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Analysis of Size and Momentum Anomalies in CAPM

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ABSTRACT
By assuming a linear relationship between expected returns and Beta (Beta is always positive), CAPM provides a powerful and direct prediction on how to measure the relationship between expected returns and risk [1]. However, CAPM still has drawbacks. On average, smaller companies have higher risk-adjusted returns than larger companies [2], which proves that CAPM is wrong. De Bondt and Thaler also find that CAPM cannot explain the abnormal returns between "winner" and "loser" stocks [3]. Based on the above information, this study aims to analyze size and momentum anomalies, and evaluate the performance of CAPM based on different sizes and momentum stocks.

Keywords: *size, momentum, stocks, CAPM.*

1. INTRODUCTION

This report aim utilize statistic testing approach including cross section and Fama MacBeth regression testing, to find the anomalies which existed in different size and momentums stocks by using CAPM.

2. DATA COLLECTION

All data comes from Ken French's online database between 1927 and 2020. LO 10 represents small size stocks and HIGH-PRIOR signifies high momentum stocks. To maintain the uniform format of the data, all the data expressed as percentage type will be divided by 100 to remove percentage sign.

3. RISK RETURN OF FOUR STOCKS

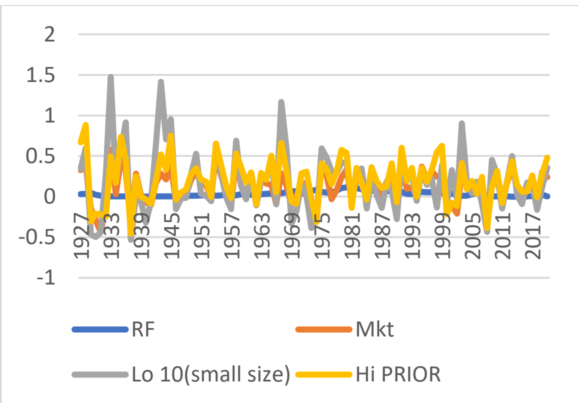

Figure-1-Annual return for four investments

According to Figure 1, during the period of 1927-2020, small size stocks have the largest volatility, which is the riskiest of the other three investments. In contrast, risk-free stocks (T-bills) has the lowest risk.

For cumulative returns, high-momentum stocks are much higher than the other three investments. To accurately represent each stock's return change over time, the four stocks are divided into two charts. Figure 2 contains the cumulative returns of risk-free, small-scale and market portfolio stocks, and Figure 3 depicts the data of high-momentum stocks.

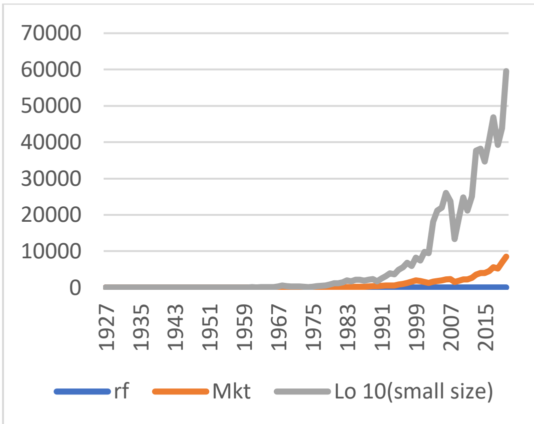

Figure-2-Cumulative return (1)

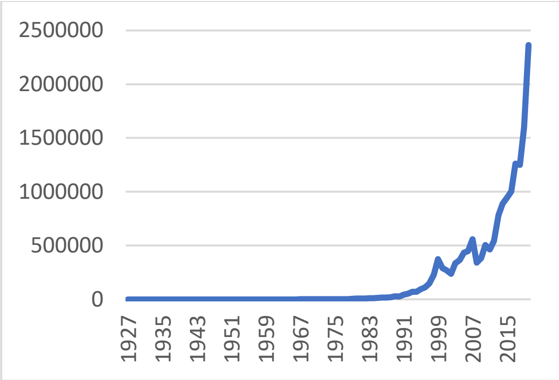

Figure-3-Cumulative returns (2)

3.1. Statistical observation of four stocks

Table-1- Statistical observation

	T-bills	Marker portfolio	Small stock	High momentum stock
mean	0.3334	0.1205	0.1839	0.2012
volatility	0.0310	0.1996	0.3847	0.2716
skew	1.0821	-0.4470	0.8658	0.0169
Excess kurtosis	1.1244	0.0818	1.5003	-0.2228
Max-drawdown	- 0.0002	-0.4404	- 0.5353	-0.4585
Beta	0	1	1.5214	1.1911
Sharp ratio	0	0.4364	0.3912	0.6180

Sharp ratio measures volatility-adjusted performance [4], defining excess return as the numerator, return standard deviation as the denominator. From table 1, high momentum stock has the largest sharp ratio, which means that this stock could provide the highest return in given risks among other stocks.

