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Prospective Elementary Teachers' Difficulties in Solving Conceptual Tasks on Representation of Addition and Subtraction of Fractions

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***Abstract*—This research is motivated by prospective teachers' challenging in solving mathematical problems about fractions, including addition and subtraction of fractions. Their difficulties have been reported by several studies, but not specific on conceptual tasks based on representation of addition and subtraction of fractions. Therefore, this study aims to investigate prospective elementary teachers' difficulties in solving mathematical task on representation of addition and subtraction of fractions. To collect the data, we developed 6 mathematical tasks based on diagram representations of addition and subtraction of fractions. The participants of this study were 101 third-year prospective elementary teachers from a teacher education study program in Riau province, Indonesia. The findings show that prospective elementary teachers have difficulties in solving mathematical task on representation of addition and subtraction of fractions. More than 60% of respondents could not give appropriate answers to the given tasks. They provided unreasonable reasons to explain their incorrect answers to the given mathematical tasks on representation of addition and subtraction of fraction.**

Keywords—addition and subtraction of fractions, conceptual task diagram representation, prospective elementary teachers

I. INTRODUCTION

Fractions have been known as a challenging mathematics topics not only for students but also for prospective elementary teachers and in-service teachers [1]. Many previous studies have showed that prospective and in-service teacher could solve procedural tasks correctly but challenge to explain the meaning underlying the procedure used [2]–[5]. They tend to rely on the standard algorithm when explaining how such a procedure use to solve the tasks related to fractions. For instance, when prospective teachers is given a situation where a student adds two fractions based on the position $\left(\frac{a}{b}+\frac{c}{d}=\frac{a+c}{b+d}\right)$, many of them only explain that it is incorrect procedure, and then they tend to explain the standard algorithm to find the correct answer [6].

A previous study conducted by Toluk-Ucar [7] has showed that prospective teachers were good at performing procedures but most had difficulty producing conceptually correct views of a given fraction statement.

This study inspired us to explore what make it difficult for prospective teachers to explain the conceptual of fractions. We focus this study on addition and subtraction of fractions. Although this topic is not so challenging comparing to multiplication and division of fractions, students' understanding of fractions is confused and incomplete reflected through their difficulties in representing addition and subtraction of fractions using diagram representation [8], and this situation makes teachers have a challenge to explain the meaning of addition and subtraction of fractions such as using diagram representation. Thus,

the research question posed in this study is formulated as "how prospective elementary teachers' difficulties in solving mathematical task on the representation of addition and subtraction of fraction?"

II. METHOD

This study was conducted using a survey and it is part of the authors' main project about prospective elementary teachers' mathematical, didactical, and technological knowledge. The participants were 101 third-year students, 95% female and 5% male. The participants have taken some courses on mathematics and mathematics education, e.g., numbers and algebra, geometry and measurement, mathematics instruction for lower primary school, and mathematics instruction for upper primary school. In this specific paper, we present the data from prospective elementary teachers' answers to six conceptual tasks about addition and subtraction of fractions. Each question asks the respondents to choose a correct diagram representation to the given task, and then they have to explain why such a diagram suitable to explain the task (Figure 1 & Figure 2). The tasks were conducted online using Google form. Some tasks were adapted from Liu [1].

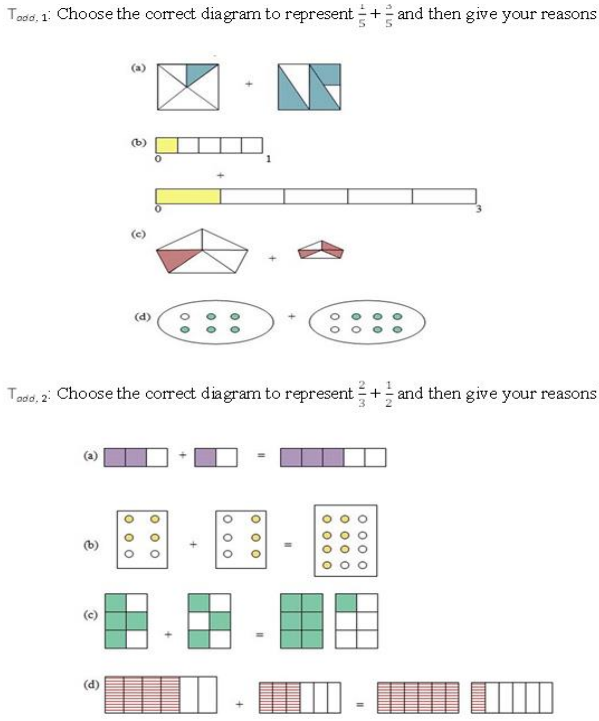

Fig. 1. Three examples of conceptual task on addition of fractions

The data analysis focused on prospective elementary teachers' reason to the given tasks on the representation of addition and subtraction of fraction. First, we consider the affirmation of the representation of addition and subtraction of fraction for several reasons. Second, we see the disagreement with difficulties in solving mathematical task on the representation of addition and subtraction of fraction for several reasons. Finally, we score prospective elementary teachers' answer as 0, 1, and 2. Score 0 is given to incorrect answers and reasoning, score 1 is correct answers with incorrect reasoning, and score 2 for correct answers and reasoning. After categorization prospective elementary teachers' perspectives, we analyses the extent prospective teachers' difficulties in solving mathematical task on the representation of addition and subtraction of fraction.

