The Application of Queueing Theory in the Parking Lot: a Literature Review

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Abstract

Waiting lines or queues are common in life, we often encounter a lot of queueing systems to wait for service regarding both crowd-based and random. Indeed, queing system is ubiquitous from customers queueing to pay at the supermarket, customers coming to make transactions at banks or customers waiting for medical examination at the hospital, etc. Queueing theory is the mathematical study of waiting for lines or queues and is one of the most commonly used mathematical tools for the performance evaluation of systems. Applications of queueing theory are increasing in various fields of life for example the banking sector, healthcare, traffic control, and computer science. This paper reviews the contributions and applications of queueing theory in parking lots. Applications of queueing theory in modeling parking lots processes are reviewed and categorized.

Research purpose:
This paper reviews the contributions and applications of queueing theory in parking lots. Applications of queueing theory in modeling parking lots processes are reviewed and categorized. Our purpose was to identify the leading areas of parking problems as modeled by queueing theory.

Research motivation:
In order to understand the importance, the application of queueing theory in parking lot, this paper aims to investigate research papers to gain insights into queueing theory and queueing models in parking lots.

Research design, approach and method:
This paper is designed as follows. Section 1 of this paper presents the research background. The search approach and papers selection are described in Section 2. In Section 3, we provide the different classifications in using queueing theory in parking lot management. Section 4 includes the list of literature on simulation-based queueing models in the parking lot. In Section 5, a descriptive analysis of the review is presented. Finally, Section 6 presents the summary and conclusions.
We presented a review of 65 papers searched in the database of Google scholar. We collected high-quality papers that were peer-reviewed and published between 1972 and 2021 using structured keywords search and cross-referencing.

Main findings:
In this paper, applications of queueing theory in modeling parking processes have been reviewed and categorized. This paper reviewed the use of queueing theory for the analysis and design of different types of parking systems. Also, we have reviewed the simulation-based queueing models that have been presented. From this review, we found that most queueing theory application articles in parking management were published after 2000. A possible reason for the upward trend since 2000 is the advancement in computational power and software availability.

Practical/managerial implications:
Since parking is an essential component of today's transportation system., improving the system performance is a very important goal. Queueing theory provides an effective and powerful modeling technique that can assist managers in achieving the aforementioned goal.

Keywords: queueing theory, waiting line, parking lots, queueing model

1. RESEARCH BACKGROUND

1.1. Queueing theory overview

1.2.1 History

The history of queueing theory goes back more than one hundred years. The first person to put forth research on queueing theory was Agner Krarup (A. K) Erlang (1/1/1878 – 3/2/1929) - a Danish mathematician, statistician, and engineer who in 1909 published *"The Theory of Probabilities and Telephone Conversations"*, (Erlang, 1909), introduced the world to the concept of telephone queueing theory. In 1917, he published his most important work (Sztrik, 2010), *"Solution of some Problems in the Theory of Probabilities of Significance in Automatic Telephone Exchanges"*, (Erlang, 1917) which contained his classical formulas for call loss and waiting time. Another important contributor to Queueing Theory was David George (D. G) Kendall (15/1/1918 – 23/10/2007) – a British statistician and mathematician, who in 1951 published an article *"Some Problems in the Theory of Queues"*, (Kendall, 1951). In this paper, he refers to a "simple queueing system". Despite there were a huge number of articles on the subject much earlier. However, the research of D. G. Kendall has given a systematic and mathematical approach to the problem of the queue (waiting line). In 1953, D. G. Kendall introduced the A/B/C type queueing notation. Queueing theory has been further developed to this day and various aspects of the queueing theory were discussed by many authors, so much so that it has generated numerous review papers over the years, for example, Saaty (1966), Bhat (1969), Koenigsberg (1982), Bitran & Dasu (1992), Medhi (1997), Worthington (2009), Mandelbaum & Hlynka (2009), Wang et al. (2010), C. Lakshmi & Appa Iyer (2013).

