

# Transit Ridesharing System Using Tradable Credit Under Asymmetric Transaction Costs

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## ABSTRACT

Tradable credit as a new type of toll has the advantages of optimizing the transportation system, changing travel time and relieving congestion pressure, and has received much attention in recent years for research. The economic cost is defined as a fixed percentage of the transaction ratio, and under the asymmetric transaction cost, the economic cost land is asymmetrically split, which produces extremely different effects between buyers and sellers. In this paper, we first focus on the bottleneck model, through which we calculate the toll rate of the optimal traffic control system, and then introduce asymmetric transaction costs to consider the impact and significance of the Pareto effect. We find that the effect of the asymmetric transaction system is optimal when the commuter bears a higher cost.

**Keywords:** *Asymmetric trading, bottleneck model, travel costs, Pareto effect*

## 1. INTRODUCTION

In recent years, with the rapid economic development, cities have become more and more crowded, but the bus network system is still one of the most popular ways to travel, but the heavy bus tasks during the morning and evening commuting hours have caused great problems for the transportation system. In order to solve the increasingly congested urban bus network and meet the demand of different levels of passenger flow, managers use a series of tools in the bus system, such as introducing intra-bus congestion costs, selecting departure times according to different utility functions, establishing dynamic travel equilibrium models [1], and performing multi-modal bus network rapid construction and passenger flow distribution among cities [2].

The bottleneck model assumes that the bus stop is in front of the home, in that work and home are connected by a single road, and a large number of commuters form a bottleneck by waiting in line at the bus hub during their commute. While everyone wants to get to their destination in the fastest, most time-efficient way, this is not possible due to objective factors such as passenger capacity. There will always be some people who can get on the train and arrive on time without any problems, and some people who are late because of delays in the queue and traffic jams. When transaction credits are not considered, the cost of travel for each person consists of

the cost of traveling and the cost of schedule delays. The travel cost is equal for the same distance, and the planned delay cost is related to the expected arrival time and departure time at the destination. Under the bottleneck model, dynamic and static tolling theories have been proposed, respectively. Dynamic tolling means that commuters traveling at different intervals during peak periods are charged in segments, so that commuters traveling at different times pay different travel costs. Although the bottleneck model cannot eliminate queuing at all, it can minimize the total travel cost and thus achieve socially optimal efficiency.

Tradable credits have become a policy that many regulators have vigorously pursued in recent years. Surveys conducted in the Netherlands and China have shown that the public perceives tradable credits as more beneficial than pricing [3]. Tradable credit has the advantage of not redistributing income because it does not involve money and does not go through the government. Its basic idea is to link the travel time and condition of different time periods and roads through points issued by the government to each person, and these points move among commuters based on travel time, and those who are outside the congested time get the credit and those who are inside the congested time need to spend the credit to pay for it, and the transaction costs include: point allocation costs, transaction conduct costs, and check transaction costs. The study of credit costs for asymmetric transactions has become a

buzzword on this topic in recent years, and Sovacol found that transaction costs range from 1% to a staggering 25% of the value of trade, resulting in 10% of efficiencies being cancelled due to high transaction costs, resulting in needless losses to society [4].

In this paper, by combining the bottleneck model and the tradable credit model, we simulate and study the optimal state of bus tolls during the peak commuting hours in China, from which we derive the relevant data in equilibrium to provide a corresponding basis for the policy formulation of the transportation management.

## 2. LITERATURE REVIEW

The bottleneck model has been studied by several scholars at home and abroad since Vickrey, a Nobel laureate in economics, first proposed the classical bottleneck model using deterministic queuing theory, which can derive the endogenous departure time that makes all travelers have the same transportation cost [5]. Since then, Wu Zixiao has studied that travel time cost (queuing time cost) as a pure loss can be transformed into a gain in the form of road usage fee payment. Dynamic tolling achieves social optimality by changing the distribution of trips and keeping individual travel costs constant [6]. Huayan Shang applied the activity-based bottleneck model to the study of evening peak residential commuting at transit hubs, and innovatively incorporated the internal congestion cost of transit to study the equilibrium dynamic travel model with constant and linear marginal activity [7]. Ti Chen extended Vickrey's bottleneck theory model with dual target moments to study the fundamentals of traffic staggering management based on the analysis of departure time choice behavior of two target arrival moments and homogeneous travelers [8]. Chao-Ting Li investigated the optimal dynamic toll and step toll problems under constant and linear marginal activity utilities, and the results showed that the dynamic bottleneck toll curve under linear marginal activity utility is no longer a segmented linear function, but a segmented quadratic curve [9].

