

Metacognition of Senior High School Students' in Solving Mathematical Problems

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ABSTRACT

This research is descriptive qualitative research that aims to describe the metacognition of senior high school students in solving mathematical problems regarding students' academic abilities. The subjects were students in the eleventh-grade students selected based on certain criteria with snowball sampling technique at a Senior High School. The data were collected through a problem-solving test and interview. The research result showed that students with high and low academical abilities had an awareness of the facts they had and how to use a strategy. While only high-ability students had an awareness of when and why a strategy should be applied. It implies that in solving mathematical problems, students with high academic ability involve three aspects of metacognitive knowledge, namely, declarative knowledge, procedural knowledge, and conditional knowledge nicely, while students with low academical ability only involve two aspects of metacognitive knowledge, namely, declarative knowledge and procedural knowledge.

Keywords: *Metacognition, Metacognitive Knowledge, Mathematical Problem-Solving.*

1. INTRODUCTION

Mathematics has a basic abstract object that causes most students to find mathematics difficult, tedious, and confusing. Mathematics is a frightening spectre for most students because it is difficult to solve [1]. In the learning mathematics process, teachers usually present mathematical problems to be solved by students in the form of questions that require answers or assignments to be completed. The mathematical problems given are expected to train students to mature their abilities in understanding, planning, performing, and obtaining solutions to any question they face.

Through mathematical problem solving, students are directed to develop their abilities, such as new mathematical knowledge and solving problems in various contexts related to mathematics. The main purpose of teaching problem-solving in mathematics is not only to equip students with a set of skills or solving procedures but also to enable students to think [2]. Thinking about what one thinks relates to self-awareness, in this case, is metacognition.

Metacognition is a person's awareness of thinking about his own thinking process, where thinking awareness is a person's awareness of what is known and what will be done [3] [4] [5]. By involving metacognition in solving mathematical problems, it allows students to build a strong and complete understanding of the problem accompanied by logical reasons [6].

Metacognition includes two components, metacognitive knowledge and experience or regulation of metacognition [3]. In addition, there are two components included in metacognition, what we know or do not know, and the regulation of how we learn [7]. Nevertheless, Metacognition is a combination of metacognitive knowledge and hierarchical metacognitive skills, where metacognitive knowledge is a prerequisite for activating metacognitive skills [8].

Recently, the most common difference in metacognition is to separate metacognitive knowledge from metacognitive skills [5]. In general, the metacognitive components can be seen in table 1.

Table 1. Metacognitive Component

Component	Indicator	description
Metacognitive Knowledge	Declarative Knowledge	awareness of a person's mathematical facts and concepts or the factors that influence his thinking and attention
	Procedural Knowledge	one's awareness of how to use a strategy
	Conditional Knowledge	a person's awareness of the conditions that affect his learning, namely: when a strategy should be applied, why apply a strategy, and when the strategy applied is appropriate
Metacognitive Skills	Prediction	a person's cognitive activity to distinguish between which problems are easy or difficult and which problems may require more skill or effort to solve
	Planning	The activity of a person's initial thinking about how, when, and why to take action to achieve goals through a series of specific goals leading to the main purpose of the problem (task).
	Monitoring	A person's supervision of the cognitive strategies he uses during learning activities to identify problems and modify plans
	Evaluation	The retrospective is done by looking back at the strategies that he has used and whether these strategies lead to the desired results or not in learning.

Several studies have shown the process of metacognition in students [9] [10] [11] [12]. The reality that occurs in many mathematics classes is that students do not use their metacognition when solving problems, so they do not understand what they are learning [9]. A significant relationship between metacognition and academic ability has been shown in [10]. Students with high, moderate, and low abilities are influenced by the metacognitive activities carried out. Furthermore, the metacognition process of students with high, moderate, and low academic abilities showed different results in preparing a plan, monitoring, developing solutions, and evaluating. Students with high and moderate academic abilities are able to realize their thinking processes when developing action plans, making solution plans, monitoring actions, and assessing their actions. In contrast to students having low academic abilities, it isn't easy to realize their thinking processes in these four aspects [12].

Therefore, this research aims to provide an overview of students' metacognitive knowledge for high and low academic levels. The metacognitive knowledge referred to in this research is metacognitive

grouping based on the characteristics of students' awareness in problem-solving.

2. METHOD

The type of research used is descriptive research with a qualitative approach. The analysis was carried out in class XI in one of the high schools. The subjects in this study consisted of students with high and low academic abilities. The subject selection technique was snowball sampling, where one student was selected from each level of intellectual ability. Therefore, two students were chosen as the first informants. After the data was saturated, four subjects consisting of two students from each ability level were selected. The instruments used were problem-solving tests and interview guidelines which two experts have validated.