1.2.2 Definition

The queueing theory is concerned with the mathematical modeling and analysis of systems that serve random demands. Queueing theory is mostly regarded as a branch of applied probability theory, the mathematical study of queues or waiting lines; uses mainly the mathematical models to analyze the relationship between arrival interval, service intensity, queueing time, queue length, and other parameters under specific service regulations in the system (Willig, 1999, Sundarapandian, 2009). Queueing theory is one of the oldest and most commonly used quantitative analysis techniques (Chowdhury, 2013) and an important study in modern society (Aronu et al., 2021).

Waiting lines or queues are a common phenomenon in life, in many fields, for example, in a supermarket, in a hospital, at a petrol station, at computer systems, etc. Waiting is one of life's most unpleasant experiences (Sztrik, 2012), and queueing theory addresses it. A waiting line (or queue) is a line or list of customers still waiting to receive certain goods or services from a service center. A queue is formed when a system is overloaded, the number of requests received exceeds the number of requests that the system can handle per unit of time (Aronu et al., 2021). Waiting lines (or queues) help establishments or businesses provide services in an orderly manner (Bhat, 2015). Waiting time and queue length are issues problems related to queueing mechanism (Stasiak, 2016). Queueing model is built to be able to predict queue length and waiting time (Sundarapandian, 2009).

1.2.3 Basic queueing model

Figure 1 illustrates the fundamental queueing model. It can be used to model, e.g., machines or operators processing orders or communication equipment processing information (Adan & Resing, n.d., p. 23):
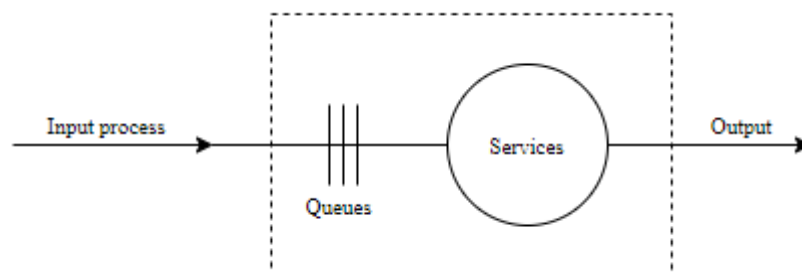

Figure 1: Basic queueing model

The basic queueing model includes three main components: (i) the input process; (ii) the service mechanism; (iii) the queue discipline (Cooper, 1990, Bhat, 2015):

i. The input process is also known as the arrival process. The input process is the process of objects that go to the system and ask to satisfy a certain requirement. The input process can be characterized by the distribution of the

interarrival times of the customers. The input process of requests to the system is a stream of random events and follows certain probability distributions such as fixed-length distribution (D), Poisson distribution, Erlang distribution, and other major distributions according to the statistical laws that it obeys under the stable state.

ii. The service mechanism determination of service rules and statistical rules of service time. The service times are usually assumed to be independent of the arrival process and each other, and service times of servers to be identically distributed, which can be divided into fixed-length distribution (D), exponential distribution (M), general shifted distribution (G), Erlang distribution (Ek).

iii. The queue discipline describes the behavior of customers arrive and find all servers busy. They can leave immediately or wait in a queue. When customers are queueing, they are called to serve depending on the nature of the customer. There are different ways in which customers are called to serve: First in First Out/First Come First Served – FIFO/FCFS; Last in First Out/Last Come First Served – LIFO/LCFS; Service in Random Order – SIRO; Priority service – PNPN; Processor Sharing – PS and so on.

The full queueing system consisting of six components as A / B / m / K / n / D, where (Sztrik, 2012):

- A: probability distribution function for the arrival process
- B: probability distribution function for the service process
- m: number of channels/number of servers
- K: capacity of the system, the maximum number of customers allowed in the queueing system including either being served or waiting for service
- n: population size, number of sources of customers
- D: service discipline

1. 2 Application of queueing theory in parking lot

1.2.1 Parking lot status

Parking areas are an essential component of today's transportation system. With rapid economic growth, the vehicle in urban areas has increased dramatically around the world, creating a greater demand for parking infrastructure, particularly in megacities (Cherian et al., 2016). The scarcity of parking or jams at the parking lots contributes to traffic congestion, pollution, and dissatisfaction among drivers, degrade the quality of life in a variety of ways (Wu et al., 2014). Therefore, they are a big source of concern for government officials, business people, shoppers, workers, and everybody who drives (Weant, 1978). With the significant expansion in the number of private vehicles, it has become increasingly vital to provide users with appropriate parking spaces, which would prevent vehicles from becoming a hazard on the streets and disrupting people's transportation and daily lives (Yuan et al., 2018).

