Urban Expansion Survey by Fractal Dimension (Case of Khovd Town, Mongolia)

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ABSTRACT

This study summarizes the dynamics of urban expansion and urbanization in the recent 20 years in Khovd town, Khovd province of Mongolia. The study used fractal geometry to describe urban land use in Khovd using a geographic information system (GIS), and applied the relation of area and perimeter index to measure urban expansion and demonstrate the potential for improving urban planning studies. The urban planning process takes into account the shape of the city, the area perimeter ratio, each land use spatial distribution, and the geometry of urban extent. The shape, land patches distribution, and expansions of the urban area were analyzed based on the created feature layers of the GIS. The GIS feature layers were developed based on the data sets from cloud-free Landsat 5,7,8 images acquired for the nearest same month time frame 2000, 2010, and 2020. The expansion of Khovd town showed relatively stable development and formed a correct shape extent and the areal expansion of urban area was 3238 ha or average annual growth of 162.4 ha/year in 20 years, while perimeter growth was 43325 m (2166 m/year), representing quite compact and relatively stable expansion. Overall, the research indicated that the GIS and remotely sensing methods were more effective to analyse the urban expansion (especially fractal geometry) as well as the urban spatial metric study.

Keywords: *Density index, urban growth, urban envelop, area perimeter relationship*

1. INTRODUCTION

Urbanization is a phenomenon of increasing the number of populations living in cities and the role of cities in social development, and the transition from the social organization of land use in the form of agriculture to the form of urban use. Many of the world's largest cities have expanded into agglomeration as they merge with neighbouring cities. Changes in urban forms and land use are often the direct or indirect result of economic and social change in a country or region. Although cities cover less than 5% of the world's land surface, more than half of the world's population lives in urban areas. In 40 years, 66.2% of the 3.2 million people have settled in cities, and the scale and intensity of urban expansion are significant in Mongolia. These changes have led to an increase in the number of motorized vehicles, air and water pollution, electricity consumption, a reduction of agricultural land, and the loss of biological diversity [1].

Mongolia is a post-communist country, and like many other countries (such as post-socialist countries), planning was very simple, fast, and uninterrupted in each stage of the government-lead project, without any public participation. Physical planning and scientific research using fractal dimensions and density indices are just beginning in Mongolia. Most land use planning studies are conducted without urban fractal analysis, and such studies are very rare in Mongolia. Since the early 1990s, remote sensing (RS) and GIS data sets along with other statistical data have been used as the basis for urban land use planning in Mongolia. [1].

In general, many researchers have discovered and modified many metrics to measure urban expansion, depending on the definition of urban expansion and density. Expansion measurements were a common method, but basic measurements were used, and the trend of scientists is beginning to study the internal structure of the city and its dynamics [2; 3; 4; 5]. The spatial patterns of each city are unique and cannot be effectively described by conventional measures such as length and height. From a RS images, the city shape resembles a unique form and often shows irregularities and complexity with several different dimensions [6].

The fractal dimension index is an accepted method in international urban research and is especially important for accurately determining urban expansion. Concepts from fractals can be used to optimize the spatial structure of cities in future urban planning [7]. Existing urban development information, land use change processes, and fractal dimensions of land use are essential to urban planning and provide a "decision key" to future development [8; 9].

Urban forms and urban land use are different and have a diverse structure compared to the general cartographic shape on a small-scale map. Geometrically, it should be presented in fractal terms. The word ‘fractal’ is derived from ‘frangere’, which means ‘breaking’ or creating uneven fragments [10; 11; 12; 14; 15]. Euclidean geometry defines only integer dimensions (i.e., 0, 1, 2,.) while fractal geometry is using fractional dimensions [16; 17; 18]. Simple dimensions are not sufficient to study urban expansion, internal land use patterns, suburban changes, morphology, and territorial boundaries [19; 20; 21; 22; 23; 24; 25]. Natural objects have fractal dimensions, such as the Koch curve, and urban coverage represents two or three dimensions [26; 27].

Urban expansion generally leads to an increasing cost in the economy, social unfair and environmental problems [28; 29], and one of the possible solutions is the “compact city” development and planning. Compact urban shape extent is one of the most effective solutions for sustainable development under the rapid growth of urbanization [30].

The adjective word “compact” means closely settled together within a comparatively minor space. In two dimensions, a compact shape is a numerical quantity degree to which a shape is compact [31; 30]. The concept of a “compact city” was first proposed by Dantzig and Saaty in 1973 [32], and since then a methodology of measuring urban compactness has been an interesting topic in urban study [33; 30].