In terms of solving road congestion, many scholars have also proposed corresponding solutions. Lin Xiao Song uses a "dynamic combination" combination charging strategy, which combines road congestion charging and parking charging strategies in congested areas, and establishes a two-tier pricing model with a combination charging strategy [10]. Li Zhichuan proposed a realistic two-tier planning model for congested road pricing, introducing the concept of user surplus to construct the upper-level objective, and the lower-level model is a logit-based SUE model with elastic demand [11]. Chao Sun combined macroscopic road network traffic state evaluation with microscopic traffic state analysis, established a quantitative road network traffic state evaluation model, and proposed a

new road network traffic state analysis method [12]. Qing-Yu Luo made a qualitative division and quantitative measurement of the composition of the cost of urban road congestion charging, and gave a specific measurement formula regarding the extra time cost, environmental pollution cost and traffic accident cost caused by congestion [13].

In summary, it can be seen that a considerable number of scholars have been studied on the issue and assessment of road congestion. Among these studies, we choose to combine the bottleneck model and tradable credit into the field of public transport travel to find the equilibrium solution of the public transport sector in the optimal state.

## 3. BOTTLENECK MODEL

### 3.1. *Bottleneck model for morning and evening peaks*

Assume that the workplace and home are connected by a road with no traffic lights and intersections. There are a total of  $N$  commuters riding the bus every day. Assume that each commuter, the bus is homogeneous, i.e., the commuters have the same time cost, the same destination, and the bus is of the same size and driving status. During the morning peak, commuters take the bus from home to work and from work to home during the evening peak. The peak period bus bottleneck capacity is  $s$ , which is the ratio of the bus car volume to the number of people in line who need to use the bus. In the morning and evening peak periods, bus departures are frequent and can be approximated as continuous and uninterrupted, and from the commuter a ride on the bus departure is considered to enter the bottleneck area, and arrival at the destination is considered to leave the bottleneck area.  $\alpha$  is the value per unit of time. At the peak of commuting, as soon as the commuter leaves his location. Then they can get on the bus and enter the bottleneck mode. Once the bottleneck mode ends, it means arriving at the destination. Because there is a bottleneck, the actual time spent by each person does not match its time, so there is a time delay cost.  $\lambda(t)$  denotes the queuing time at moment  $t$  when commuters start queuing. The travel time of the bus is  $t$ , at which point the commuting time cost per person is:  $C(t) = \text{total system delay cost} + \text{total system travel cost} + \text{congestion cost}$ . This is shown in equation 1.

$$C(t) = \alpha\lambda(t) + \alpha\sigma + g(t + \lambda(t)) \quad (1)$$

Among them:

$$\lambda(t) = \frac{D(t)}{s} \quad (2)$$

The delay at the bottleneck is the number of people who can only queue up due to the limited capacity of the bottleneck and cannot pass smoothly.  $d(t)$  refers to the

number of people queuing up at moment  $t$  and  $r(t)$  denotes the departure rate.

$$\frac{dD(t)}{dt} = \begin{cases} r(t) - s & D(t) > 0 \\ 0 & D(t) \leq 0 \end{cases} \quad (3)$$

The departure rate is  $l(t)$ , then.

$$l(t) = \begin{cases} r(t) & D(t) = 0 \\ s & D(t) \neq 0 \end{cases} \quad (4)$$

When the number of people in the queue at the bottleneck is 0, all the people departing are equal to the people leaving and the roadway is clear. When there is a queue at the bottleneck, the number of people leaving the bottleneck is equal to the maximum capacity of the bottleneck.

$$g(t) = \begin{cases} \delta \left( \frac{r(t)}{s} \right) \sigma & D(t) = 0 \\ \delta \sigma & D(t) \neq 0 \end{cases} \quad (5)$$

$\delta$  is the congestion parameter,  $\sigma$  is the bus running time.

Therefore, when the number of people in line is 0:

$$C_a(t) = \alpha \sigma + \delta \left( \frac{r(t)}{s} \right) \sigma \quad (6)$$

When there is a peak:

$$C_b(t) = \alpha \frac{D(t)}{s} + \alpha \sigma + \delta \sigma \quad (7)$$

$$\int_0^{t_c} r(t) = N \quad (8)$$

So, from the beginning to the end of the morning or evening rush, everyone can leave their location and take the bus to their destination.