To optimize the traffic organization of the parking lot plays a positive role in alleviating traffic congestion and parking difficulties. Lots of scholars have researched the optimization design of parking facilities and traffic organization. Many models have been developed to evaluate the performance of parking lots, analyze parking systems, and design parking systems for different purposes. In 2003, Sattayhatewa & Smith (2003) developed parking choice models for special events by using the logit function and based on three major factors—driving time, parking cost, and walking time. Benenson et al. (2008) presented a model of parking – PARKAGENT, to study residential parking in the evening hours. In 2013, Wilson proposed a 12-step toolkit for reforming parking requirements, and this toolkit a very useful to parking requirements for multifamily housing, workplaces, and mixed-use developments (Willson, 2013). Ji et al built up a short-term APS forecasting model based on the wavelet neural network (WNN) to forecast available parking spaces (Ji et al., 2015). Cai *et al.* suggested a parking space allocation method (PSAM) at the network level to allocate the parking demand to a specific parking lot and then the parking space (Cai et al., 2018). Shao et al., (2019) studied parking availability prediction with the Long Short Term Memory model (LSTM) in their paper. However, the queueing theory was not used in the above-quoted works.

1.2.2 The usefulness of queueing theory in the parking lot

The queueing theory is a tremendously useful tool because queueing models require relatively little data and are simple and fast to use. The modelers can be used to swiftly analyze and compare numerous options for providing service due to this simplicity and speed. Queuing models can be useful in gaining insights into the appropriate degree of specialization or flexibility to use in organizing resources, as well as the impact of various priority schemes for determining service order among patients, beyond the most basic issue of determining how much capacity is needed to achieve a specified service standard. With the rising power of information technology, numerical approaches and simulation models

can be utilized alongside classical queueing theory to gain a better understanding of real-world queueing systems (Shortle et al., 2018).

The parking systems can be the analyzed as mass service systems (Krpan et al., 2017) due to which have key features are occupancy, turnover, duration of stay per user, acceptability of parking charges, as well as potential identification of the modal split across alternative means of transportation (Basarić et al., 2013). At the same time, the arrivals of vehicles and length of parking time can be seen as random (stochastic). Corresponding theoretical distributions can be applied for studying the arrivals of vehicles and the parking duration. For that reason, it is possible to apply the analytical approach, i.e. using formulas set out by the queuing theory to calculate the parking functioning ratio (Basarić et al., 2013, Maršanić et al., 2011), to analyze the problems of waiting lines at a parking system entrance, with the cars that arrive at random and ask for parking tickets (Maršanić et al., 2010). The use of queueing theory is an attempt to save costs by reducing inefficiencies and delays. There are many problems in parking systems that can be solved using queueing theory in operational research. The parking area is a complex system, which represents a queuing system with the following structure: vehicles form a waiting line or not (depending on the current situation) to be served (parked) in a parking section and after the service has been completed (certain length of parking time), they exit the system.

The application of queuing theory to parking problems has not yet been thoroughly explored and presented to the public in the international scientific and professional literature. In the existing literature, the application of queuing theory focuses to reduce the construction cost, evaluate and improve the garage performance, optimize the use of parking space, estimate the appropriate parking price, availability of the parking, and parking demand.

1.3 Summary

Understanding the importance the application of queueing theory in parking lot, this paper aims to investigate research papers to gain insights into queueing theory and queueing models in general in parking lots. The rest of this paper is laid out as follows. The search approach and papers selection are described in Section 2. In Section 3, we provide the different classifications in using queueing theory in parking lot management. Section 4 includes the list of literature on simulation-based queueing models in the parking lot. In Section 5, a descriptive analysis of the review is presented. Finally, Section 6 presents the summary and conclusions.

