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# Description of Mathematical Reasoning Ability of Students in Solving TIMSS Questions in the Cognitive Domain of Reasoning on Algebraic Content Based on Thinking Styles

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**ABSTRACT**
This study aimed to describe the mathematical reasoning ability of students in solving TIMSS questions in the cognitive domain of reasoning on algebraic content based on thinking styles. This type of research was qualitative research with a descriptive approach. The research subjects consisted of four students representing each thinking style of concrete sequential (SK), abstract sequential (SA), abstract random (AA), and concrete random (AK). The research instrument used consisted of a thinking style questionnaire, mathematical reasoning ability test questions, and interview guidelines. The results of this study indicate that: SK subjects can meet four indicators of mathematical reasoning ability, namely: a) the ability to make conjectures, b) the ability to conclude, compile evidence, provide reasons or evidence for the correctness of the solution, c) the ability to draw the validity of an argument, and d) the ability to find patterns or properties of mathematical phenomena to generalize. SA subjects can meet four indicators of mathematical reasoning ability, namely: a) the ability to make conjectures, b) the ability to conclude, compile evidence, provide reasons or evidence for the truth of the solution, c) the ability to draw the validity of an argument, and d) the ability to find patterns or characteristics mathematical phenomena to make generalizations. AA subjects are able to fulfill three indicators of mathematical reasoning ability, namely: a) the ability to make conjectures, b) the ability to draw the validity of an argument, and c) the ability to find patterns or characteristics of mathematical phenomena to make generalizations. AK subjects can fulfill two indicators of mathematical reasoning ability: 1) the ability to conclude, compose evidence, provide reasons or evidence for the correctness of the solution, and 2) the ability to draw the validity of an argument.

***Keywords:*** *Mathematical Reasoning, TIMSS Questions, Algebraic Content, Thinking Style.*

## 1. INTRODUCTION

The results of TIMSS Indonesia based on the International Benchmarks of Mathematics Achievement are at the low international benchmark level. Based on the data of TIMSS in 2011, the mathematics ability of students in 8th-grade has decreased from the TIMSS 2007 [1]. The results of the TIMSS become one of the external challenges in the development of the Curriculum 2013 in Indonesia, namely to understand more deeply about achievements and the education system [2].

After implementing the Curriculum 2013, Indonesia did not include grade 8 in the TIMSS 2015 and TIMSS 2019, so there was no data for those years. Based on the latest data, namely the TIMSS 2011 results for grade 8, Indonesia is ranked 38th out of 42 countries with the lowest percentage of correct answers on the dimensions of algebraic content (22%) and cognitive reasoning (17%).

The low abilities obtained in timss 2011, including the content domains of algebra and cognitive reasoning, have been highlighted in the curriculum 2013. Algebra is one of the materials in mathematics

lessons in accordance with the character of the curriculum 2013. The reasoning is one standard process in the scientific approach applied in the curriculum 2013 [3].

Algebra has a relationship with mathematical reasoning since to understand mathematical material, we need mathematical reasoning. Therefore, teachers need to train and hone mathematical reasoning through learning mathematics [4]. Reasoning has a structure that can be continuously improved through cognitive, social, and even effective learning [5]. Therefore, it is necessary to look at the description of students' mathematical reasoning abilities with the curriculum's application 2013 in solving timss algebra and reasoning domain questions in terms of students' thinking styles.

As stated by deporter & hernacke, each student has a different thinking style in solving the same problem [6]. The thinking style is not an ability but a preference for performing abilities [7]. The thinking style used by the individual adapts to the current situation, and everyone is aware of the act of thinking [8].

According to Sternberg, style can be measured, socialized, taught [9]. Therefore, to measure the thinking style, the highest score is described as the thinking style most often adopted by an individual. Based on this score, everyone will tend to lean towards one of the types of thinking styles proposed by grogerc, namely concrete sequential thinkers (SK), concrete random thinkers (AK), abstract random thinkers (AA), and abstract sequential thinkers (SA) [6].