A compactness ratio is a key urban geography index for research on urban spatial expansion extent. As quoted by Fan [34] from Chu, changes in urban density will reflect the trend of urban spatial expansion regimes: “as the city develops rapidly, the compactness ratio will decrease; The compactness ratio will increase as the city is in the process of internal filling and reconstruction” [34].

Researchers are beginning to focus on the use of RS and GIS, one of the most advanced methods available today to track urban change over time and predict the future trends [35; 25; 36; 37].

The aim of this study is to measure the shape of the Khovd town and prove the 20-year time-series change of spatial expansion through the fractal geometry of urban land use with the help of mathematical procedures and GIS analysis.

2. MATERIAL AND METHOD

The study area is located in the north-western part of Mongolia (i.e. geographic point location N48^{0}00'15" of latitude and E 91^{0}38'26" of longitude.), at an altitude of 1380 meter above sea level. Khovd town is considered as a major hub for the western economical region and political administration.

The shape, land patches distribution, and perimeter of the urban area were analysed based on the created feature layers of the GIS. The GIS feature layers were developed based using data from Landsat 5 TM, 7 ETM, and 8 OLI satellites for the same time frame 2000, 2010, and 2020. The time-series satellite images were downloaded from the website of the US Geological Survey.

GIS spatial analysis is the most important part of urban expansion measurement, because it uses basic parameters and shape information [1; 36; 37]. The spatial analysis contains conventional overlay, weighted overlay, buffering, network analysis, principal component analysis, etc. GIS spatial analysis is particularly easy and innovative to use for urban shape study and more interactive and demonstrative for the presentation of end results to the decision-makers and public [1; 36]. In this study, we used conventional overlay, weighted overlay, visual comparison methods for data interpretation.

To extract land use classes of the study area, a maximum likelihood classification method [8], one of the well-known parametric supervised classification algorithms was used. For the accurate representation of the urban land use, Landsat imageries (30 m resolution) were reclassed by onsite land surveyed map layers for the detailed classification of land uses, such as low density residential and sparse suburban land.

There are a several methods for estimating fractal dimension and the ‘box-counting dimension’ is used in this research [10; 11; 12; 13; 14; 15]. The box-counting method calculates the number of grid cells required to cover an object entirely. In practice, regular grids should be applied over an object and the number of occupied cells counted. The logarithm of N(r) means the number of occupied cells, versus the logarithm of size of one cell 1/r, and this gives a "Richardson–Mandelbrot plot" [21; 25], a line whose gradient corresponds to the box dimension D [7]. If the trend is linear, an observed object should be counted to be fractal [21]. The box-counting method is used for data—subset X of the map in r scale. The box-counting dimension of a subset *X* of the map is defined by counting the number of unit boxes which intersects *X:* for any $r > 0$, let $N(r)$ denote the minimum number of *n*-dimensional cells of linear scale *r* (side length) needed to cover *X* [38]. Then *X* has dimension *D* if *N(r)* with constant *c* Equation (1):

$$N(r) \approx c\,(1/r)\,D \quad (1)$$

then *X* will with dimension *D* in Equation (2):

$$D = \lim r \rightarrow 0\ [-\log N(r)/\log r] \quad (2)$$

Using this principle, we employ the most general Equation (3) for the calculation of land-use fractal dimension [38; 8]:

$$D = \frac{2\ \log P/4}{\log A} \quad (3)$$

where *D* is fractal dimension, *P* is the external perimeter of urban land-use area, and *A* is the area of urban land use.

To compute the perimeter of each land-use object, first, we need to count each chord length of the object. We used the calculation form according to Batty and Longley [11] and found chord length using the standard triangle equality (Equation 4), where chord has pair of *(x_i,y_i)* and *(x_(i+1),y_(i+1))* coordinates.

$$d_{i,i+1=}[(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2]^{1/2}, i = 1, n-1 \quad (4)$$

Then, after finding each chord length, the perimeter *L* at scale *r* is summed (according to Batty and Longley,[12]) in Equation (5) and using the *L(r)* the area of the land-use *A* is computed:

$$L(r) = \sum_{i=1}^{n-1} d_{i,i+1} \quad (5)$$

In 1822 for the first time Ritter proposed to measure a shape's compactness using a simple ratio of the perimeter *(P)* to the area *(A)* of the shape: *(P/A)* [39]. Since then, many alternatives and modified forms of geometry have been proposed and the most commonly accepted is the "circularity ratio" *4A/P2* by Miller [6] and the compactness ratio $2\sqrt{\pi A}/P$ by Richardson for a compactness measurement [29].