2. RESEARCH METHODOLOGIES

Different research papers on the topic published between 1972 and 2021 have been reviewed. In the initial stage, all articles (except for literature review papers) were reviewed relating to the application of queueing theory in parking lots searched in the database of Google scholar. The keywords applied to search for the articles were Queueing Theory (or Queuing Theory), Queue Model or Queueing Model, Queue (or Queueing, Waiting line), Parking Lots or Parking or Parking Spaces. During the second stage, the articles on queuing models and their applications in parking lot management were studied. Most collected articles were related to the stated criteria. After the collection of the initial set of papers, there have been works of classification, titles, abstracts, and conclusions screening to choose the appropriate papers to review. The reviewed articles are (directly or indirectly) related to queueing theory and its applications research in parking lots. We selected and reviewed 65 papers reported during 1972 – 2021.

3. RESEARCH VARIETIES AND CLASSIFICATION

In the existing literature, various queueing models have been studied for parking systems, among which the M/M/s queue is the most widely used model. In this section, the classification methods are classified into two parking groups: parking systems design and parking system analysis. These works are studied and implemented in the parking problem.

3.1 Parking systems design

3.1.1 Automated parking system

Li & Miao (2020) proposed a parking system - automated stereo-garage to reduce the construction cost and improve the garage performance. To achieve the goal, they designed two scheduling strategies: garage efficiency-oriented strategy and customer waiting time-oriented strategy. The queueing theory used to analyzed service performance after the operation processes of the garage during busy periods is simulated. **Senapati & Khilar (2020)** proposed an automated parking service with the help of vehicles and the Road Side Unit (RSU). The M/M/1 queueing model was used to compute total service time (TST) for the overall parking service. Total service time is the total searching time, parking fee collection time, and vehicle entry time into a predefined parking spot. **Pypno & Sierpi (2013)** presented the issues of parking in the cities and proposed the idea of a multi-story, overground garage - Modeling of The Operation of The Multi-Story Automated Garage with A

Big Capacity. The queuing theory has been used in the modeling process of the operation of the vehicle, description of the process of entering and exiting. From there, the efficiency of the parking operation in a multi-story garage was assessed. **Min & Shidong (2015)** constructed a mathematical model on queuing theory for a lift-sliding solid garage, and some important performance parameters are analyzed based on the queuing model. They analyzed the performance of the garage by a M / M / n / ∞ / FCFS queuing model. The purpose was the planning and designing a reasonable and efficient garage. The authors also conducted studies and made a comparison the system's operation strategies in order to achieve an optimized operating strategy. The entire analysis and conclusions are important for solid garage design as well as improving the garage's service performance.

3.1.2 Smart parking

Lu et al. (2009, 2010) proposed a new VANET-based intelligent parking scheme for large parking lots. They analyzed performance through extensive simulations which demonstrate the efficiency and practicality of the proposed scheme. They used the M/G/c/c queuing model to estimate the blocking probability B. The blocking probability B is a stable statistic, which denotes the probability that a vehicle could be blocked, i.e., the parking lot is full when the vehicle arrives. As so as, the parking lot's capacity and blocking probability can be disseminated to the vehicles that run on the road. **Geng & Cassandras (2011, 2012, 2013)** proposed a dynamic resource allocation model based on queuing theory and user objective function of destination distance and parking cost, but various uncertainties in the real environment may impair system performance. **Belkhala et al. (2019)** studied and set up a smart parking system. Queue modeling helped them to test the performance of the system and its ability to manage requests. **Du & Gong (2016)** proposed a decentralized and coordinated online parking mechanism. Mitigated the competition among vehicles helps alleviate parking congestion at numerous parking facilities. They modeled the service of the parking facility as an M/M/c queuing model. **Nugraha & Tanamas (2017)** designed a new reservation parking system and built a simulator to test that system performance and compare it to conventional parking systems and conventional reservation parking systems based on queueing theory. **Sutjarittham et al. (2019)** outlined experiences in designing and deploying a monitoring system for a real car park at their university campus. They developed a car park model to determine the potential of new offerings. Given the rate of arrival

and departure, they model the car park as an M/M/1/C queue and simulator in R to quantify the probability of rejection considering a futuristic scenario. The primary aim would be to optimize the use of parking space for generating revenue from shared cars with minimal impact on private car users.