SK thinkers hold to reality and process information in an orderly, linear, and sequential manner. SK gets and remembers from what they get from the senses and remembers facts, information, formulas, and special rules easily. AK thinkers have an experimental attitude that is accompanied by less structured behaviour based on reality but wants to take a trial and error approach. AA minds absorb ideas, information and organize impressions by reflection; they remember very well when information is personified. Feelings can also further enhance or influence their learning of AA.

Meanwhile, reality for SA thinkers is a world of metaphysical theory and abstract thought. They like to think about concepts and analyse information. They are easy to spot important things, like key points and important details. They as think logically, rationally, and intellectually [6].

## 2. RESEARCH METHOD

This research is qualitative research with a descriptive approach. The research was conducted in class VIII SMP in Makassar. The subjects in this study consisted of 4 subjects, namely one subject each for each thinking style. Data were collected using a thinking style questionnaire, mathematical reasoning ability test, and an interview guide that have been validated by two experts.

The instrument of this research was a thinking style questionnaire by John letellier that has been modified; the mathematical reasoning ability test is a timss question and an interview guide to digging deeper information about the subject's mathematical reasoning ability.

The data analysis of this research was data reduction, data presentation, and conclusion drawing. Data reduction was selecting data from test results and interview quotes related to mathematical reasoning abilities. The next step was presenting data from the results of the mathematical reasoning ability test in the form of narrative text and presenting interview quotes in an organized manner. The last step concludes the results of the analysis of the reasoning ability test, interviewing, and verifying it.

## 3. RESULTS AND DISCUSSION

This section presents a description of students' mathematical reasoning abilities based on indicators of mathematical reasoning abilities. The indicators are a) the ability to make conjectures, b) the ability to conclude, compile evidence, provide reasons or evidence for the correctness of the solution, c) the ability to draw the validity of an argument, and d) the ability to find patterns or properties of mathematical phenomena to make generalizations.

### *3.1. Indicator of Ability to Make Conjectures*

To find information about this indicator, a question about equilibrium was given to the subjects where the weight of each object in the picture has been known. There was an illustration of a fraction between the weight of an object and a metal rod/piece. The object was lighter than one metal rod/piece in the first illustration, and the second illustration was heavier than three metal rods/pieces. The problem requires students to determine the weight of a metal rod/piece by selecting the answer options provided. The following is the result of the subject's test on indicator 1:

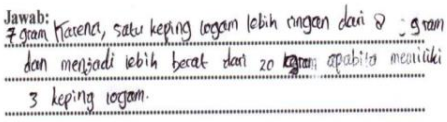


**Figure 1 Answers to indicator 1 by subjects SK**

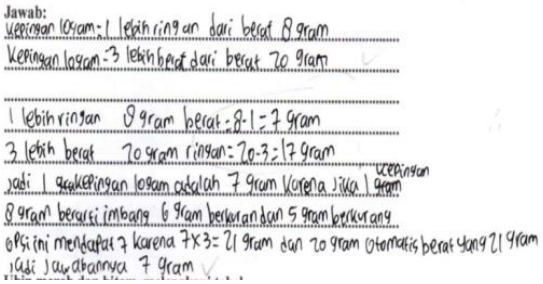


**Figure 2 Answers to indicator 1 by subjects SA**

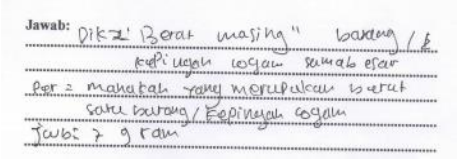


**Figure 3 Answers to indicator 1 by subjects AA**

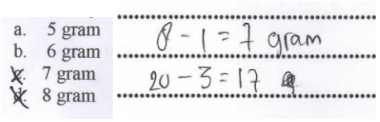


**Figure 4** Answers to indicator 1 by subjects AK

Four subjects, namely SK, SA, and AA had similarities in giving the correct conclusion, while AK chose two answers in the options provided in the question. No arguments or reasons were given by AA and AK. On the other hand, SK wrote down the answer and then explained it based on the nature of the equilibrium regarding the suitability of choice. SA wrote down information known from the problem and conducted the first trial by finding the difference between the weight of one side and the amount of metal and the second trial by testing each option with the principle of equilibrium and concluding the correct answer. AA wrote down what was known, asked, and answered without any process. AK did the same thing with SA in trial 1.