The compactness ratio is calculated as follows:

$$C = 2\sqrt{\pi A} / P \quad (6)$$

Where, *C* is the compactness of urban area; *P* is the perimeter of built area; *A* is the area of built district.

3. RESULTS

The general scope of this study is to measure the shape of the city and to prove the fractal geometry of land use in Khovd with the help of mathematical procedures [40] and GIS of urban analysis [41]. The shape, land patches distribution, and expansion of the urban area were analysed based on the created feature layers of the GIS. The GIS feature layers were developed based on data from cloud-free Landsat images acquired for the nearest same time frame 2000, 2010, and 2020.

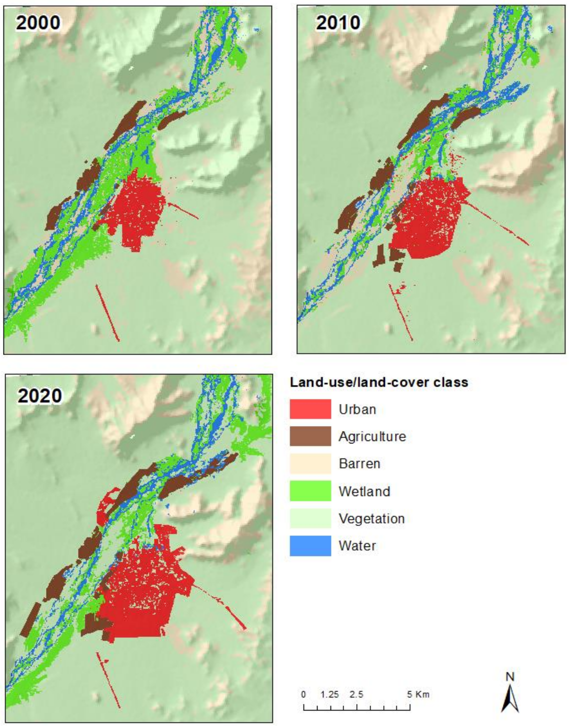

Figure 1. Main land use/land cover classes of Khovd town (2000-2020)

The time-series RS images were downloaded from the website of the US Geological Survey. The Landsat images were additionally rectified to datum WGS84, and resampled using the cubic convolution algorithm with a pixel size of 30 by 30 m for all time-series 3 satellite images. The supervised classification with sampled objects was used for the classification of the land use classes and urban expansion extent. Maximum likelihood classification is one of the well-known parametric algorithms and it was used for supervised classification in our study (Figure 1).

Initially, the Landsat images have been rectified to a UTM/WGS84 system, and resampled using a cubic convolution with a pixel size of 30 by 30 m for all-time series satellite images [36; 37]. Then, the maximum likelihood classification method was applied to extract land use classes of the study site (Figure 1).

In fact, the D-fractal dimension is an index that measures the spatial size, spatial complexity, spatial consistency, or density of an urban area. The greater the value, the greater the expansion of the city: the spatial structure of the city becomes more complex and irregular. On a two-dimensional digital map, $D = 2$ indicates an irregular spatial distribution in the plane of the object. The other limit, $D = 0$, indicates a high local concentration in centre with uniform shape, such as compact circle or square (Figure 2).

Table 1. Time series and spatial change of Khovd town

Year	Popu-lation	Area (ha)	Peri-meter (м)	Fractal dimen-sion	Compactness index
2000	29,911	907	13108	1.0105	0.8142
2010	29,645	1053	13929	1.0087	0.8256
2020	31,626	1288	16288	1.0154	0.7808
ΔT2000-2010	-266	146	821	0.0018	-0.0113
ΔT2010-2020	1981	235	2359	-0.006	0.0447
Mean year change_{10}	-266	14.6	82.1	0.0009	-0.0056
Mean Year change_{20}	198.1	23.5	235.9	-0.003	0.0223

Table 1 shows an increase in the density index and a slight decrease in the fractal dimension (DΔt = 0.006), indicating that in 2000 the urban boundaries were already uniform. Compactness metrics were calculated to determine to what extent of urban footprint approximates circle shape. The urban extent is an outer expansion from built-up areas, surrender natural area, which is pre-urban agriculture, forest land uses within administrative boundary [29]. The compactness index in Khovd has increased (0.01-0.04 or 0.02) and is relatively stable, rigorous in the city centre (Figure 2).