Data collection in this study began with the provision of a problem-solving test instrument, then analyzed and categorized based on the criteria of students' metacognitive knowledge. The test result is described based on the level of students' academic ability. The next step was interviewing informants to explore data on students' metacognitive knowledge to clarify test result data that could not be explained through the analysis of written test answers. The data

obtained during the interview was then described. Finally, the test and interview data are presented in the form of a description.

3. FINDING AND DISCUSSION

This section describes the data, the results of problem-solving tests, and the types of metacognitive knowledge possessed by students based on high and low levels of academic ability.

3.1. Description of Students Metacognition with High Academic Ability (ST)

3.1.1. Declarative Knowledge

In the following, the test results and interview with one of the students show that there is an application of declarative knowledge in solving mathematical problems.

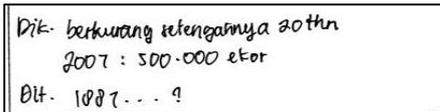


Figure 1 . ST test results

Figure 1 showed that ST could identify information on the question by writing down what was known and what was being asked. ST also wrote down formulas that could be used to solve problems, namely the formula for the nth term and the ratio of geometric series. This is confirmed in transcript 1.

TRANSCRIPT 1

- W-19 : What do you think after looking at the problem?
- ST12-W19 : The problem is related to the geometric series.
- W-33 : What is geometric series?
- ST12-W33 : Geometric series is a series with fractions or ratios. From the problem, the number of animals decreases to half of the previous year.
- W-23 : So, what kind of information do you find from the problem?
- ST12-W23 : In 2007, there were 500 thousand animals, and every 20 years, this number decreased to its half. What is being asked from the problem is the number of animals in 1887.
- W-24 : What for do you write the information given in the problem?

ST12-W24 : With this, I do not need to reread the problem, making it easy to solve.

In transcript 1, it can be seen that ST was aware of the concepts and initial knowledge needed to solve the problem by using the geometric series concept (ST1 2-W19). ST thought about the concept by considering the information in the problem, namely because of the ratio (ST12-W33). This shows that ST utilizes previously acquired knowledge to understand the problem. Furthermore, ST also explained the reason for writing down the information he got on the problem to make it easier for him to solve it (ST12-W24). This proves that ST was aware of the essence of writing down what is known and asked to achieve the goal.

Based on Figure 1 and transcript 1, it can be seen that ST tends to be aware of mathematical facts and concepts or prior knowledge needed to help them solve problems. Therefore, it can be said that ST applies metacognitive knowledge related to declarative knowledge in solving mathematical problems.

3.1.2. Procedural Knowledge

In the following, the test results and interview with one of the students show that there is an application of declarative knowledge in solving mathematical problems.

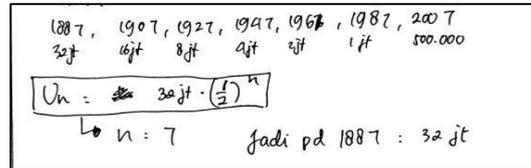


Figure 2. ST test results

Figure 2 shows that ST looked for a solution by building his own steps without using a formula in solving the problem. ST registered the number of animals that existed every year by counting down the number of animals from 2007 to 1887. ST used a formula to evaluate the results of his work. This is confirmed in transcript 2.

TRANSCRIPT 2

- W-26 : What are the steps to solve this problem?
- ST12-W26 : Since every 20 years, the number of animals decreases to its half (pointed to 500 thousand), then every 20 years later, the number increases twice.

In transcript 2, ST explains the alternative solutions applied by giving clear reasons (ST12-W26). By using simple logic, ST obtains answers correctly and quickly. This shows that ST applies a method to solve the problem.

Based on Figure 2 and transcript 2, ST tends to be aware of how to implement a comprehensive strategy to solve problems. So, it can be said that ST applies metacognitive knowledge related to procedural knowledge in solving mathematical problems.

3.1.3. Conditional Knowledge

Subject ST conditional knowledge can be seen in transcript 3.

TRANSCRIPT 3

- W-41 : Why do you use the formula of geometric series?
 ST22-W41 : Because this series does not involve subtraction but use ratio or fraction.
 W-42 : Why should it be a ratio?
 ST22-W42 : Since it decreases by 1/5 every year.
 W-44 : From which do you know that this problem is related to geometric series?
 ST22-W44 : Since geometric series generally involve ratios in the form of a fraction.