To dig deeper into the conjectures from the arguments made, the researcher conducted interviews with the subjects, namely SK, SA, AA, and AK. The researchers interviewed SK, SA, AA, and AK, working on the questions based on the interpretation of the images on the questions. SA and AK provided arguments according to the answers written but felt doubtful about the answer of the first trial and did not realize the error by subtracting something that was not appropriate. SA and AK made the mistake of subtracting the weight of an object by the number of objects. SK and AA explained more broadly than what was written; SK conducted trials on each option and gave a correct conclusion, while AA tested the options sequentially and stopped at the right answer. The following is an excerpt from the interview subject SK and AK:

*Q* : *How can you guess the correct answer is 7?*

*SK* : *Because for 8 grams, this is also 8 grams (shown in the picture), so it must be the same and will be balanced.*

*Because it's balanced, and here it's 8 grams heavier (referring to the picture), there are three options 5, 6, 7 (pointing to options), and here there are 3 metal pieces that are heavier than 10. If* 53 *is the result 15, so no may weigh more than 20 grams. If 6,* 63= *18, so it can't be heavier than 20 grams. Only 7,* 73 = 21, *so it's heavier than 20 grams.*

*AA* : *It's 8 (look at the first picture), so it can't be above 8 because it's heavier than metal. So… so 7, eeee….me time…. times 3, 7 times 3=21, it is impossible that 21 is lighter than 20.*

From the results of tests and interviews, SK, SA, and AA could make conjectures. The way the subjects made assumptions was that the subjects used and interpreted the image on the problem. The subjects used the principle of equilibrium in the figure to find the correct value. The value was obtained by substituting each option in the question.

### *3.2. Indicators of Ability to Draw Conclusions, Compile Evidence, Provide Reasons or Evidence for The Correctness of The Solutions*

Questions related to number patterns were given to the subjects in the form of pictures of square patterns from the arrangement of tiles. It was known that two color combinations of tiles formed a square pattern. There was also a table in which each column contained a number of the shape of a square pattern, tiles, and color combinations of tiles. The information asked was how many tiles of a certain square pattern were on the empty table. The following is the result of the subject's test on indicator 2:

| Bentuk | Banyaknya ubin hitam | Banyaknya ubin merah | Jumlah ubin |
|---|---|---|---|
| 3 × 3 | 1 | 8 | 9 |
| 4 × 4 | 4 | 12 | 16 |
| 5 × 5 | 9 | 16 | 25 |
| 6 × 6 | 16 | 20 | 36 |
| 7 × 7 | 25 | 24 | 49 |
| . . . | | | |
| n × n | $(n-2)^2$ | [illegible] | [illegible] |

**Figure 5 Answers to indicator 2 by subjects SK**

| Bentuk | Banyaknya ubin hitam | Banyaknya ubin merah | Jumlah ubin |
|---|---|---|---|
| 3 × 3 | 1 | 8 | 9 |
| 4 × 4 | 4 | 12 | 16 |
| 5 × 5 | 9 | 16 | 25 |
| 6 × 6 | 16 | 20 | 36 |
| 7 × 7 | 25 | 24 | 49 |
| . . . | | | |
| n × n | $(n-2)^2$ | 28 | 54 |

**Figure 6 Answers to indicator 2 by subjects SA**

| Bentuk | Banyaknya ubin hitam | Banyaknya ubin merah | Jumlah ubin |
|---|---|---|---|
| 3 × 3 | 1 | 8 | 9 |
| 4 × 4 | 4 | 12 | 16 |
| 5 × 5 | 9 | 16 | 25 |
| 6 × 6 | 16 | 22 | 37 |
| 7 × 7 | 25 | 26 | 51 |
| 8×8 | [illegible] | [illegible] | [illegible] |
| n × n | $(n-2)^2$ | ... | ... |

**Figure 7 Answers to indicator 2 by subjects AA**

| Bentuk | Banyaknya ubin hitam | Banyaknya ubin merah | Jumlah ubin |
|---|---|---|---|
| 3 × 3 | 1 | 8 | 9 |
| 4 × 4 | 4 | 12 | 16 |
| 5 × 5 | 9 | 16 | 25 |
| 6 × 6 | 16 | 20 | 36 |
| 7 × 7 | 25 | 24 | 49 |
| . . . | | | |
| n × n | $(n-2)^2$ | $(2n-2)^2$ | $(n^2-2)^2$ |

**Figure 8** Answers to indicator 2 by subjects AK

Each subject concluded the answer without any arguments. They could not translate the relationship in the table into algebraic form (the last column in the table). Each answer written by AA was wrong, while the answers of the subjects SK, SA, and AK were correct except in the last column, which was the algebraic form of the pattern in the table.