3.1.3 Normal/another parking system

Maršanić et al. (2010) applied queueing theory to defined the optimal number of servers (ramps) and the required capacity (number of parking spaces) in closed parking areas. **Yan et al. (2015)** used an M/M/c/c queueing model to estimate the availability of the parking spots in the parking lot, model this factor based on the arrival and service rate of the cars and the capacity of the parking lot. **Larson & Sasanuma (2007)** developed an M/M/1 queueing model of the parking problem after reviewed various road pricing (RP) and parking pricing (PP) schemes for implementing congestion pricing (CP). **Hauer & Templeton (1972)** presented queueing model in parking system analysis for applied to a parking lot design problem. **Guo (2020)** optimized of traffic organization of a parking lot; proposed improvement measures from three aspects of parking lot entrance and exit, internal traffic organization, and external traffic organization. The parking lot's entrance and exit were regarded as a queueing model. To measure the entrance and exit queuing model, the authors used the queueing model parameters. **Caliskan et al. (2007)** built a parking model based on queueing theory and used a continuous-time homogeneous Markov model to estimate the future occupancy of parking lots to enable each vehicle to choose an appropriate parking lot. **Zhu et al. (2018)** presented a method to optimize the speed of vehicle access for three-dimensional garage used queueing model M/M/s.

3.2 Parking system analysis

Queueing models are used to gain information about activities within a parking system. We classified these into three subareas: availability of parking and parking demand, parking performance, appropriate parking price estimation.

3.2.1 The availability of parking and parking demand

Abdeen et al. (2021) used an M/M/c/c queueing model to estimate the availability of the parking spots in the parking lot. They model this factor based on the arrival and service rate of the cars and the capacity of the parking lot. **Kawakami et al. (1994)** proposed a method for forecasting parking demand at individual parking lots by traffic assignment model used M/M/s(∞) to estimate

the parking waiting time. The proposed model can be used to forecast parking demand for each parking lot, as well as to assess the effect of parking policies. **Jung & Lee (2014)** conducted parking demand-supply status of attached car parks analysis of 13 big discount stores' in Busan based on queueing theory and simulation using Arena 10.0. **Chiara & Cheah (2017)** described the parking facility at a mall as a queueing system. **Millard-Ball et al. (2014)** derived an empirical relationship between average occupancy and the probability that a driver finds a block full, as well as between average occupancy and the number of blocks cruised, using queueing theory as a foundation. **Sukmana & Lesmono (1999)** built the M/M/29/29 model that describes the stochastic nature of the parking situation to determine the probability that all parking bays are occupied. **Pel & Chaniotakis (2017)** proposed a G/G/c queuing model for computing endogenous parking probability, as well as admissible maximum search times. They used simulation to approximate the parking probability for any particular parking site. **Klappenecker et al. (2014)** proposed a method to predict the availability of parking resources based on the M/M/n/n queueing model. **Xiao et al. (2018)** suggested a model-based practical framework for predicting future occupancy based on historical occupancy data. They used to M\ M\C \C queueing model to describe the stochastic occupancy change of a parking facility. **Atif et al. (2020)** used M/M/C model to the estimated arrival times are used to predict the current parking lot states to infer the future parking state, considering the expected arrival time to each parking lot. **B. Li et al. (2017)** proposed a smart parking guidance algorithm by considering various preferences of drivers and three representative decision factors: walk duration, parking fee, and the number of vacant parking spaces. They used to queue model to quantity the availability degree of vacant parking spaces in a parking facility. **Debnath & Serpen (2015)** presented a study for the development of a real-time scheduling algorithm for a multi-story and fully automated parking structure with a group of elevators. They used to queueing model of the queueing theory to calculate elevator count and parking spaces on each floor under an assumed customer arrival rate and mean service rate. **Ma et al. (2017)** introduced a smart on-street parking system to predict parking occupancy and provide a routing strategy using cloud-based analytics. They developed a prediction algorithm based on transient queueing theory and Laplace transform to predict parking occupancy thus predicting open parking locations. The queueing model was applied to predict the probability of finding a parking space as well as the future occupancy level based on historical

arriving rate data and parking rate for each bay. **Mohd Bukari et al. (2019)** identified the availability of parking spaces to produce a quantitative assessment of parking and to analyze the sufficiency of parking spaces in Muar town's chosen area. They used Erlang's loss model to calculate the likelihood that a client may abandon the system or facility due to insufficient parking lots.

