 3$. Hence we have the following two cases:

Case 1: If $n = 3$, then $C_3 \cong K_3$, and hence by Theorem 3.6 $P(C_3, x) = x^2(x+3)$. ....(4)

Case 2: If $n \geq 4$, then by Proposition 2.2, $r(C_n, n) = 1$ and $r(C_n, n-1) = n-1$. To select a nonsplit resolving dominating set D of cardinality $n-2$ such that $V-D$ is connected and $|V-D| = 2$. Then $V-D$ is an edge in C_n, and since C_n has size $m = n$ and $V(C_n) - e$, for any edge $e = uv, u, v \in V(C_n)$ is a nonsplit resolving dominating set of C_n. Then we can choose D by n ways and hence, $r(C_n, n-2) = n$. Therefore,

$$P(C_n, x) = \sum_{i=n-2}^{n} r(C_n, i)x^i$$
$$P(C_n, x) = r(C_n, n-2)x^{n-2} + r(C_n, n-1)x^{n-1} + r(C_n, n)x^n$$
$$P(C_n, x) = nx^{n-2} + nx^{n-1} + x^n$$
$$P(C_n, x) = x^{n-2}(x^2+nx+n) \qquad (5)$$

Theorem 3.5. *For the star graph, for* $n \geq 2, P(K_{1,n-1}, x) = x^{n-1}(x+n)$.

Proof. Let $K_{1,n-1}$ be the star graph. Then by Lemma 1.1, $\gamma_{nsr}(K_1, n-1) = n-1$ and hence by Proposition 2.2, parts (1) and (2), $r(K1, n-1, n) = 1$ and

$r(K1, n-1, n-1) = n$.

Therefore $P(K_{1,n-1}, x) = \sum_{i=n-1}^{n} r(K_{1,n-1}, i)x^i$

$$P(K_{1,n-1}, x) = r(K_{1,n-1}, n-1)x^{n-1} + r(K_{1,n-1}, n)x^n$$

$$P(K_{1,n-1}, x) = x^{n-1}(x+n) \qquad (6)$$

Theorem 3.6. For a complete bipartite graph $K_{r,s}$ for $r \geq s \geq 2$, $P(K_{r,s}, x) = x^{r+s-2}(x^2+(r+s)x+rs)$.

Proof. Let $K_{r,s}$ be a complete bipartite graph with $n = r+s$ vertices and let $V_1 = \{u_1, u_2, \ldots, u_r\}$ and $V_2 = \{v_1, v_2, \ldots, v_s\}$ be the vertex partite sets of $K_{r,s}$. Then by Lemma 1.1, $\gamma_{nsr}(K_{r.s}) = r+s-2 = n-2$ and hence by Proposition 2.2, parts (1) and(2), $r(K_{r,s}, n) = 1$ and $r(K_{r,s}, n-1) = n = r+s$. To select a nonsplit resolving dominating set D of $K_{r,s}$ with cardinality $n-2$ such that $V-D$ is connected and $|V-D| = 2$. Then $V-D$ is an edge in $K_{r,s}$, and since $K_{r,s}$ has size $m = rs$ and $V(K_{r,s}) - e$, for any edge $e = u_i v_J, i = 1,2,\ldots,r$ and $j = 1,2,\ldots,s$ is a nonsplit resolving dominating set of $K_{r,s}$. Then we can choose D by rs different ways and hence, $r(K_{r,s}, n-2) = rs$. Therefore,

$$P(K_{r,s}, x) = \sum_{i=r+s-2}^{n} r(K_{r,s}, i)x^i = \sum_{i=r+s-2}^{r+s} r(K_{r,s}, i)x^i$$
$$P(K_{r,s}, x) = r(K_{r,s}, r+s-2)x^{r+s-2} + r(K_{r,s}, r+s-1)x^{r+s-1} + r(K_{r,s}, r+s)x^{r+s}$$
$$P(K_{r,s}, x) = rsx^{r+s-2} + (r+s)x^{r+s-1} + x^{r+s}$$
$$P(K_{r,s}, x) = x^{r+s-2}(x^2+(r+s)x+rs) \qquad (7)$$

Corollary 3.7. *For a complete bipartite graph $K_{\frac{n}{2},\frac{n}{2}}$ for $n \geq 4$,* $P\left(K_{\frac{n}{2},\frac{n}{2}}, x\right) = x^{n-2}\left(x^2+nx+\frac{n^2}{4}\right)$.

