# Students' Reflective Thinking Ability Based on Their Prior Knowledge in Solving Geometry Problems

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**ABSTRACT**
Mathematics is an important subject in education. Mathematics learning in schools focused on problem-solving ability. One of thinking skills necessary for solving the math problem is reflective thinking. Reflective thinking is one component of higher-order thinking skills. The purpose of this study was to describe students' reflective thinking skills in solving geometry problems with reference indicators based on Surbeck, Han, & Moyer. This research is a qualitative descriptive study with three subjects in class VIII students of one of the state junior high schools in Wonogiri. The subject selection technique used purposive sampling. The research instrument was a test and interview guideline. The validity of the data using triangulation technique. The data analysis used the steps of data reduction, data presentation, and drawing a conclusion. The results of the study concluded that students with high prior knowledge can think reflectively well, this was indicated by students can reach the three components of reflective thinking. Students with moderate and low prior knowledge can reach indicators on reacting component, and can't reach several indicators on elaborating and contemplating components.

***Keywords:*** *Geometry, Prior knowledge, Problem-solving, Reflective thinking.*

## 1. INTRODUCTION

Mathematics is an important subject of education because it plays a role in the development of technology and other disciplines [26]. Mathematics is a systematics subject, where the concepts range from simple to more complex [22]. Mathematics can help humans to solve problems encountered in life. This makes mathematics a compulsory lesson at all levels of education.

Problem solving is the main focus in the learning process of mathematics in schools [14]. Solving math problems is an important aspect of mathematics learning and is a necessity in math curriculum around the world [12]. Problem solving is a process that involves understanding problems, creating problem solving strategy planning, implementing problem solving strategy, and re-examining [20]. The student is said to have problem solving skills if can understand the problem, plan the problem solving, carry out the problem solving, and interpret the solution [13].

Thinking skills are required in problem solving, one of which is reflective thinking. Reflective thinking is one of the must-have abilities and is developed in mathematics learning [4]. Reflective thinking helps students to make sense of experience at the highest level of critical thinking [9]. Reflective thinking encourages students to explore effective and efficient strategies for problem solving. This is related to the need for students to solve problems in daily life [29].

[5] In his research concluded that reflective thinking is important for students to solve math problems. Reflective thinking just not depend on the knowledge of the student, but how to utilize the knowledge that has been possessed to solve the problem at hand. So far, students' mathematical reflective thinking skills have not been developed well by students and teachers [16]. Students' tendency to study mathematics focuses only on memorizing formulas [2].

This condition is supported based on the answer of one of the state junior high school math teachers in Wonogiri, mathematics learning today still tends to use conventional approaches. Teachers emphasize the delivery of materials, formulas, sample question, and procedures for their work. Assessments are generally still oriented to the students' learning outcomes, not on the learning process. The question used for the assessment of the majority of learning outcomes are also still routine questions whose work procedures are clear. This causes students' thinking ability still not be well trained.

Reflective thinking is a way of thinking students actively and carefully in performing complex tasks to find the right solution [6]. Reflective thinking is also defined as an active and rigorous activity referring to knowledge, whereby a person who has reflective thinking will use his attitude and knowledge to make decisions [11]. [27] Explains that reflective thinking is the persistent use of memory to provide important information, connect with personal experience, and draw conclusions on problem solving faced. Reflective thinking can make students think directedly and precisely in connecting the knowledge possessed to solve math problems and draw conclusion.

Reflective thinking in this study refers to the three components presented by [30] namely reacting, elaborating, and contemplating. Reacting is the student's initial response when faced a problem. This response is the ability to mention what information is known and what is not yet understood from the problems encountered. Elaborating is the core of reflective thinking. Students are able to explain what they think, considering the knowledge they have to draw up a problem-solving plan based on experience or knowledge they have had before. Contemplating is a combination of initial reactions with follow-up investigations that prioritize deep and constructive personal understanding. Students can explain the steps of working on question, double-check the steps that have been done and draw the right conclusion from the answers obtained. The reflective thinking ability indicators were described in Table 1.

One of the mathematical competencies that junior high school students must master is geometry. Geometry studies points, lines, fields, and spaces along with their properties, sizes and relation to each other [17]. Geometry is one of the branches of mathematics that helps students related abstract mathematical concepts with something more concrete as a stimulus for deeper understanding [8]. [25] Stated that the majority of students have difficulty in applying formulas to solve geometry problems, not infrequently students don't understand the meaning of the given problem. On the other hand, the purpose of studying geometry isn't just to understand the concept, but can apply it to help solve problems in everyday life.