3.2.2 The parking performance

Kokolaki et al. (2012) used queueing models for the analytical investigation of the parking system's performance to assess the effectiveness of the parking search process such as the parking search time and route length, and the proximity of the found/assigned parking spot. **Bajpai & Maneesha (2015)** assessed the situation and made recommendations to reduce the queue in the parking areas lots of one of Dubai's major shopping malls using Little's Theorem, the M/M/s queuing model along with simulation software to prepares for expo 2020. According to the simulation results, increasing the number of servers (entry points) to at least three times present capacity reduces the wait time to under a minute during peak hours. The parking lot will be even less congested as a result of this. **Maršanić et al. (2011)** presented the application of the queuing theory in the optimal dimensioning of parking areas. They have calculated the number of parking spaces and therefore defined the required parking area capacity. The objective of the paper was to demonstrate that by applying the queuing theory the optimal number of servers (ramps) and the required capacity (number of parking spaces) in closed parking areas can be defined. **Abhishek et al. (2021)** proposed a queueing model for an urban parking system consisting of delivery bays and general on-street parking spaces. They obtained the performance measures from the analysis of the queues. **Cejka & Šedivý (2021)** analyzed car park occupancy based on queuing theory to investment decision-making. **Ahda et al. (2019)** focused on analyzing the implementation of Active RFID as an effort to reduce queues and waiting times. **Fuentes et al. (2017)** used the queue model in analysis evaluates the measures of performance for parking spaces of parking by visitors in a national park. The managers, then, can facilitate utilization estimates based on visitor changes and different seasons, and make decisions relating to the expansion of parking facilities or management of parking. **Tamber (2015)** optimized the queueing system of a fast-food restaurant. The author analyzed this restaurant under a queuing model to assess the behavior of the system for improving its performance. Parking was also part of the queuing system at this restaurant. **Abdel-Aal (2020)** researched to calibrate a mathematical

queuing model for parking lot entries with one queue and single-server of the parking system major shopping malls in Alexandria and Giza. Based on the comparison between the acquired data and different analytical methods, the authors showed that neither the assumed models (M/M/1 and M/D/1) are accurate. But the M/D/1 model appears to perform more closely and to be a reasonable assumption for modeling a queuing system. The author proceeded with calibration average service time (one of the queue model's important parameters) for the M/D/1 queueing model. The result showed that when the application of the calibrated M/D/1 model, its performance measures are significantly better than those of the original model that wasn't calibrated. **Setiawani et al. (2020)** determined the queuing model, measures the performance of the queue model, and determine whether or not this model's application was effective in a motorcycle parking area.

3.2.3 Estimate the appropriate parking price

Given that the arrival rate and staying time are dependent on the parking fee, **Keren & Hadad (2021)** used the M/G/N/N queueing model to estimate the appropriate parking price. **Babic et al. (2015)** calculated of parking duration at the parking. They used an M/M/c/0 queue with time-varying parameters to model arrival and stay in the parking lot, allowing different timeslots (hours) to have different arrival and service rates. **Gong et al. (2013)** used a queueing model to predict the PEV flow pattern into the parking lot for the PEV charging control development; estimate the waiting time under a different number of chargers. The charging requirement for the parking lot, then, could be predicted and estimated; providing a way for the transformer capacity design; the charging station design, or as guiding information for the customers.

4. SIMULATION-BASED QUEUEING MODELS IN PARKING SPACES

Simulation is an efficient method to solve complicated problems; providing a replica of the exact model with the behavior of the system and helps in delivering some important decisions for the system. Simulation modeling has long been part of queue modeling. Many applications and some research mentioned earlier in this paper have been used in simulation models.