Bistar graph, shown in figure 2, is a graph constructed from P_2 by attaching r edges in one vertex and t edges in the other vertex, and is denoted by $S(r,t)$.

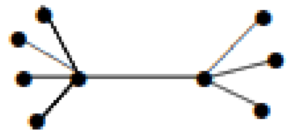

Figure 2 Bistar graph $S(4,3)$

Theorem 3.8. For a bistar graph $S(r,s)$, for $r \geq s \geq 2$, with $n = r+s+2$ vertices, $P(S(r,s), x) = x^n - 2(x^2+nx+1)$.

Proof. Let $S(r,s)$ be the bistar graph with $n = r+s+2$ vertices. Then by easy check one get that $\gamma_{nsr}(S(r,s)) = n-2$. It is clear that there is only one nonsplit resolving dominating set of $S(r.s)$ with size $n-2$, that is containing all the pendant vertices. Thus, $r(S(r,s), n-2) = 1$, and by Proposition 2.2, $r(S(r,s), n-1) = n$ and $r(S(r,s), n) = 1$. Therefore,

$$P(S(r,s), x) = x^n - 2(x^2+nx+1) \qquad (8)$$

In the following result, we compute the nonsplit resolving domination polynomial of the friendship graphs F_p, for $p \geq 2$, where the friendship graph is a graph formed by joining p copies of K_3 with a common vertex, as shown in figure 3. Thus, F_p has $n = 2p+1$ vertices, and $m = 3p$ edges [30-33].

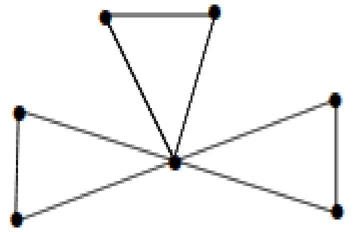

Figure 3 Friendship graph F_3

Theorem 3.9. For a friendship graph F_p, for $p \geq 2$, with $n = 2p + 1$ vertices,

$$P(F_p, x) = x^{2p+1} + 2px^{2p} + \sum_{i=0}^{p} \binom{p}{i} 2^{p-1} x^{p+1}.$$

Proof. Let F_p, for $p \geq 2$ be a friendship graph with $n = 2p + 1$. Then by Lemma 1.1, $\gamma_{nsr}(F_p) = p$. It is clear that the central vertex of F_p does not belong to any nonsplit resolving dominating set D with cardinality at least or equal to $n - 2$, because $V - D$ in this case is disconnected. Thus we can chose a nonsplit resolving dominating set of size p, by chose one vertex from two for every copy of K_3. Then we have 2^p deferent ways and hence, $r(F_p, p) = 2p$ [40]. To chose a nonsplit resolving dominating set of size p + 1, firstly, we chose the both vertices of any copy of K_3 from the p copies of F_p, so we have $\binom{p}{1}$ ways and by chose one vertex from two vertices of the reminding copies of K_3 of F_p. Thus we have 2^{p-1} deferent ways. Hence, $r(F_p, p + 1) = \binom{p}{1} 2^{p-1}$. By continuous in the same way we obtain that, $r(F_p, p + i) = \binom{p}{i} 2^{p-i}$, for every $2 \leq i \leq n - 2 = 2p - 1$ and by Proposition 2.2 $r(F_p, n - 1) = n = 2p + 1$ and $r(F_p, n) = 1$. Therefore

$$P(F_p, x) = \sum_{i=\gamma_{nsr}}^{n} r(F_p, i)\, x^i = \sum_{i=p}^{2p+1} r(F_p, i) x^i$$

$$P(F_p, x) = r(F_p, p)x^p + r(F_p, p + 1)x^{p+1} + r(F_p, p + 2)x^{p+2} + \cdots \ldots \ldots + r(F_p, 2p - 1)x^{2p-1} + r(F_p, 2p)x^{2p} + r(F_p, 2p + 1)x^{2p+1}$$

$$P(F_p, x) = 2^p x^p + \binom{p}{1} 2^{p-1} x^{p+1} + \binom{p}{2} 2^{p-2} x^{p+2} + \binom{p}{p-1} 2^{p-(p-1)} x^{2p-2} + (2p + 1)x^{2p-1} + x^{2p+1}$$