Problem solving in addition to involving reflective thinking skills is also influenced by students' prior knowledge. This is in line with [19] which states that prior knowledge are abilities that have been attached to a person and related to new things that he will learn. Prior knowledge became the basis of the formation of new concepts in learning. Learning is said to be meaningful when it is able to facilitate students to associate mathematical concepts with real-life applications.

Prior knowledge is one of the factors that determine the success of students in learning. Students are directed to learn through a gradual process from simple concepts to more complex understandings. Until finally the student understands, understands, mastered and is able to apply it in solving the problems of daily life. [1] explains that prior knowledge can be

**Table 1**. Reflective thinking ability indicators

| Components | Reflective Thinking Ability Indicators |
|---|---|
| Reacting | 1. Students has been able to identify the information in the question<br>2. Student has been able to formulate problems that exist in the question<br>3. Students has been able to choose relevant and irrelevant information to solve problems |
| Elaborating | 1. Students has been able to choose the formula or mathematical concept involved to complete question<br>2. Students has been able to connect the given information with the concepts that have been learned to solve problems on the question |
| Contemplating | 1. Students has been able to evaluate the veracity of the completion answers obtained by<br>2. Students has been able to make correct conclusions |

distinguished in three categories, namely high, medium, and low.

Reflective thinking has been widely discussed in various literatures. This study emphasizes that prior knowledge plays an important role in students' reflective thinking ability in problem solving. However, this prior knowledge isn't considered in the learning process. In addition, the indicators of reflective thinking used in this study were developed from previous research, adjusted to the characteristics of the subject and research needs.

Based on the description above, researchers feel the need to follow up on problems that occur by conducting studies related to students' reflective thinking ability in solving geometry problems based on prior knowledge. The results of this study are expected to provide an overview related to reflective thinking of junior high school students in solving geometry problems as well as input for teachers in developing students' reflective thinking skills in the learning process.

## 2. RESEARCH METHODS

The type of research used is descriptive qualitative that describes all facts, phenomena, and symptoms without manipulation [24]. This study describes students' reflective thinking ability to solve geometry problems based on reacting, elaborating, and contemplating components. Subject selection using purposive sampling techniques. The subjects used in this study were grade VIII students of the 2020/2021 School Year one of the State Junior High Schools in Wonogiri, Central Java. The subject determination in this study was classified based on the prior knowledge of high to low using the daily test results of students conducted by the teacher. Classification is conducted using benchmark assessment (PAP) as in Table 2.

**Table 2.** PAP student prior knowledge

| Score | Category |
|---|---|
| 70 < score ≤ 100 | High |
| 30 < score ≤ 70 | Moderate |
| 0 < score ≤ 30 | Low |

Based on the PAP criteria were selected three subjects consisting of one subject for each category. The instrument was test and interview guideline. The geometry problem solving test used to measure students' reflective thinking ability. Test instrument consist of one question as follows.

Fahira has a block water container without a lid. The container is filled with water half the height of the prism. The prism is 28 cm heigh and has a base area of 64cm$^2$. Then Fahira inserts a 6 cm high square pyramid-shaped solid object into the water container to raise the water level by 1 cm. Can Fahira know the volume of a solid object in the shape of a square pyramid? If so, please confirm the volume! If not, please give your reasons?

The validity of the data using triangulation technique performed with how to check data with the same source but different techniques. The results of the test answers are compared with the results of interviews. The data were analyzed with stages of data reduction, data presentation, and conclusion withdrawal.

## 3. RESULT AND DISCUSSION

The research was conducted on Tuesday, June 8, 2021 at one of the state junior high schools in Wonogiri. Reflective thinking component in this study using components stated [30]. Source: (modification [21])

The results of the description of students' reflective thinking ability based on the prior knowledge category are described as follows.

### *3.1. Subjects With High Prior Knowledge*

The answer to Subject S1 with high prior knowledge can be seen in Figure 1.

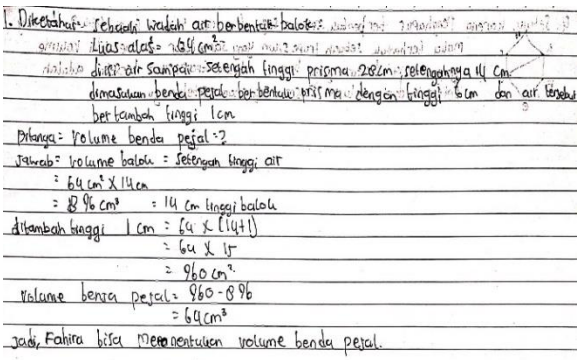


**Figure 1** Sheet S1

Based on Figure 1, S1 rewrote given information. Subject S1 writes down the height of the block is 28 cm, the area of the base is 64 cm$^2$, the height of the pyramid is 6 cm, and the height of the water is 1 cm. Then, S1 uses the block volume formula to find the volume of the water. And then, S1 calculate the volume of the water inside the block after inserting the pyramid. Lastly, S1 determines the volume of the pyramid by

subtracting the volume of water after inserting the pyramid with the volume of water before inserting the pyramid. After calculation, subject S1 got a result of 64 $cm^3$. S1 concludes that Fahira can determine the volume of the pyramid, which is 64 $cm^3$.