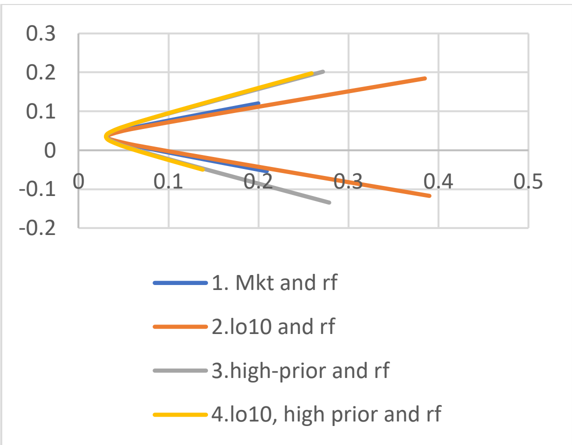

Figure-4-Efficient frontier

Since all investors hope higher returns with lower risk, they would like to choose the portfolio on the upper left edge. It can be seen from Figure 4 that, compared with the high-momentum and risk-free stock portfolio, the small-size, high-momentum and risk-free stock portfolio has higher returns and lower volatility (lower risk). Therefore, this combination performs best, while small-size stocks and risk-free stocks portfolio performs poorly. Based on this information, Table 2 is created.

Investment opportunities refer to all possible investment portfolios surrounded by effective frontiers. Obviously, the combination of market portfolio and risk-free stocks has a relatively larger effective frontier than the combination of small-size and risk-free stocks, indicating that size anomaly cannot be used to expand the set of benchmark investment opportunities. However, the opportunities set could be expanded with small size, high momentum stocks and Treasury bonds portfolio, as well as the combination of high momentum stocks and Treasury bonds.

Table-2- Performance ranking

Performance ranking of four investment	
1	Combination of small size, high momentum stocks, and T-bills
2	Combination of high momentum stocks and T-bills
3	Combination of market portfolio stocks and T-bills
4	Combination of small size and T-bills

3.2. Result analyzes

For the level one portfolio, investing in several stocks could diversify the risk. For the level two combination, the sharp ratio of high momentum is much higher than that of small-size level four combination. Although the average returns of high-momentum and small-size stocks are similar, the standard deviation of high-momentum stocks is significantly smaller than that of small-size stocks. High-momentum and risk-free stocks perform better.

4. ANALYSIS OF DIFFERENT SIZE STOCK

SML equation is as below

$$E_{(i)} = r_f + \beta * (r_m - r_i) \qquad (1)$$

By taking the excess return of each size stock/each momentum stock as the dependent variables, the excess return of the market portfolio as independent variables, the corresponding Beta can be calculated with the "slope" function. Obtain the average return of each size stock/each momentum stock by using the "mean" function. The stock market line can be drawn as follows.

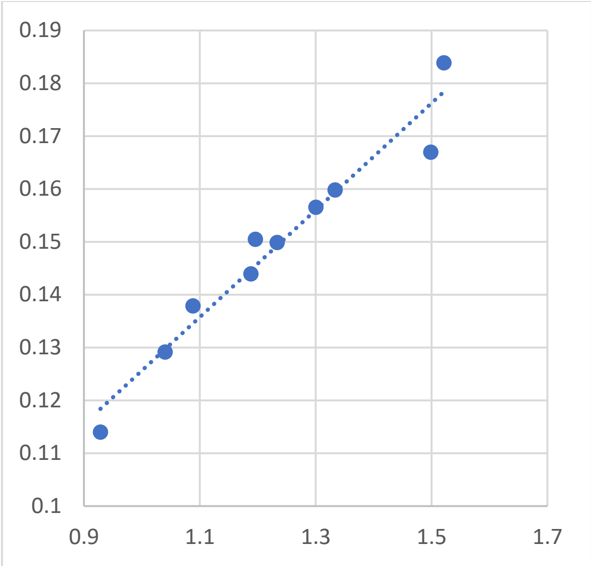

Figure-5-Security market line for each size stock

Figure 5 indicates that the trend line is upward and consistent with the implication of CAPM. In addition, the distance between each point and the trend line is very small, so that it seems possible for CAPM to work when dealing with stocks of different sizes.

4.1. Empirical Performance testing for size

4.2. Cross sectional testing

$$E(R^{ei}) = \lambda_0 + \lambda * \beta_i + \alpha^i \quad (2)$$

Statistic testing could be made by utilizing this equation.