4. DISCUSSION AND CONCLUSION

The study indicates that RS data sets can be efficiently used for urban sprawl study. As it is known new technology can provide new tools for effective urban geography research. Many scholars have invented or modified many particular metrics to measure urban sprawl, depending on the main goals and own points of view on sprawl vs. compact city. Sprawl fractal metrics are a common method, however, they mostly used it like basic measurement and the trend shows scholars start "deeper dig" into the internal structure of the city and its dynamic.

The novelty of our study is the application of a compactness index along with the fractal dimension index which is not widely used in the urban study. The rapid urbanization and fast urban sprawl globally have triggered countless social negative impacts and environmental problems [30]. The urban expansion has accelerated and it adversely impacts the green belt areas, wetlands, water buffer zones, open space, and public land [1].

The compactness index in Khovd has increased over 20 years (0.01-0.04) and is relatively stable and the densest urban land patches are concerted right behind and around the downtown centre. As seen, the expansion of Khovd town shows relatively stable and correct shape form extent and the areal expansion of urban area was 3238 ha or average annual growth of 162.4 ha/year in 20 years, while perimeter growth was 43325 m (2166.25 m/year) representing quite compact and relatively stable expansion.

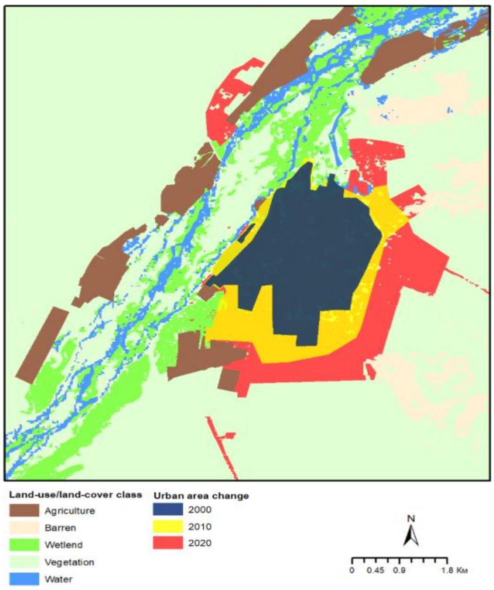

Figure 2. Main land use/land cover change of Khovd town (2000-2020).

Cities are not real natural fractals, but their shape has fractal character, so it has been proved to be a fractal dimension. Numerous empirical studies have shown that fractals meet conditions based on a certain range of scales, indicating that the city can be considered a fractal dimension.

The basic properties of fractal objects are limited by their scale, and its fractal dimension values are based on the scale. A slight decrease in fractal dimension (DΔt = 0.006) indicates that in 20 years the urban envelope was already started to extend but in the right geometrical shape forms. In this paper, the territorial boundary perimeter and built-up area parameter measurement for the fractal dimension and compactness index indicated the city area segregation or fragmentation at in a low rate and leap frog and strip development is not occurred. In short, Khovd town development is sustainable in sense of land management. However, there's a need to pay attention to the minor growing increase of the urban expansion. There's any significant readjustment plan for the low-density slum district reconstruction and therefore urban expansion sustains the most possible way to land adverse exploitation. For that reason, at the municipal level, planners should look at the rural area sustainable development in parallel to the town readjustment.

The results of our study indicated that the GIS and RS methods were more effective to analyse the urban expansion (especially fractal geometry), as well as the urban spatial metric study.

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REFERENCES

[1] Myagmartseren, P., Buyandelger, M., Brandt, S.A., 2017. Implications of a Spatial Multicriteria Decision Analysis for Urban Development in Ulaanbaatar, Mongolia. Mathematical Problems in Engineering. DOI: https://doi.org/10.1155/2017/2819795

[2] Liu Jisheng and Chen Yanguang. 1999. Fractal dimensions of spatial structure of an urban system and the methods of their determination [J]. Geographical Research, 18 (2): 171-178.

[3] De Keersmaecker M L, Frankhauser P, Thomas I. 2003. Using fractal dimensions for characterizing intra-urban diversity: the example of Brussels J. Geographical Analysis, 35(4): 310-328. DOI: https://doi.org/10.1111/j.1538-4632.2003.tb01117.x

[4] Jiang Shiguo and Zhou Yixing. 2006. The fractal aggregating feature of the urban form of Beijing and its practical significance[J]. Geographical Research, 25(1): 204-212. [3] C. Baier, J-P. Katoen, Principles of Model Checking, MIT Press, 2008.