$$P(F_p, x) = \binom{p}{0} 2^p x^p + \binom{p}{1} 2^{p-1} x^{p+1} + \binom{p}{2} 2^{p-2} x^{p+2} + \cdots \ldots + \binom{p}{p-1} 2^{p-(p-1)} x^{2p-2} + (2p + 1)x^{2p-1} + x^{2p+1}$$

$$P(F_p, x) = \sum_{i=0}^{p-1} \binom{p}{i} 2^{p-i} x^{p+i} + (2p + 1)x^{2p} + x^{2p+1}$$

$$P(F_p, x) = x^{2p+1} + (2p + 1)x^{2p} + \sum_{i=0}^{p-1} \binom{p}{i} 2^{p-i} x^{p+i} \quad (9)$$

4. NONSPLIT RESOLVING DOMINATION POLYNOMIAL OF GRAPHS UNION

This section will provide formula for the nonsplit resolving domination polynomial of the graphs union [34-39] and we will apply it on the union of some standard graphs. The following result required to prove our main result.

Theorem 4.1. [14] Let $G_1, G_2, \ldots, G_k$, for $k \geq 2$, be pairwise vertex-disjoint graphs with n_i vertices, $i = 1,2, \ldots, k$. Then for $G = G_1 \cup G_2 \cup \ldots \cup G_k$,

$$\gamma_{nsr}(G) = \left(\sum_{\substack{i=1 \\ i \neq j}}^{k} n_i \right) + \gamma_{nsr}(G_j)$$

such that $n_j - \gamma_{nsr}(G_j) = \max\{n_i - \gamma_{nsr}(G_i): i = 1,2, \ldots, k\}$, for some $1 \leq j \leq k$.

Theorem 4.2. Let $G_1, G_2, \ldots, G_p$, for $p \geq 2$ be pairwise vertex-disjoint connected graphs with vertices $n_1, n_2, \ldots, n_p$, respectively. If G_j, for some $j = 1,2, \ldots, p$, is the Graph with $n_j - \gamma_{nsr}(G_j) = \max\{n_i - \gamma_{nsr}(G_i): 1 \leq i \leq p\}$, *and* $k = \sum_{\substack{i=1 \\ i \neq j}}^{p} n_i$, *then*

$$P\left(\bigcup_{i=1}^{p} G_i, x\right) = x^k P(G_j, x).$$

Proof. Let $G_1, G_2, \ldots, G_p$, for $p \geq 2$ be pairwise vertex-disjoint graphs with $n_1, n_2, \ldots, n_p$ vertices, respectively, let G_j, for some $j = 1,2, \ldots, p$, is the graph with

$$n_j - \gamma_{nsr}(G_j) = max\{n_i - \gamma_{nsr}(G_i): 1 \leq i \leq p\},$$

and let $k = \sum_{\substack{i=1 \\ i \neq j}}^{p} n_i$ Then by Theorem 4.1,

$$\gamma_{nsr}\left(\bigcup_{i=1}^{p} G_i\right) = k + \gamma_{nsr}(G_j) \quad (10)$$

If D is the nonsplit resolving dominating set of $\bigcup_{i=1}^{p} G_i$, and D_j is the nonsplit resolving dominating set of a graph G_j for $i = 1,2, \ldots, p$ and $i \neq j$, then

$$D = \left(\bigcup_{\substack{i=1 \\ i \neq j}}^{p} V(G_i)\right) \cup D_j \quad (11)$$

Thus, the different ways to choose a nonsplite resolving dominating set $\bigcup_{i=1}^{p} G_i$ of *size* $k + \gamma_{nsr}(G_j)$ are the same ways to choose a nonsplit resolving dominating set of Size $\gamma_{nsr}(G_j)$, hence

$$r\left(\bigcup_{i=1}^{p} G_i, k + \gamma_{nsr}(G_j)\right) = r\left(G_j, \gamma_{nsr}(G_j)\right) \quad (12)$$

By continuing with similar discussion, we obtain that

$$r\left(\bigcup_{i=1}^{p} G_i, k + \gamma_{nsr}(G_j) + t\right) = r(G_j, \gamma_{nsr}(G_j) + t),$$

for $1 \leq t \leq n_j - \gamma_{nsr}(G_j)$.