Table-3- Statistical testing for size (Cross-sectional)

Intercept testing	*Slope testing*
H0 λ_0 =0	*H0* λ =0
H1λ_0 <>0	*H1*λ <>0
Significance level =5%	*Significance level =5%*
T-stat =-0.8889	*T-stat =12.4885*
T-test=1.9885	*T-test=1.9858*
T-stat< T-test	*T-stat> T-test*
Null hypothesis should not be rejected	*Null hypothesis should be rejected*

$$E(R^{ei}) = -0.0090 + 0.1013 * \beta_i + \alpha^i \quad (3)$$

According to table3, λ_0 =0 and λ <>0 could be accepted. Besides, the R-square of the model is equal to 0.9512, showing that the model can explain 95.12% of the change in the dependent variable variance. The intercept coefficient is -0.0090, which is approximately 0, and the slope coefficient is 0.1013 that approximates each excess return. The maximum difference is 0.049, which is acceptable. Based on this result, CAPM is feasible in different inventory sizes.

4.3. Fama MacBeth regression testing

As is shown in table 4, which is another method to run cross-sectional statistics by constructing cross-sectional regression and estimating the time series averages $\lambda_0$and λ each time.

Table-4- Statistical testing for size (Fama MacBeth)

Intercept testing	*Slope testing*
H0 λ_0 =0	*H0* λ =0
H1λ_0 <>0	*H1*λ <>0
Significance level =5%	*Significance level =5%*
T-stat =-0.2114	*T-stat =2.1702*
T-test=1.9885	*T-test=1.9858*
T-stat< T-test	*T-stat> T-test*
Null hypothesis should not be rejected	*Null hypothesis should be rejected*

Based on this result, intercept coefficient is equal to 0 while slope coefficient is not equal to 0, same result would be obtained.

5. ANALYSIS OF DIFFERENT MOMENTUM STOCK

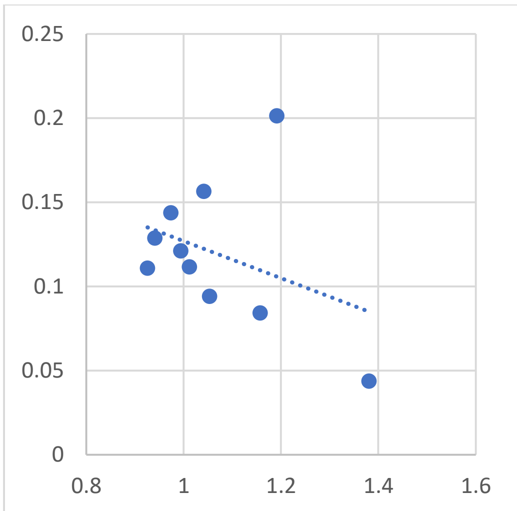

Figure-6-Security market line for each momentum stock

It can be seen from Figure 6 that the trend line is downward, and there is a negative correlation between beta and expected returns. This is inconsistent with the implication of CAPM. Compared with Figure 5, the distance between the points and the trend line is very large. Therefore, when dealing with different momentum stocks, CAPM does not work well.

5.1. Empirical Performance testing for momentum

5.2. Cross sectional testing

Table-5- Statistical testing for momentum (Cross-sectional)

Intercept testing	*Slope testing*
H0 λ_0 =0	*H0* λ =0
H1λ_0 <>0	*H1*λ <>0
Significance level =5%	*Significance level =5%*
T-stat =1.8698	*T-stat =-1.0862*
T-test=1.9885	*T-test=1.9858*
T-stat< T-test	*T-stat< T-test*
Null hypothesis should not be rejected	*Null hypothesis should not be rejected*

$$E(R^{ei}) = 0.2033 - 0.1099 * \beta_i + \alpha^i \qquad (4)$$

In table 5, since all T-stat values are less than T-test values, all null hypotheses cannot be rejected, that is, the intercept and slope coefficient are equal to 0. This result is inconsistent with CAPM's implication. R-square is equal to 0.1285, which means that the model can only account for 12.85% of the dependent variable variance, which is too low. CAPM is not feasible in different momentum stocks based on this result.

5.3. Fama MacBeth regression testing

Table-6- Statistical testing for momentum (Fama MacBeth)

Intercept testing	*Slope testing*
H0 λ_0 =0	*H0* λ =0
H1λ_0 <>0	*H1*λ <>0
Significance level =5%	*Significance level =5%*
T-stat =5.0638	*T-stat =-2.4735*
T-test=1.9885	*T-test=1.9858*
T-stat> T-test	*T-stat<- T-test*
Null hypothesis should be rejected	*Null hypothesis should be rejected*

Table 6 explicitly illustrates that both λ_0=0 and λ =0 cannot be accepted, since the T-stat of the intercept is larger than the T-test and the T-stat of the slope is smaller than the negative T-test. Therefore, CAPM does not play a good role in dealing with stocks with different momentum.

6. CONCLUSION

CAPM is effective in handling size anomalies, but it cannot resolve momentum anomalies. However, there are some drawbacks in this report. Bias may exist since the whole dataset is not big enough and only US data is used. And there is no normality assumption in the standard deviation calculation process, which indicates that the standard deviation of each stock may be an imperfect risk measure [5].

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