Amari et al. (2019) simulated the proposed parking system using the queuing theory which aims to improve the quality of service of a smart parking. **Dowling et al. (2018)** simulated curbside parking as a network of finite capacity queues. **X. Li et al. (2015)** investigated the "non-shaped" tower parking garage's vehicle scheduling

strategy. They simulated various access processes based on queuing theory, then thoroughly analyzed the experimental findings of the MATLAB simulation, demonstrating that efficiently enhances the efficiency of the parking garage system. **Portilla et al. (2009)** applied an M/M/∞ queueing model to simulate the effect of parking on average link journey times due to the presence of parking maneuvers. **Xie & Hlynka (2019)** built a mathematical model to simulate the parking directions in the parking lot and discuss the relation between parking direction and arrival time and service time. **Pandey & Hanchate (2018)** proposed an intelligent parking management system using queueing theory and IoT. Simulation in Python to analyze system performance and improve user experience. **S. Li et al. (2012)** studied the queue system in the vertical circulation-type stereo garage through a queue theory to obtain the management strategy of scheduling optimization for parking and driving. The simulation experiment revealed that applying the optimization method described in this work has various advantages, such as an average short time for parking and taking a car one time, and high efficiency for average service rate. **Lu et al. (2009, 2010)** proposed a new VANET based mart parking scheme (SPARK) for large parking lots. Its efficiency and practicality are demonstrated by simulation. **Guo et al. (2018)** made a simulation in advance to evaluate the construction project of a seven-level automated parking lot for a hospital. They proposed and simulated three optimization solutions to mitigate the issue of hospital traffic congestion based on an analysis of hospital traffic characteristics and the estimation of the possible maximum of cars reaching the hospital. The average queue length to estimate the traffic congestion situation was caculated based on queueing model M/M/C.

5. DESCRIPTIVE ANALYSIS

In this paper, the simple meta-analysis used only provides descriptive information and does not include any statistics; it is intended to provide a better understanding of the development and evolution of queueing theory research in parking management, as well as to identify potential research areas for future research and improvement. We identified and analyzed the 65 articles related to the application of queueing theory research in parking management by (1) year of publication and (2) using our classification groups.

5.1. Distribution of articles by year of publication

Figure 2 shows that there is limited research output before 2000 and an increasing research trend on queueing

theory application in parking spaces from 2000 onward. Another possible reason for the upward trend since 2000 is the advancement in computational power and software availability.
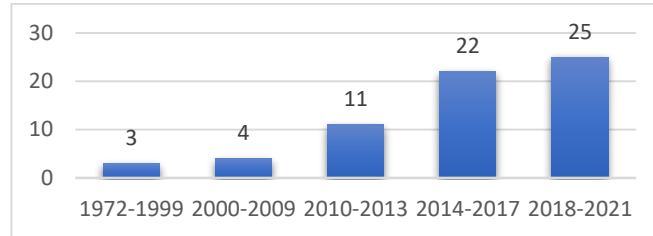

Figure 2: Distribution of reviewed papers by published year

5.2 Classification groups

5.2.1 Parking system design and system analysis

The following is a classification table of the papers on the application of queueing model in the parking lot (Table 1), which is divided into two groups: parking system design and parking system analysis.

Where: The parking system design group is divided into three categories: Automated parking system, Smart parking system, Normal/another parking system; The parking system analysis group is divided into three categories: The availability of parking and parking demand, The performance of the parking, Estimate the appropriate parking price.

Table 1. Classification scheme: parking system design and system analysis

Classification methods		References
Parking systems design	Automated parking system	Z. Li & Miao (2020), Senapati & Khilar (2020), Pypno & Sierpi (2013), Min & Shidong (2015)
	Smart parking system	Lu et al. (2009), Lu et al. (2010), Geng & Cassandras (2011a), Geng & Cassandras (2011b), Geng & Cassandras (2012), Geng & Cassandras (2013), Belkhala et al. (2019), Du & Gong (2016), Nugraha & Tanamas (2017), Sutjarittham et al. (2019)
	Normal/another parking system	Maršanić et al. (2010), Yan et al. (2015), Larson & Sasanuma (2007), Hauer & Templeton (1972), Guo (2020), Caliskan et al. (2007), Zhu et al. (2018)
Parking system analysis	The availability of parking and parking demand	Abdeen et al. (2021), Kawakami et al. (1994), Jung & Lee (2014), Chiara & Cheah (2017), Millard-Ball et al. (2014), Sukmana & Lesmono (1999), Pel & Chaniotakis (2017), Klappenecker et al. (2014), Xiao et al. (2018), Atif et al. (2020), B. Li et al. (2017), Debnath & Serpen (2015), Ma et al. (2017), Mohd Bukari et al. (2019)
	The performance of the parking	Kokolaki et al. (2012), Bajpai & Maneesha (2015), Maršanić et al. (2011), Abhishek et al. (2021), Cejka & Šedivý (2021), Ahda et al. (2019), Fuentes et al. (2017), Tamber (2015), Abdel-Aal (2020), Setiawani et al. (2020)
	Estimate the appropriate parking price	Keren & Hadad (2021), Babic et al. (2015), Gong et al. (2013)