### 3.2. Bottleneck Model Generation

The bottleneck model is shown in Figure 1, where: the vertical coordinate is the commuter arrival rate and the horizontal coordinate is the moment.

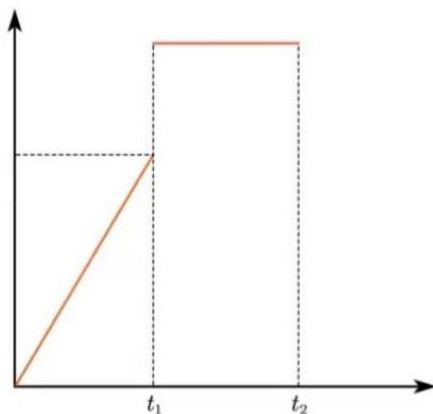


Figure 1 Dynamic charge model diagram

According to the generation type of assumptions, we divided the existed work into two categories.

In the bottleneck model, commuters gather at bus stops, which leads to queues. Let the start time of the morning and evening peak be 0.

(1) Between moments 0 and  $t_a$ ,  $D(t) = 0$ , when the bus stop is relatively relaxed,  $r(t) = l(t) < s$ , and all commuters arriving at the stop can get on the bus immediately.

(2) At moments  $t_a$  to  $t_b$ , queues occur because commuters gather in increasing numbers and the passenger arrival rate is greater than the capacity of the bus.

### 3.3. Bottleneck Model Generation

Tradable credits (TCS) are now proposed to alleviate congestion at bus stops. Let each person's expected arrival time point be  $t_2$  and there are a total of  $K$  credits, with each person assigned  $K/N$ . The first commuter arrives at time 0 and the peak end time is  $t_f$ . The credits paid by commuters for road congestion are event-dependent and are set to  $h(t)$ .  $h(t)$  varies with  $t$  because the closer the time is to  $t_2$ , the more congested the road is, the more people are on the bus, and the congestion per person the higher the cost. Because the farther away from  $t_2$  the fewer people, the closer to  $t_2$  the more people, so the need to motivate everyone to arrive at the point in time as far away from  $t_2$  the better. Therefore, located before  $t_1$ , commuters at this time are credit sellers, and after  $t_1$  are credit buyers, so  $(t_2) = K/N$ .  $P_c$  is the credit price, and the ratio of commission paid by buyers and sellers is  $\theta$ . The travel costs of commuters include (1) total system queuing delay cost; (2) total bus travel time cost; and (3) early arrival cost with a cost parameter of  $U$ ; (4) congestion cost at the bus stop; (5) payments associated with tradable credits; and (6) transaction costs. Therefore, the cost functions differ for whether a person sells or buys credits. In addition, even the last person to depart can arrive within the late time, so all people have the early arrival cost parameter. Thus the following equation is derived.

$$C_1(t) = \alpha \lambda(t) + \alpha \sigma + g(t) + P_c (1 - \gamma T_c) \left( k(t + \lambda(t)) - \frac{K}{N} \right) \quad t \in [0, t_1] \quad (10)$$

$$C_2(t) = \alpha \lambda(t) + \alpha \sigma + U (t_f - t - \lambda(t)) + g(t) + P_c (1 + (1 - \gamma) T_c) \left( k(t + \lambda(t)) - \frac{K}{N} \right) \quad t \in [t_1, t_2] \quad (11)$$

The departure time of the last commuter is before the first commuter arrives at the destination.

In order to minimize the system cost, we need to find the optimal point allocation scheme, where the total cost is minimized in the optimal state of the system, when the queuing delay cost is 0. Let the number of people leaving at this point is exactly equal to the maximum number of people passing at the bottleneck. Therefore, the bus journey should be smooth, with the same travel time for each vehicle.

$$\frac{dC_1(t)}{dt} = -U + P_c(1 - \gamma T_c) \frac{dk(t)}{dt} = 0 \quad (12)$$

$$\frac{dC_2(t)}{dt} = -U + P_c(1 + (1 - \gamma)T_c) \frac{dk(t)}{dt} = 0 \quad (13)$$

From the model we have:  $k(0)=0$ ,  $k_1(t_1)=K/N, k_2(t_1)=K/N$ .

$$K_1^* = \frac{U}{P_c(1 - \gamma T_c)} t \quad (14)$$

$$K_2^* = \frac{U}{P_c(1 + (1 - \gamma)T_c)} t + \frac{KT_c}{N(1 + (1 - \gamma)T_c)} \quad (15)$$

This leads to:

$$s \left( \int_0^{t_1} K_1^*(t) dt + \int_{t_1}^{t_2} K_2^*(t) dt \right) = K \quad (16)$$

This results in a credit charge for tradable credits in the optimal state of road transit. At this point, the road is at full capacity, exactly uncongested, and the road bottleneck passes exactly the number of people departing.