Therefore,

$$P\left(\bigcup_{i=1}^{p} G_i, x\right) = \sum_{t=\gamma_{nsr(\cup G_i)}}^{n} r\left(\bigcup_{i=1}^{p} G_i, t\right) x^t$$

$$P\left(\bigcup_{i=1}^{p} G_i, x\right) = r\left(\bigcup_{i=1}^{p} G_i, k\, \gamma_{nsr}(G_j)\right) x^{k+\gamma_{nsr}(G_j)} + r\left(\bigcup_{i=1}^{p} G_i, k + \gamma_{nsr}(G_j)\right) x^{k+\gamma_{nsr}(G_j)+1} + \cdots \ldots \ldots + + R\left(\bigcup_{i=1}^{p} G_i, n - 1\right) x^{n-1} + r\left(\bigcup_{i=1}^{p} G_i, n\right) x^n \quad (13)$$

Since

$r(\bigcup_{i=1}^{p} G_i, k + \gamma_{nsr}(G_j) + t) = r(G_j, \gamma_{nsr}(G_j) + t), for\ 0 \le t \le n_j - \gamma_{nsr}(G_j)$,

$n = n_1 + n_2 + \cdots .. + n_p, and\ n = k + n_j$.

Then

$$P(\bigcup_{i=1}^{p} G_i, x) = r\left(G_j, \gamma_{nsr}(G_j)\right) x^{k+\gamma_{nsr}(G_j)} + r(G_j, \gamma_{nsr}(G_j) + 1) x^{k+\gamma_{nsr}(G_j)+1} + \cdots + r(G_j, n_{j-1}) x^{k+n_{j-1}} + r(G_j, n_j) x^{n_j}$$

$$P(\bigcup_{i=1}^{p} G_i, x) = x^k \left(r\left(G_j, \gamma_{nsr}(G_j)\right) x^{\gamma_{nsr}(G_j)} + r(G_j, \gamma_{nsr}(G_j) + 1) x^{\gamma_{nsr}(G_j)+1} + \dots .. + r(G_j, n_{j-1}) x^{n_j - 1} + r(G_j, n_j) x^{n_j}\right)$$

$$P(\bigcup_{i=1}^{p} G_i, x) = x^k \sum_{t=\gamma_{nsr}(G_j)}^{n_j} r(G_j, t) x^t$$

$$P(\bigcup_{i=1}^{p} G_i, x) = x^k P(G_j, x) \tag{14}$$

We obtain the *pH graph G* by Theorem 4.2, empty and *p*-components of some standard graphs.

Proposition 4.3 If H is a connected graphs of order n and let G = pH, then $P(G, x) = x^{(p-1)n} P(H, x)$.

Proof. Let H be a connected graphs of order $n$and let $G = pH$. Since

$G = \bigcup_{i=1}^{p} H$.

Then by Theorem 4.1, $\gamma_{nsr}(G) = (\sum_{i=1}^{p-1} n) + \gamma_{nsr}(H) = (p-1)n + \gamma_{nsr}(H)$ and henceas in Theorem 4.2, $k = (p - 1)n$. Therefore, by using Theorem 4.2, we obtain,

$$P(G, x) = x^{(p-1)n} P(H, x) \tag{15}$$

Corollary 4.4. *For* $n \ge 2$,

(1) $P(\overline{K_n}, x) = x^n$.
(2) $P(pK_n, x) = x^{pn-1}(x + n)$.
(3) $P(pP_n, x) = x^{pn-2}(x^2 + nx + n - 3)$.
(4) $P(pC_n, x) = x^{pn-2}(x^2 + nx + n)$.

5. CONCLUSION

Studies on the concept of domination and also dominating sets plays a predominant role in graph theory with enormous applications to the networks in real-world.

Connected dominating polynomials have applications in networks such as WSN (wireless sensor networks), WAN (wireless ad hoc networks) and also in linked with few broadcast problems, recently it has found the application in network reliability as well.

Here we have considered one such domination called nonsplit resolving domination. In particular the nonsplit resolving domination polynomial representation of certain graphs namely paths, cycles, complete graphs, complete bipartite graph, star graph, bi-star graph and friendship graph. The same polynomial representation is considered for Union of graphs and corona product of graphs. This work has some applications in the field of biology, biomedicine and biochemistry.

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