The researcher conducted interviews to dig deeper into each subject's work and find out how the subject concluded, provided, and compiled pieces of evidence for the correctness of the solution. All subjects solved the problem based on the table provided without paying attention to the picture on the question. SK explained that the pattern in the table was that the value in the number of tiles column was the product of the values in the shape column, while the difference between the values in the number of tiles column and the number of black tiles was the value for the column of the number of red tiles. SA and AK had the same reason that the value in the number of tile columns was the product of the values in the shape column, while the value in the number of red tiles columns had a pattern plus 4. AA was confused by the numbers in the table and just wrote numbers randomly. In the last column in the table, SK and AA did not write down the answer because they did not understand the meaning of the question, while SA wrote a continuation of the previous pattern and AK only wrote something without knowing the meaning and purpose of the question.

Based on the results of tests and interviews for each subject, it was found that SK, SA, and AA subjects were able to conclude, compile evidence, provide reasons or evidence for the correctness of the solution. The way the SK subject concludes is by analogy. Subjects draw conclusions based on the similarity of data or processes. SA and AA subjects conclude trans inductive, that is, conclude a particular case or trait that is applied. Each subject then generalizes the solution method to determine the value in question, but each subject does not understand the value of n×n.

### *3.3. Indicator of Ability to Draw the Validity of an Argument*

A question given was about a number pattern in the form of an image of a tile arrangement containing numbers. There was one tile whose value was unknown and symbolized by the letter x. Asked information was the value of x in the pattern and the reason for getting the answer. The following is the result of the subject's test on indicator 3:

Jawab: 24 karena

**Figure 9** Answers to indicator 3 by subject SK

Jawab:
nilai x=24 menyapa?

**Figure 10** Answers to indicator 3 by subject SA

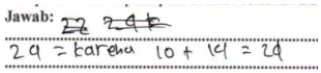


**Figure 11** Answers to indicator 3 by subject AA

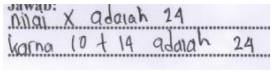


**Figure 12** Answers to indicator 3 by subject AK

Each subject gave the same correct answer. SK and SA did not write down the reason for the solution, while AA and AK wrote the reason by linking the pattern connected to the value contained in the picture provided.

To find out more about the evidentiary process carried out, interviews were conducted to find out the arguments of each subject. Each SK, SA, AA, and AK concluded the correctness of the solution based on the previous interrelated patterns. Still, SK and SA showed the previous patterns and conducted trials with the results obtained in the next patterns. The following are the results of the interviews for each subject:

*Q* : *Explain how you got the answers?*

*SK* : *Because of this….because this (pointing to number 6 in the question) is obtained from this (pointing to number 2 in the question) add this (pointing to number 4 in the question), this (pointing to number 10 in the question) is obtained from this (pointing to number 4 in the question) add this (pointing to 6 in the question). So this is also 6+10 16 (referring to the numbers 6, 10, and 16 in the problem). So if you want to get this (pointing to the letter x in the question), this must first add 10 (pointing to the number 10 in the question) + 14 (pointing to the number 14 in the question) 24. Then this is also the definite result of 16 (pointing to the number 16 in the question) +24(points to the letter x in the question). This can (points at the number 40 in the question)*

*SA* : *If this is the case, this again (points to question no.4) is added. This is 2+4=6, 6+8=14, 6+10=16. So 10+14=24. Here again 24+16, surely the answer is 40*

*AA* : *Because this is 10+14*
*Because of this (referring to the numbers in the picture) 2+4, 2+4=6, 4+6 = 10, 6+8=14*

*AK* : *because 2+4= 6 , 4+6 = 10, 6+8= 14 so this is also in total.*

Based on the results of tests and interviews of each subject, each subject can draw the validity of an argument. Each explanation of the answers was based on the pattern in the picture. They also generalized the patterns and then solved the given problem. SK and SA subjects re-tested the patterns and proved the correctness of the solution.