### 3.4. Activity-based travel choice

The welfare effect changes in the activity-based travel of commuters, and the objective function can be expressed as:

$$W = W^A(t) - \varphi C(t) \quad (17)$$

$$W^A = \int_t^{\lambda(t)} W_1(u) du + \int_{\lambda(t)}^{\lambda(t)+\sigma} W_2 u du \quad (18)$$

$W^A(t)$  denotes the total utility of commuters, and  $\varphi$  denotes the parameter that converts commuting costs into welfare utility.  $u_1$  is the utility of waiting for a car, and  $u_2$  is the utility of taking a car. When  $r(t) > 0$ ,  $E^*(t) = E(t)$ , and when  $r(t) = 0$ ,  $E^*(t) \geq E(t)$ . So equilibrium exists with commuters when they have complete information.

Under the social optimum, the utility of all commuters is equal and maximum.

$$\begin{cases} r(t)(E^* - E(t)) \\ r(t) \geq 0 \\ E^* - E(t) \geq 0 \end{cases} \quad (19)$$

At the time of equalization:

$$\frac{dW(t)}{dt} = 0; D(t) = 0 \quad (20)$$

### 3.5. Dynamic Tolling

Under tradable credit, in order to urge commuters to change their time consciously, the tolling system needs to change with time, by which the purpose of urging is achieved. Dynamic tolling actually converts the cost of waiting into a toll, making the implicit cost visible as a fee. From a societal perspective, welfare increases because the entire system tends to an optimal level because of the presence of tolls. At this point, commuters travel, as shown in Figure 2.

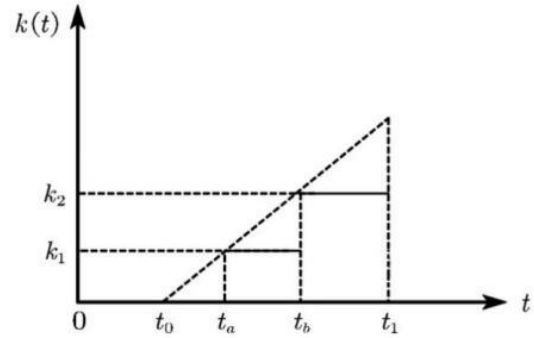


Figure 2 Dynamic charge model diagram

But because the continuous dynamic cost is too high, the technology is more complex, and the implementation is more troublesome, so the dynamic charge is fixed at a certain time interval in the change, the charge costs in order  $k_1, k_2, \dots$ . Charges start from the moment  $t_0$ , the cost is equal at a certain time interval, at this time, the only decision-making behavior is the departure time.

$$\begin{aligned} C_1(t) = & \alpha \lambda(t) + \alpha \sigma + U \left( t_f - t - \lambda(t) \right) + g(t) \\ & + P_c(1 - \gamma T_c) \left( k(t + \lambda(t)) - \frac{K}{N} \right) \quad t \in [0, t_1] \end{aligned} \quad (21)$$

$$\frac{dC_1(t)}{dt} = \alpha \lambda'(t) - U \lambda'(t) + g'(t) = 0 \quad (22)$$

At this point the number of people in the queue  $D(t) \neq 0$ .

$$g(t) = \delta \left( \frac{r(t)}{s} \right) \sigma, g'(t) = \delta \left( \frac{r'(t)}{s} \right) \sigma \quad (23)$$

$$\lambda'(t) = \frac{\delta \left( \frac{r'(t)}{s} \right) \sigma}{U - \alpha} \quad (24)$$

$$\lambda(t) = \frac{\delta \sigma r(t)}{s(U - \alpha)} \quad (25)$$

$$\int_0^t r(a) da = s(t + \lambda(t)) \quad (26)$$

$$r(t) = s(1 + \lambda'(t)) \quad (27)$$

$$r(t) = s \left( 1 + \frac{\delta \left( \frac{r'(t)}{s} \right) \sigma}{U - \alpha} \right) \quad (28)$$

In the case of dynamic step tolls, the commuter travel rate as above, at this time, society is not optimal, there are certain queues and traffic jams, although the queues can not be completely eliminated, but the implementation is easier and favored by many government managers. The optimal step charge needs to determine the amount and timing of the step charge, and in previous studies, the step charge is broadly divided into three types: ADL model, Laih model. The difference between these two types of models lies in the assumptions about the behavior of non-tolled travelers at the end of the step-toll period.