[5] Thomas, I.; Frankhauser, P.; Biernacki, C. 2008. The Morphology of Built-Up Landscapes in Wallonia (Belgium): A Classification Using Fractal Indices. Landsc. Urban Plan. 2008, 84, 99–115. DOI: https://doi.org/10.1016/j.landurbplan.2007.07.002

[6] Herold, M.; Scepan, J.; Clarke, K. 2002. The use of remote sensing and landscape metrics to describe structures and changes in urban land uses. Environ. Plan. A 2002, 34, 1443–1458. DOI: https://doi.org/10.1068/a3496

[7] Chen Yanguang. 2005. The fractal cities and the urban planning[J]. Planning Research, 29: 33-51.

[8] Myagmartseren Purevtseren, Bazarkhand Tsegmid, Myagmarjav Indra and Munkhnaran Sugar. 2018. The Fractal Geometry of Urban Land Use: The Case of Ulaanbaatar City, Mongolia. Land 2018, 7, 67; DOI: https://doi.org/10.3390/land7020067

[9] Zhan, Q.; Wang, J.; Gong, P.; Shi, P. Urban built-up land change detection with roads density and spectral information from multi-temporal Landsat TM data. Int. J. Remote Sens. 2002, 23, 3057–3078. DOI: https://doi.org/10.1080/01431160110104728

[10] Mandelbrot, B. 1983. The Fractal Geometry of Nature; Freeman: San Francisco, CA, USA, 1983.

[11] Batty, M.; Longley, P.A. 1986. The Fractal Simulation of Urban Structure. Environ. Plan. A Econ. Space, 18 DOI: https://doi.org/10.1068/a181143

[12] Batty M, Longley P A .1994. Fractal cities: a geometry of form and function. London: Academic Press.

[13] Addison, P.S. 1997. Fractals and Chaos: An Illustrated Course; Institute of Physics

Publishing: Bristol, UK DOI: https://doi.org/10.1201/9780849384431

[14] Batty M .1991. Cities as fractals: simulating growth and form//Crilly A J, Earmshaw R A, Jones H (Eds.). Fractals and Chaos. New York: Springer-Verlag DOI: https://doi.org/10.1007/978-1-4612-3034-2_4

[15] Frankhauser, P. 1988. Fractal Aspects of Urban Systems. Sonderforschungsbereich 230, 67-76.

[16] Kaye, B.H. 1989. A Random Walk through Fractal Dimensions; VCH Publishers: New York, NY, USA, 1989.

[17] Hastings, H.M.; Sugihara, G. 1994. Fractals: A User's Guide for the Natural Sciences; Oxford University Press: Oxford, UK.

[18] Peng Ruidong, Xie Heping, Ju Yang. 2004. Computation method of Fractal dimension for 2-D digital image[J]. Journal of China University of Mining & Technology, 33(1): 19-2.

[19] Benguigui L. 2000. When and where is a city fractal? J. Environment and Planning B, 27: 507-519. DOI: https://doi.org/10.1068/b2617

[20] Benguigui, L.; Czamanski, D.; Marinov, M.; Portugali, Y. 2000. When and Where Is a City Fractal. Environ. Plan. B Plan. Des. 27, 507–519. DOI: https://doi.org/10.1068/b2617

[21] Terzi, F.; Kaya, H.S. 2008. Analyzing Urban Sprawl Patterns through Fractal Geometry: The Case of Istanbul Metropolitan Area; Paper 144; Centre for Advanced Spatial Analysis: London, UK, 2008. DOI: https://doi.org/10.1080/13658810210137013

[22] Shen, G. 2002. Fractal Dimension and Fractal Growth of Urbanized Areas. Int. J. Geogr. Inf. Sci. 2002, 16, 437–519. DOI: https://doi.org/10.1080/13658810210137013

[23] Feng Jian. 2003. Spatial-temporal evolution of urban morphology and land use structure in Hang Zhou J. Acta Geographica Sinica, 58: 343-353. [24] Ge, M. Lin, Q. 2009. Realizing the Box-counting Method for Calculating Fractal Dimension of Urban Form Based on Remote Sensing Image. Geo-Spat. Inf. Sci., 12, 2652–2670. DOI: https://doi.org/10.1007/s11806-009-0096-1

[24] Ge, M. Lin, Q. 2009. Realizing the Box-counting Method for Calculating Fractal Dimension of Urban Form Based on Remote Sensing Image. Geo-Spat. Inf. Sci., 12, 2652–2670.