From transcript 3, it can be seen that ST understood the concepts related to the given problem (ST22-W41), so ST could explain the reasons for using the geometric series formula in solving the problem (ST22-W44).

Based on transcript 3, ST tended to be aware of when and why a strategy should be implemented. This shows that the subject involves metacognitive knowledge related to conditional knowledge in solving mathematical problems.

3.2. Description of Students Metacognition with Low Academic Ability (SR)

3.2.1. Declarative Knowledge

The test results and interview with one of the students show that there is an application of declarative knowledge in solving math problems.

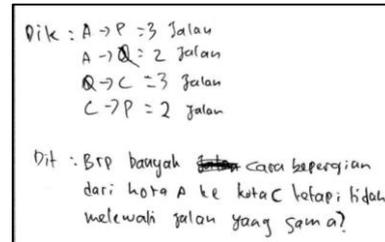


Figure 3. SR test results

Figure 4 shows that SR can identify the problem by writing down what is known and what is asked in the problem. This is confirmed in transcript 4.

TRANSCRIPT 4

- W-03 : According to you, is there any formula or concept that can be applied to solve this problem?
 SR11-W03 : I think so, but I do not know which one.
 W-06 : How do you understand the problem?
 SR11-W06 : I read the question many times.
 W-08 : After reading the question, could you explain what do you know about the problem?
 SR11-W08 : We are required to find a number of ways. Since there are two alternatives, namely, through city P or Q. A, P, C, and Q are cities, while the lines are the routes. At the first time, I thought there were only two ways.

In transcript 4, it can be seen that SR did not realize the concepts related to the given problem (SR11-W03). SR also explained how he identified information given in the problem to understand the problem (SR11-W06). Next, the subject presented orally the info he got (SR11-W08).

Based on Figure 3 and transcript 4, SR tended to have awareness about the information given in the problem but did not yet realize the concepts needed to solve the problem. This shows that the subject applies metacognitive knowledge related to declarative

knowledge in solving mathematical problems but is not perfect.

3.2.2. Procedural Knowledge

The test results and an excerpt from an interview with one of the students show an application of declarative knowledge in solving mathematical problems.

Renge : $A \rightarrow P = 3 + (P) P = 2$
 $A \rightarrow Q = 2 + Q \rightarrow C = 3$
 $= 3 + 2 : 5$
 $2 + 3 : 5$
 $= 5 + 5 = 10$

Figure 4. SR test results

Figure 4 shows that in solving problems, SR solved problems without applying certain concepts. The subject immediately added up all the existing paths. This is confirmed in transcript 5.

TRANSCRIPT 5

- W-17 : *Could you explain the steps you use to solve the problem?*
 SR11-W17 : *From city A to city P, there are three routes, city C to city P there are two routes and city A to city Q, there are 2 routes, and from city Q to city C, there are three routes. So, I added it and found $3+2=5$ and $3+2=5$, and it is 10 in total.*

In transcript 5, SR described the alternative solutions that are applied. SR solved the problem using a simple analysis by trial and error (SR11-W17). This shows that SR applies a method to solve problems but still uses a trial-and-error strategy with a simple analysis.

Based on Figure 4 and transcript 5, SR tended not to understand how to implement a strategy to achieve goals fully. This shows that SR subjects apply metacognitive knowledge related to procedural knowledge in solving mathematical problems.

3.2.3. Conditional Knowledge

The application of SR conditional knowledge can be seen in transcript 6. In transcript 6, SR took action based on intuition (SR11-W13) and did not know the reason for implementing the strategy he used to solve the problem (SR11-W18).

TRANSCRIPT 6

- W-13 : *Why did you add it?*
 SR11-W13 : *Because I think so immediately after writing the information from the problem.*
 W-18 : *Why do you use addition operations? Why is it not multiplication or others?*
 SR11-W18 : *Because if I multiply it, the solution is too large.*

Based on the transcript SR did not yet have awareness about when and why to implement a strategy. This shows that SR does not involve metacognitive knowledge related to conditional knowledge in solving mathematical problems.

4. CONCLUSION

High and low math ability students are aware of the facts they have and know how to use a strategy. Still, only high-ability students have an understanding of when and why a strategy should be applied. Therefore, it can be said that in solving mathematical problems, students with high academic abilities involve three aspects of metacognitive knowledge, namely, declarative knowledge, procedural knowledge, and conditional knowledge. Meanwhile, students with low intellectual ability can only involve two aspects of metacognitive knowledge, namely declarative knowledge and procedural knowledge.

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