3.5.1. ADL Model

The ADL model was proposed by Amott et al in 1990. The model divides the whole time into four parts, in the beginning and end of the tolling period, commuters can pay no toll, in the second time zone travel, commuters travel to pay a certain early arrival cost, etc., so no one will choose to travel, and in the third time zone, commuters will pay a toll to complete the trip. The specific model curves are shown in Figure 3. Taking  $t_c$  moment as an example, the curves from top to bottom are, in order: cumulative departure curve without toll, cumulative departure curve after toll, cumulative arrival curve without toll, and cumulative arrival curve after toll. Where: The vertical coordinate represents the cumulative number of departures.

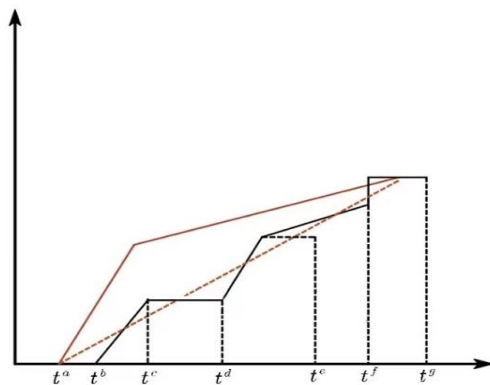


Figure 3 ADL Model Curve Chart

3.5.2. Laih Model

The model is a tolling strategy proposed by Laih, where when equilibrium is reached, the tolls are collected with some commuters passing the intersection on the main artery and some commuters avoiding the tolls on the side roads. Like ADL, the model divides the whole time into four parts, where commuters can not pay the toll at the beginning and end of the tolling period, and travel in the second time zone, where commuters travel to pay a certain early arrival cost and other costs, so no one will choose to travel, and in the third time zone, commuters will pay the toll to complete their trips. The specific model curve is shown in Figure 4. Where: The vertical coordinate represents the cumulative number of departures.

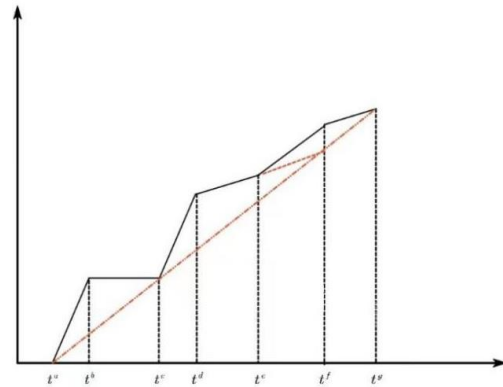


Figure 4 Laih Model Curve Chart

4. DEVELOPMENT SUGGESTIONS

Based on the research in this paper, the multidisciplinary integration of research ideas can also be advanced to contribute solutions to alleviate peak traffic pressure. The current available programs are described below.

4.1. Solution considering variable capacity and parking constraints

The research is carried out for the morning peak commuting problem by introducing bottleneck capacity variability and parking constraints into the classical Vickery bottleneck model. The main work is as follows: In the actual traffic system, when the traffic congestion is serious and the queue length is long, the traffic management may increase the capacity at the traffic bottleneck by taking some measures to relieve the congestion. In this case, the bottleneck model of capacity affected by queue length can be studied.

In this model, the bottleneck capacity increases from  $s_1$  to  $s_2$  when the queue length exceeds the threshold  $D_1$ , and returns to  $s_1$  when the queue length decreases and is smaller than another threshold  $D_2$  ( $D_2 < D_1$ ). It has been found that multiple user equilibria exist when  $D_2 < D_1$ . The stability of these equilibria can be further investigated by a time-evolving dynamic model. When  $D_2 = D_1$ , there is no multiple user equilibrium, and the capacity does not change more than twice. In addition, there is a parameter range in this case in which there is no equilibrium solution, unless the bottleneck capacity is allowed to switch back and forth in an instant.