### *3.4. Indicator of Ability to Find Patterns or Properties of Mathematical Phenomena to Make Generalizations.*

From the question, it was known that the sequence consisted of numbers in the form of fractions. Students were required to find the next terms, the number in a certain sequence term, and the number in the nth term of the sequence accompanied by the reasons for obtaining the answer. The following is the result of the subject's test on indicator 4:

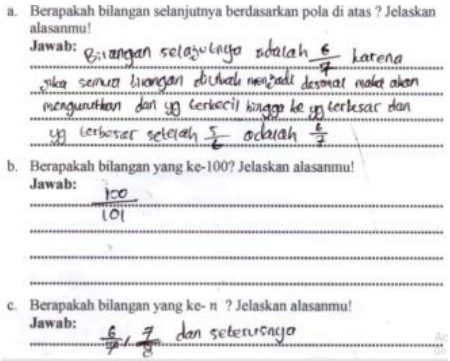


**Figure 13 Answers to indicator 4 by subjects SK**

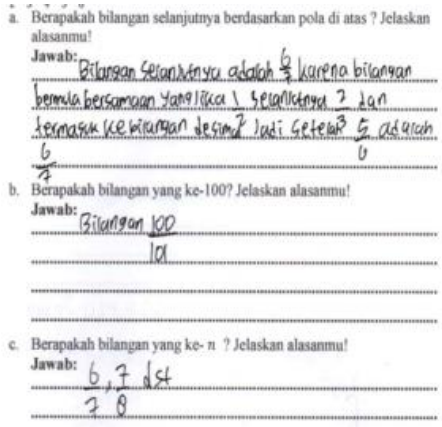


**Figure 14 Answers to indicator 4 by subjects SA**

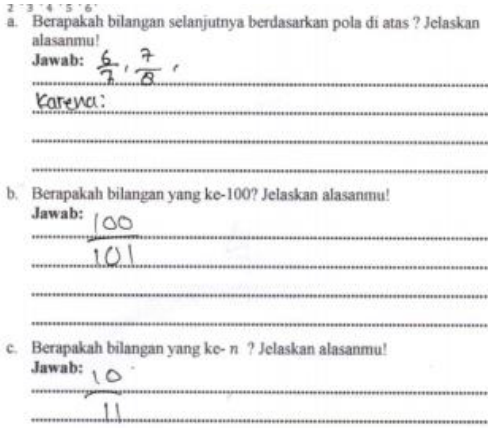


**Figure 15 Answers to indicator 4 by subjects AA**

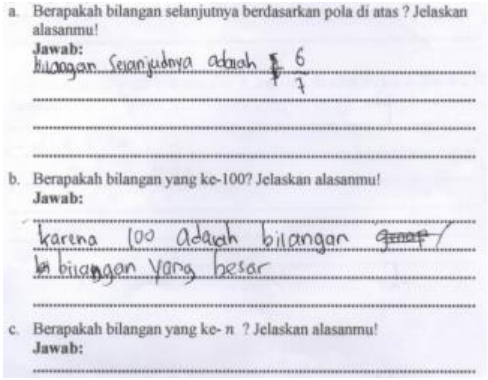


**Figure 16.** Answers to indicator 4 by subjects AK

There are questions with three problems that must be solved. Each subject concluded the correct answer to the first problem; SK and SA wrote the reasons for the pattern between each fraction while AA and AK did not write down the reasons. SK, SA, and AA concluded the correct answer to the second problem without any reasons, while AK gave the wrong answer. The third problem was finding algebraic forms that could not be answered correctly by every subject. In the third problem, SK and SA wrote a continuation from the previous pattern, and so on; AA wrote the 10th pattern of the question, while AK did not write down the answer.