In phase reacting, subject S1 has been able to identify and write down the information on the question. Subject S1 has been able to select relevant information to solve problems on the problem. This is in line with the interview with S1 as follows.

*Q: Please explain with your own language what do you understand from the question?*
*S1: It is known that the height of the block is 28 cm, the area of the base is 64$cm^2$, the height of the pyramid is 6 cm, and the height of the water after inserting the pyramid raise 1 cm.*
*Q: Then, what is the problem with this problem?*
*S1: Asked to determine the volume of pyramid*
*Q: Is the given information sufficient to solve the problem?*
*S1: Yes, the given information is enough to solve the problem.*

In phase elaborating, subject S1 has been able to transform problems into mathematical models on the concept of Pythagorean. Subject S1 has been able to choose the right concepts and formulas and is able to connect the information available to solve the problem. This is in line with the interview with S1 as follows.

*Q: Remember what formulas was used in doing this problem?*
*S1: The volume of a block*
*Q: Why the volume of a block? Why not the volume of a pyramid?*
*S1: Because there is known the base area of the block, the height of the block, and the height of the water. We can't use the volume of pyramid because the base area of pyramid is unknown.*

In phase contemplating, subject S1 rechecks the written answer. This can be seen from the answer of the subject S1 who is able to write down what is known, what is asked, and apply the right formulas and concepts to solve the problem. Subject S1 has also been able to write the conclusion correctly. This is in line with the interview with S1 as follows.

*Q: Please explain, how do you solve the problem!*
*S1: The first one is to find the volume of the water before inserting the pyramids. And then, calculate the volume of the water in the block after inserting the pyramids.*
*Q: Then what are you doing?*
*S1: Lastly, subtracting the volume after and before inserting the pyramids.*
*Q: Are you trying to double check what is obtained?*
*S1: Sure*

Based on the result above, noted that subject S1 has been able to think reflectively well.

### *3.2. Subjects With Moderate Prior Knowledge*

The answer to Subject S2 with moderate prior knowledge can be seen in Figure 2.

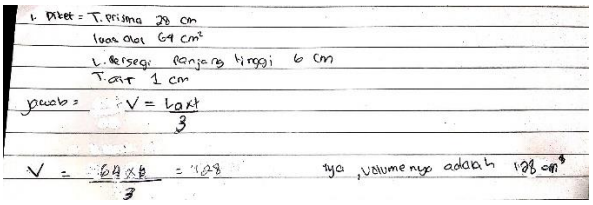


**Figure 2.** Sheet S2

Based on Figure 2, subject S2 writes down given information, the height of the block is 28 cm, the area of the base is 64 $cm^2$, the height of the pyramid is 6 cm, and the height of the water is 1 cm. S1 calculate the volume of pyramids with the existing formula. S2 assume that the base area of pyramids is equal to the base area of the block. After doing calculations, S2 finds that the volume of the pyramids is 128 $cm^3$. S2 concludes that Fahira can determine the volume of pyramids, which is 128 $cm^3$.

In phase reacting, subject S2 has been able to correctly identify and record information about the problem. Subject S2 can correctly identify and write down the information in the question, but S2 was confused and couldn't choose relevant information to solve the problem. This is in line with the interview result with S2 as follows.

*Q: Please explain what do you understand from the question?*
*S2: It is known there is height of the block is 28 cm and the base area is 64$cm^2$. And then, the height of the pyramid is 6 cm, and the height of the water before and after inserting the pyramids.*
*Q: Okay, then what is the problem with this problem?*
*S2: Asked to find the volume of pyramids*
*Q: Is the given information sufficient to solve the problem?*
*S2: I'm actually a little bit confused with this problem. Because to find the volume of pyramids there must be the base area, but it is unknown. So, I assume that the base area of pyramids is equal to the base area of the block.*

In phase elaborating, subject S2 is confused to connect the given information with the concept that have been learned. This is in line with the interview result with S2 as follows.

*Q: Why you assume that the base area of pyramids is equal to the base area of the block?*

*S2: Because I think that isn't assumed the same means we can't find the volume.*
*Q: Regarding to the problem, what formulas was used in doing this problem?*
*S2: The volume of pyramids.*
*Q: Why? Are there other formulas to solve the problem?*
*S2: I don't know, for me isn't.*

In phase contemplating, subject S2 not sure about the correctness of the answer and conclusions. This is in line with the interview result with S2 as follows.