On this basis, the constraint effect of public parking spaces can be considered. Since commuters must compete for parking spaces on a first-come, first-served basis and the number of spaces is limited, some commuters have to choose to travel by public transportation. Under this constraint, there is an internal equilibrium in which both car and public transportation modes are used. The number of people willing to drive is the potential maximum number of people willing to drive in the equilibrium, and parking is only a constraint

when the total number of spaces is below this value. In the bottleneck model with  $D_2=D_1$ , the parking constraint does not result in multiple equilibria, and the capacity does not change more than twice.

In general, the parking constraint can be extended to situations where there are both public spaces and reserved spaces. Some commuters reserve parking spaces in advance, while others have to compete for public spaces or take public transportation. In the bottleneck model with both parking constraints, when the total number of parking spaces is greater than the number of potential willing drivers, commuters with reserved spaces have no advantage over commuters competing for spaces, and the two can mix. Conversely, when the total number of parking spaces is less than the number of potential self-drivers, commuters competing for public spaces must travel before commuters in reserved spaces, and the former have higher travel costs than the latter. In this model, the bottleneck capacity can vary up to four times, but again there is no multiple user equilibrium.

In addition, we can analyze the impact of queue length threshold, total number of spaces, and space allocation ratio on the total travel cost of the system by several arithmetic examples. Existing studies have found that allocating the right number of spaces can increase the number of commuters using public transportation, while dividing the right allocation ratio can drive commuters competing for spaces to leave their homes earlier, both of which are effective in reducing the total system cost. Considering the potential risk costs, queue length thresholds are no longer as small as possible, but should be based on the system as a whole.

#### ***4.2. Activity-based evening peak commuting solution for bus hubs***

In China's metropolitan areas, especially in the CBD, the demand for evening peak commuting is greater than the capacity of public transportation, resulting in traffic congestion and even traffic accidents. The bottleneck of transportation hubs directly restricts people's travel activities. The solution to this problem is to provide more travel options by expanding road traffic supply; secondly, to induce commuters' choice of departure time and travel mode from the perspective of traffic demand, so as to avoid traffic bottleneck congestion, improve travel efficiency, and achieve easy travel. Therefore, we can study the evening peak commuting problem of residents at transportation hubs from the theory of maximizing commuters' utility.

Based on such considerations, we can combine the bottleneck model with an activity-based approach to innovatively introduce the cost of transit congestion into the chain study of late-peak residential commuting at transit hubs in order to investigate the issues related to

the time allocation of late-peak commuters between their activities and trips and the bottleneck congestion. The bottleneck model is used as a basis to consider internal transit congestion and to correlate travel behavior with activity. A dynamic travel equilibrium model is developed by introducing the cost of congestion within the bus and choosing the departure time according to different utility functions, and from this, the correlation properties under equilibrium conditions can be derived to explain the traffic phenomenon of commuters queuing in front of the bottleneck entrance in the evening peak. It has been found that the more sensitive commuters are to congestion inside the bus, the more they will try to avoid peak trips. In order to achieve greater net utility, commuters choose to stay longer at their place of work and delay the evening peak hours.

### **5. CONCLUSION**

In this paper, the bottleneck model is applied to the transit domain, creatively combined with tradable credits. Using the cost function of the bus bottleneck, the parameters related to public transportation are discussed. The bottleneck model is a fundamental tool for studying public transportation travel congestion. This paper reviews the bus applications of the classical bottleneck model and analyzes and compares the advantages and disadvantages of related dynamic charges.

Subsequent research can also be extended in the following directions (1) Considering the social optimum, the waiting cost per person in the case of discrete bus departures (2) The bus bottleneck model in more complex situations such as the non-peak situation due to unexpected traffic jams or traffic jams due to weather. (3) A calculation that combines the benefits generated by commuters at the origin and destination under the socially optimal total benefits, i.e., the commuter's welfare is maximized. (4) For government decision makers, the issue of how to plan the total benefits of transportation commuting with capital investment is also quite important, and how to use the least amount of money for the maximum benefit needs further consideration. (5) For flexible workers, commuting at a fixed time every day is unrealistic, and this part of commuters needs to be taken into account in the model.

In applying to the concrete reality, there is a specific goal for government managers and commuters: to exchange the least time for the maximum benefit, but subject to specific practical constraints, such as weather, income level, personal health, etc., the model also needs to be changed flexibly to make the results more realistic, applicable and universal.

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