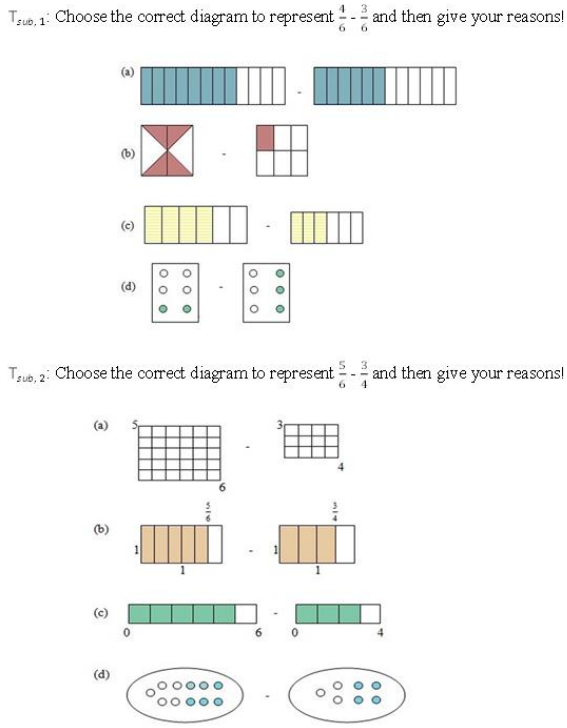

Fig. 2. Three examples of conceptual tasks on subtraction of fractions

III. RESULT

TABLE I. SUMMARY OF PROSPECTIVE ELEMENTARY TEACHERS' SCORES ON ADDITION AND SUBTRACTION OF FRACTIONS

Code	Types of task	Mean score
$T_{add,1}$	Adding fractions with a common denominator	0.04
$T_{add,2}$	Adding fractions with uncommon denominators	0.45
$T_{sub,1}$	Subtracting fractions with a common denominator	0.13
$T_{sub,2}$	Subtracting fractions with uncommon denominators	0.68
$T_{add,3}$	Representing of adding fractions with uncommon denominators	0.94
$T_{sub,3}$	Representing of subtracting fractions with uncommon denominators	0.85

The findings of this study are based on prospective elementary teachers' answers to the given tasks of addition and subtraction of fractions. The study results

generally show that third-year students have difficulties to representation of addition and subtraction of fractions (Table 1). They have more difficulties in choosing the correct answers for addition tasks of fractions than those for subtractions of fractions. It is almost three-quarters of third-year students get score 0 on the addition tasks of fractions (Table 1).

A. *Prospective teachers' responses towards the representation of addition of fractions*

Prospective elementary teachers have several reasons to support their responses toward the representation of addition of fraction (figure 1). We describe their responds based on the score gained by prospective teachers.

Prospective teachers with score 2

Only 14.5% of third-year students reached correct answers and reasoning to the addition tasks. The task of adding fractions with a common denominator becomes the most challenging task, and only 1.0% of the respondents could give correct answers and reasoning. The following example is a correct answer given by respondent 19. "B, because for the first diagram the shaded part is 1 in 5 so it represents the fraction 1/5. Whereas the second diagram, if we match the size of the first part of the diagram, is equal to the 3 boxes of the first diagram. Or if overwriting it will cover 3 parts then it can represent the fraction 3/5".

Respondent 19 could recognize that there was a need a common size of two diagrams in order to be added. To the task of adding fractions with uncommon denominators, the respondents could reach 16.8% correct answers and reasoning. We present an example of correct answer from respondent 76. "Diagram C, because the diagram illustrates the operation of the fraction 2/3 + 1/2 which has the denominator equalized so that it becomes 4/6 + 3/6 and the result is 7/6 or 1 1/6. Diagram C is also the same size and part".

The task of representing adding fractions with uncommon denominators was the most effortless task comparing to the other two tasks. Twenty-five point five percent of respondents could give correct answers and representations.

Prospective teachers with Score 1

There were 18.8% of respondents in this category. The task of adding fractions with a common denominator becomes the most challenging task for them, and only 2.0% of respondents gave a correct answer but insufficient reasoning. For example, the answer from respondent 3 is correct. She chose B, but she only wrote "because the sizes of the fractions and shapes are balanced". This explanation is not so precise what she means with sizes of the shapes are balanced.

Up to 11.9% of respondents reached this category for the task of adding fractions with uncommon denominators. While, the task of representing adding fractions with uncommon denominators reached 42.6% of correct answer but incomplete representation. For example, respondent 69 provided correct diagram representations for the two fractions, but she did not write how the two diagrams could be combined to get the result of adding the two fractions (figure 3).