5.2.2 Queueing models in parking

Table 2 is a table that categorizes articles into different queueing models based on whether they use simulation or not. The queueing models are categorized to MM, MG, MD, GM, GG models, and a handful of other models.

Table 2. Classification scheme: queueing models in parking

Techniques/Method/ Model	References	
	Simulations	Non-simulation/ unmentioned
MM	Abdeen et al. (2021), Marchi et al. (2018), Amari et al. (2019), Bajpai & Maneesha (2015), Geng & Cassandras (2011a), Geng & Cassandras (2011b), Geng & Cassandras (2012), Geng & Cassandras (2013), Dowling et al. (2018), Belkhala et al. (2019), Ratliff et al. (2016), Portilla et al. (2009), Babic et al. (2015), Xie & Hlynka (2019), Atif et al. (2020), Cho et al. (2017), B. Li et al. (2017), Sutjarittham et al. (2019), pandey & Hanchate Author (2018), Mao et al. (2019), Gong et al. (2013), Caliskan et al. (2007), Hong et al. (2013), Debnath & Serpen (2015), S. Li et al. (2012), Ma et al. (2017), Guo et al. (2018), Chukwu & Mahajan (2014), Jung & Lee (2014), Millard-Ball et al. (2014), Abhishek et	Yan et al. (2015), Krpan et al. (2017), Maršanić et al. (2011), Senapati & Khilar (2020), Kawakami et al. (1994), Larson & Sasanuma (2007), Chiara & Cheah (2017), Sukmana & Lesmono (1999), Du & Gong (2016), Cejka & Šedivý (2021), Fuentes et al. (2017), Guo (2020), Klappenecker et al. (2014), Xiao et al. (2018), Tamber (2015), Zhu et al. (2018), Shahzad et al. (2015), Min & Shidong (2015), Setiawani et al. (2020), Abdel-Aal (2020), Chiara & Cheah (2017)

	al. (2021)	
MG	Lu et al. (2009), Lu et al. (2010), Z. Li & Miao (2020)	Keren & Hadad (2021), Pypno & Sierpi (2013)
MD	Dowling et al. (2018), X. Li et al. (2015), Abdel-Aal (2020)	
GM	Zhao et al. (2020)	Maršanić et al. (2010)
GG	Pel & Chaniotakis (2017), Hong et al. (2013), Kokolaki et al. (2012)	
Others	Ahda et al. (2019), Nugraha & Tanamas (2017)	Mohd Bukari et al. (2019), Hauer & Templeton (1972)

Where, MM – The arrival process is Poisson distribution (M), service process is exponential distribution (M), MG – The arrival process is Poisson distribution (M), service process is generally shifted distribution (G), MD – The arrival process is Poisson distribution (M), service process is fixed-length distribution (D), GM – The arrival process is generally shifted distribution (G), service process is exponential distribution (M), GG – The arrival process is generally shifted distribution (G), service process is c shifted distribution (G).

6. CONCLUSION

In this paper, applications of queueing theory in modeling parking processes have been reviewed and categorized. Since parking is an essential component of today's transportation system., improving system performance is a very important goal. Queueing theory provides an effective and powerful modeling technique that can assist managers in achieving the aforementioned goal.

We presented a review of 65 papers searched in the database of Google scholar. We collected high-quality papers that were peer-reviewed and published between 1972 and 2021 using structured keywords search and cross-referencing. Our objective was to identify the leading areas of parking problems as modeled by queueing theory. This paper reviewed the use of queueing theory for the analysis and design of different types of parking systems. Also, we have reviewed the simulation-based queueing models that have been presented. From this review, we found that most queueing theory application articles in parking management were published after 2000. Another possible reason for the upward trend since 2000 is the advancement in computational power and software availability.

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