*Q: Please explain, how you completed!*
*S2: Calculate the volume of the pyramids by substituting what is given into the formula for the volume of pyramids.*
*Q: Are you trying to double check the answer? And are you sure about your answer?*
*S2: Yes, I doubled check my answer, but I'm not sure this is correct or not because I still don't understand about this material.*

Subject S2 didn't show good reflective thinking ability because it only met some reaction indicators.

### *3.3. Subject With Low Prior Knowledge*

The answer to the S3 Subject with low prior knowledge can be seen in Figure 3.

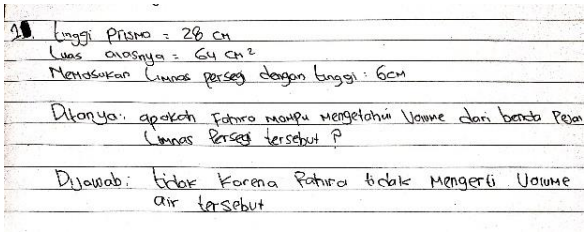


**Figure 3**. Sheet S3

Based on Figure 3, subject S3 writes down the given information, which is the height of the block is 28 cm, the area of the base is 64 $cm^2$, and the height of the pyramid is 6 cm. S3 writes that the problem is to find the volume of pyramids and concludes that the volume of pyramids couldn't be determined because Fahira didn't understand the volume of the water.

In phase reacting, subject S3 has not been able to select relevant information to solve the problem. This is in line with the interview result with S3 as follows.

*Q: What do you understand from the problem?*
*S3: It is known that the height of the block is 28 cm, the area of the base is 64$cm^2$, the height of the pyramid is 6 cm.*
*Q: Then, what is the problem with this problem?*
*S3: Asked to find the volume of pyramids*
*Q: Is the given information sufficient to solve the problem?*
*S3: No, because the volume of wates is unknown.*

In phase elaborating, subject S3 was confused and unable to understand the meaning of the problem. Subject S3 has not been able to choose what concepts and formulas are appropriate to solve the problem, so the answer written is still wrong. This is in line with the interview result with S3 as follows.

*Q: Remember what formulas was used in doing this problem?*
*S3: No, because the given information is incomplete.*
*Q: If you think that the given information is incomplete, what should be added to make it complete?*
*S3: Maybe the volume of water and the base area of the pyramids*

In phase contemplating, subject S3 doesn't double-check the written answer. The conclusion written by the subject S3 is also incorrect. This is in line with the interview result with S3 as follows.

*Q: Please explain, how you completed!*
*S3: The volume of pyramids can't be determined because the given information is incomplete.*
*Q: Are you trying to double check the answer? And are you sure about your answer?*
*S2: Yes, sure.*

The S3 subject has not shown good reflective thinking ability, as it only met some indicators on reacting.

Based on the results of the study, it can be known that subjects with high prior knowledge are able to meet all three components and indicators of reflective thinking in solving geometry problems. Subjects with high prior knowledge has been able to use their knowledge and experience to overcome difficulties in solving problems on the problem. This is because subjects with high prior knowledge always try to solve new problems, so that he is able to solve the problems faced [3].

Subjects in all three categories are able to understand the information on the problem and convert it into mathematical form. The results are relevant to [23] that students are able to turn information into math problems despite difficulties. Subjects with high prior knowledge are able to connect information with concepts that have been learned to solve the problems. This is relevant to [7], [10] students are able to connect concepts for problem solving even though the concept has been studied for

a long time. Two of the three subjects taken did not re-examine the written answer. Re-examining answers minimizes errors, so settlement steps should be carefully evaluated [18].

Based on the description above, it is known that students' reflective thinking differs in each category of prior knowledge. This is in line with Suharna's [28] that differences in prior knowledge cause differences in the ability to solve math problems. [15] stated that the higher prior knowledge of students, the higher the achievement and ability to think reflectively in solving math problems.

## 4. CONCLUSION

Students' reflective thinking differs in each category of prior knowledge. Student with high prior knowledge is able to think reflectively well, this is shown by students being able to meet all three components of reflective thinking. Students with moderate and low prior knowledge are only able to meet some indicators on the reacting component, and have not been able to meet some indicators on the elaborating and contemplating components. Based on these results it is recommended for teachers to emphasize the prior knowledge and reflective thinking ability of students in learning mathematics. Therefore, it is recommended for teachers to developed the reflective thinking skills by providing problem solving questions for the students.

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