To dig deeper into the evidence carried out, interviews were conducted. SK, SA, and AA obtained two patterns in the questions: the overall pattern (pattern 1) and the pattern in each part or tribe (pattern 2). In the first problem, each subject related each successive pattern to obtain the next patterns, while SK and SA explained in detail the patterns. In the second problem, SK, SA, and AA answered based on pattern 2, and AK did not know the question's meaning. In the third problem, SK wrote a continuation of the pattern because he did not understand the question's purpose. SA considered the intent of the question to be a continuation of each term from the existing pattern, and AA wrote down the answer originally. AK did not understand the meaning of the question. The following are excerpts from interviews for each subject:

*Q* : *Based on these questions, what information did you get, and how did you find solutions to the problems provided?*

*SK* : *(problem 1) Because the problem is, what is the next number, a fractional number. Then because* $\frac{5}{6}$ *becomes* $\frac{6}{7}$*. Because it's* $\frac{1}{2}$ *plus 1, so* $\frac{2}{3}$ *is added* $\frac{3}{4}$ *is added again…*

*(problem 2) Because the number is 100*

*if the 100th number, the one above it is 100, and the bottom is 101. Because the first number under 2 is added 1. It means that it is correct*

*(problem 3 ). If that doesn't understand*

*SA* : *(problem 1) Here* $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$*, automatically because he is added, one, one, here, and this is also added one. So it's automatic here* $\frac{6}{7}$ *because 5 plus 1 is 6 and 6 plus 1 equals 7.*

*(problem 2) Enter the 100th number, automatically here above the 100th number means that you add 100 plus 1, so 101 so,* $\frac{100}{101}$*.*

*(problem 3) If this is what I said earlier, I thought (...) and so on*

*AA* : *(problem 1) Because there it continues, because this is* $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$ *so* $\frac{6}{7}$

*(problem 2 ) Because here is 100, so because 100 is below it, add 1. So 101*

*(problem 3) Origin*

*AK* : *(problem 1)because here 1,2,3,4,5, only 6 and those below 7, because what is being asked the next number*

*(problem 2) doesn't know because 100 is a big number. Don't understand*

*(problem 3) I don't know*

Based on test results and interview results for each subject, subjects SK, SA, AA can find patterns or properties of mathematical phenomena for generalizing. They paid attention to the whole line and

to each term to see the pattern. The subject then generalized it and determined the value of the term randomly, but the subject could not formalize the formula for the nth term.

## 4. CONCLUSION

Results Based on research and discussion, the conclusions of this study are as follows:

a. In the indicator of ability to make conjectures, concrete sequential, abstract sequential, and abstract random can be performed. The three subjects took advantage of the questions' information, worked on the theory of equilibrium, and tested each option. There are mistakes made by abstract sequential and concrete random subjects, namely by looking for the difference between the weight and number of an object.

b. In the indicator of the ability to conclude, compile evidence, provide reasons or evidence for the correctness of the solution, it can be done by concrete sequential, abstract sequential, and concrete random. The concrete sequential subject connects and relates each part of the problem provided to determine a solution; the concrete abstract and random sequential subject connects several parts and finds a solution with its own pattern.

c. The indicator of ability to show the validity of an argument can be done by the subject of concrete sequential, abstract sequential, abstract random, and concrete random. Each subject made generalizations to the information obtained, but the concrete sequential and abstract sequential subjects did retest to make the answer more convincing.

d. In the indicator of finding patterns or properties of mathematical phenomena to make generalizations, the subjects can be concrete sequential, abstract sequential, and abstract random. The three subjects were able to determine the pattern in question. Still, for the nth number, the concrete sequential and abstract sequential subjects assumed that what was asked was the next number, while the abstract random subject did not understand the meaning of the nth number.

e. Each subject is explicitly unable to formalize the formula for the nth term or the n×n form on the given problem, but the nth term formula can be used in the process. Subjects have not been able to switch from pre-algebraic arithmetic reasoning about numbers to algebraic reasoning using symbols to represent variables; this is a serious challenge for many students [10].

## AUTHORS' CONTRIBUTIONS

The results of my research provide an overview of the mathematical reasoning ability of each thinking style. Therefore, by knowing this, teachers can use the results of this study as a reference in terms of compiling learning tools that adapt to mathematical reasoning abilities based on each student's thinking style as well as providing challenges for teachers to think of solutions that can solve the problems experienced by students in this study.

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