[25] Tannier, C.; Thomas, I.; Vuidel, G.; Frankhauser, P. 2011. Fractal Approach to Identifying Urban Boundaries. Geogr. Anal. 2011, 43, 211–227. DOI: https://doi.org/10.1111/j.1538-4632.2011.00814.x

[26] Theiler, J. 1990. Estimating fractal dimension. J. Opt. Soc. Am. A 1990, 7, 1055–1073. DOI: https://doi.org/10.1364/JOSAA.7.001055

[27] Triantakonstantis, D.P. 2012. Urban Growth Prediction Modelling Using Fractals and Theory of Chaos. Open J. Civ. Eng. 2012, 2, 81–86. DOI: https://doi.org/10.4236/ojce.2012.22013

[28] Seto, KC., Kaufmann PK., 2003, Modeling the drivers of urban land use change in the Pearl River delta, China: integrating remote sensing with socioeconomic data. Land Economy 79(1): 106-121 DOI: https://doi.org/10.2307/3147108

[29] Myagmartseren, P.; Myagmarjav, I.; Otgongerel, B. Urban Compactness Measurement of the Ulaanbaatar City. Geogr. Issues 2015, 14, 51–59.

[30] Lan Ting, Shao Guofan, Xu Zhibang, Tang Lina, Sun Lang. 2021. Measuring urban compactness based on functional characterization and human activity intensity by integrating multiple geospatial data sources. Ecological Indicators, Vol. 121, DOI: https://doi.org/10.1016/j.ecolind.2020.107177

[31] Gillman, R., 2002. Geometry and Gerrymandering. Math Horizons, 10 (1), 10-13. DOI: https://doi.org/10.1080/10724117.2002.11974602

[32] G.B. Dantzig, T.L. Saaty. Compact City: A Plan for a liveable Urban Environment. WH Freeman .1973.

[33] Papadimitriou F. The algorithmic complexity of landscapes. Landscape Res., 37 (5). 2012, pp. 591-611 DOI: https://doi.org/10.1080/01426397.2011.650628

[34] Fan, W.; Shi, Y.; Liu, Y. 2012. Application of GIS in quantifying the urban form: A case study of Shanghai. J. Digit. Content Technol. Appl. 2012, 6, 3443–3453. DOI:

[35] Pastzo, V.; Marek, L.; Tucek, P.; Janoska, Z. 2011. Perspectives of fractal geometry in GIS analysis. GIS Ostrav. 2011,1, 232–236.

[36] Gantumur B., F. Wu, B. Vandansambuu, B. Tsegmid, E. Dalaibaatar, and Y. Zhao, 2020. "Spatiotemporal dynamics of urban expansion and its simulation using CA-ANN model in Ulaanbaatar, Mongolia," Geocarto International, pp. 1-16. DOI: https://doi.org/10.1080/10106049.2020.1723714

[37] Myagmartseren P., Indra Myagmarjav, Nergui Enkhtuya, Gantumur Byambakhuu, and Tsegmed Bazarkhand. 2020. Estimating built-up area change in Ulaanbaatar city, Mongolia. The International Archives of The Photogrammetry, Remote Sensing and Spatial Information Sciences, Volume XLIII-B4-2020, DOI: https://doi.org/10.5194/isprs-archives-XLIII-B4-2020-143-2020

[38] Mandelbrot B B, Chen Shouji, Ling Fuhua. 1998. Fractal geometry of the nature [M]. Shanghai: Shanghai Far East.

[39] Frolov, Y.S. Measuring shape of geographical phenomena—History of issues. Sov. Geogr. Rev. Transl. 1975, 16, 676–687. DOI: https://doi.org/10.1080/00385417.1975.10640104

[40] Falconer, K.J. 1999. Fractal Geometry: Mathematical Foundations and Applications; John Wiley & Sons: New York, NY, USA,

[41] Morency, C.; Chapleau, R. 2003. Fractal geometry for the characterisation of urban-related states: Greater Montreal Case. Harmon. Fractal Image Anal. 30–34. Available online: www.fch.vut.cz/lectures/imagesci/download_ejournal/09_C.Morency.pdf (accessed on 17 April 2018).