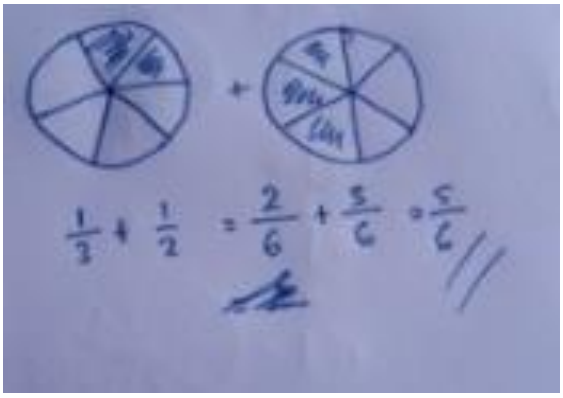

Fig. 3. Diagram representations of adding fractions by respondent 69

Prospective teachers with Score 0

There were 66.7% of respondents in this category. Those respondents neither gave correct answers nor sufficient reasoning to explain their answers. Surprisingly, the task of adding fractions with a common denominator reached the highest percentages, namely 97.0%, and then it was followed by the task of adding fractions with uncommon denominators (71.3%).

We illustrate the answers given by a respondent related to the task of adding fractions with a common denominator. Respondent 7 wrote "C, because the pentagon is 1/5 and the shaded area is 1 in 5 regions. Likewise for 3/5, there are 3 out of 5 shaded areas". This respondent did not realize that the two shapes were not in the same size. Therefore, they could not be combined as a whole.

The task of representing adding fractions with uncommon denominators had 31.7% of incorrect answers and representations. The most common mistake was the respondents represent the two fractions with different whole.

B. *Prospective teachers' responses towards the representation of subtraction of fractions*

Prospective elementary teachers have several reasons to support their responses toward the representation of subtraction of fraction (figure 2). We describe their responds based on the score gained by prospective teachers.

Prospective Teachers with Score 2

Only 16.2% of third-year students reached correct answers and reasoning to the subtraction tasks. The task of subtracting fractions with a common denominator becomes the most challenging task, and only 5.0% of the respondents could give correct answers and reasoning. The following example is a correct answer given by respondent 23. "A. The reason is because 8/12 - 6/12 is equivalent to 4/6 - 3/6".

To the task of subtracting fractions with uncommon denominators, the respondents could reach 20.8% correct answers and reasoning. We present an example of correct answer from respondent 76. "Diagram B because it has the same area but different divisions that represent 5/6 & ¾".

The task of representing subtracting fractions with uncommon denominators reached 22.8% correct answers and reasoning. The respondents could represent the symbolic tasks into diagrams with a common whole/size.

Prospective Teachers with Score 1

There were 23.4% of respondents in this category. The task of subtracting fractions with a common denominator becomes the most challenging task for them, and only 4.0% of respondents gave a correct answer but insufficient reasoning. For example, the answer from respondent 75 is correct, but she did not give any reason to her response.

Up to 26.7% of respondents reached this category for the task of subtracting fractions with uncommon denominators. While, the task of representing subtracting fractions with uncommon denominators reached 39.6% of correct answer but incomplete representation. The reason was the same with the case of adding fractions in which they could present the correct representations for the task, but they did not present the diagram for results of subtracting fractions.

Prospective teachers with Score 0

There were 60.4% of respondents in this category. Those respondents neither gave correct answers nor sufficient reasoning to explain their answers related to subtracting fractions. The task of subtracting fractions with a common denominator reached the highest percentages, namely 91.0%, and then it was followed by the task of subtracting fractions with uncommon denominators (52.5%).

We illustrate the answers given by a respondent related to the task of subtracting fractions with a common denominator. Respondent 31 wrote "C, because the numerator is shaded according to the number of the given task and the number of boxes is in accordance with the denominator of the given task". This respondent did not aware of needing a common size of the two shapes. This kind of misconception often occurred in this study.

IV. Discussion

This study aims to see how prospective elementary teachers' difficulties in solving mathematical task on the representation of addition and subtraction of fraction. The results showed that many prospective teachers difficulties in solving mathematical task on the representation of addition and subtraction of fraction along with several reasons. This finding is in line with the research by Toluk-Ucar [7] who found that prospective teachers had difficulty producing conceptually correct views of a given fraction statement.

The answers by prospective primary school teachers regarding difficulties in solving mathematical task on the representation of addition and subtraction of fraction is 66.7% in solving mathematical task on the representation with several reasons, have difficulties representation of addition of fraction. While 60.4% in solving mathematical task on the representation with several reasons, students' understanding of fractions is confused and incomplete reflected through their difficulties in representing addition and subtraction of fractions using diagram representation [8].

V. Conclusion

According to fractions have been known as a challenging mathematics topics not only for students but also for prospective elementary teachers and in-service teachers [1]. The primary purpose of conceptual knowledge in teacher education is to carry out effective teaching and better student learning.

Acknowledgment

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