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Duration Strategy for Bond Investment Based on an Empirical Study

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ABSTRACT
The duration is also known as the Macaulay duration. It takes the cash flow occurring in the future time to fold the present value according to the current rate of return, and then multiplies each present value by the number of years from the time when the cash flow occurs, and then sums it up. The duration is calculated by dividing the sum by the discounted sum of the bond's cash flows. In general, it is the weighted average of the time required for the bond's cash flows to be paid. We calculated the sum of the duration of the bonds. For example, the first bond has a duration of four years and the second bond has a duration of 1.96 years. By calculating the duration, we can help investors better know when to receive the money receivable. And immunization is a strategy that helps to reduce the impact of interest rate fluctuation. In this paper, we have studied the methods of calculating duration and how to use the knowledge of duration and the immunization strategy to hedge the risk.

Keywords: *duration, immunization strategy, bond*

1. INTRODUCTION

A bond is a security issued by the issuer to raise funds, pay a certain percentage of interest at an agreed time, and repaying the principal at maturity. According to the face value, the basic feature of a bond is a fixed coupon and the principal, which is repaid at maturity.

According to different issuers, they can be divided into government bonds, financial bonds, and corporate bonds. Investors buy bonds as if they were lending money to governments, large corporations, or other bond issuers. Of the three, government bonds, guaranteed by government taxes, carry the least risk and the least return. Corporate bonds are the riskiest and most likely to offer the highest returns. Bondholders are creditors, and issuers are debtors. Different from bank credit, the bond is a direct debt relationship. Bank credit forms an indirect debt relationship through depositor - bank, bank - lender. No matter the form of bonds, most of them can be bought and sold in the market, thus forming the bond market.

Bonds are also being sold at negative interest rates, which were first sold in 2010 as investors sought refuge from the threat of inflation. Investors pay $105.50 for every $100 of bonds they buy, willing to pay the government for the right to borrow from it. Investors still want to make money on bonds because they offer protection against inflation. This paper tries to investigate the bond valuation based on quantitative analysis.

Bond price is the sum of the present value of future coupon payments and the present value of the maturity value, which are both calculated at the interest rate prevailing in the market. Sarig and Warga [1] found that a common implicit assumption in studies of bond market data is that recorded bond prices are true. Besides, Livingston and Zhou [2] develop a new method to estimate bond price changes in response to changes in interest rates. This method is always more accurate than traditional estimation with modified duration. Brooks and Livingston [3] study the impact of duration and convexity on bond price changes. They found that the duration of a first-order approximation of the magnitude of a percentage change in a bond's price when interest rates change and convexity can be employed to improve the approximation to second order.

The duration method has been studied commonly. Kiefer [4] found econometric methods based on hazard functions provide a natural approach to analyzing economic data that can be modeled as generated by series of sequential decisions. The methods do not provide a "correct" approach to data analysis to be contrasted with an "incorrect" approach based on the specification of a density function: Specification of a hazard function is an alternative to the specification of a density function, providing a simple way to choose a specification that allows plausible behavior to be modeled. Many of the economic applications of hazard function methods have to date been in labor economics. Alps [5] have the study of interest theory includes the concept of duration and how it may be used to approximate the change in the present value of a cash flow series resulting from a small change in interest rate. Also, they found that a non-linear approximation using Macaulay duration that is more accurate than the linear approximation using modified duration, A corresponding second-order approximation using Macaulay duration and convexity is more accurate than the usual second-order approximation using modified duration and convexity. Hawawini [6] examines the behavior of a bond's duration mathematically in response to changes in the market yield, the bond's term to maturity, and its coupon rate. Although the nature of the relationships between duration and yield, duration and maturity, and duration and coupon are known to financial economists, a review of the literature on the subject reveals that the exact form of these relationships has generally been investigated via numerical evaluation, computer simulation, or casual analytical observation. The absence of a systematic mathematical approach to the problem may be partly due to the somewhat intractable form of the definitional expression of a bond's duration, particularly in the case of the relationship between duration and term to maturity.

An appropriate risk adjustment technique applied to discount rate for evaluating stochastic negative cash flows is discussed for this part for risk analysis. The proposed approach considers a future cash flow to respond to an investment or a borrowing rather than an independent cash flow [7]. As discount rates applied to evaluate investments and borrowings have different meanings, the generalized net present value method is more appropriate to value cash flows with opposite signs [8]. The given method uses two different rates: the finance rate is applied to discount present positive values (PVs) and the reinvestment rate – to discount negative PVs of a nonconventional project [9-10]. It is shown that these rates are adjusted for risk relative to their risk-free values in an opposite way. A universal relationship between risk penalty and risk premium is derived from the assumption that investment and borrowing risks are equal in their value.

For the portfolio investment, we summarized some literature. Brennan and Cao [11] develop a model of international equity portfolio investment flows. The implications of the model are tested using data on United States (U.S.) equity portfolio flows. Aggarwal et al., [12] using panel regression estimates from the IMF's CPIS survey of foreign debt and equity portfolios across 174 originating and 50 destination countries from 2001 to 2007. Durham [13] suggests that lagged FDI and EFPI do not have direct, unmitigated positive effects on growth. Moreover, extreme bound analysis (EBA) of significant results indicates that the estimates are robust compared to other empirical studies on growth. Goldstein and Razin [14] develop a model of foreign direct investments (FDI) and foreign portfolio investments (FPI). FDI enables the owner to obtain refined information about the firm. The model can explain several stylized facts regarding foreign equity flows.

Our final goal is to hedge the risk of interest rate fluctuation as time goes by in this paper. In this process, many concepts are contained, including bond, duration, modified duration, portfolio weight, and immunization. Firstly, we have studied the concepts related to our project by citing the former studies and giving the definition of them. Each of the concepts has several citations. Secondly, we are going to handle the problem. The problem is divided into pieces by us, which means we solve it step by step. In the beginning, we introduce the background information about the topic. And then, we solve the specific problems logically. Finally, we conclude the topic.

The remainder of the paper is organized as follows: section 1 calculates bond A; section 2 is the calculation about bond B; section 3 is the calculation about modified duration. Section 4 is about immunization.

2. BACKGROUND INFORMATION

Immunization is an important strategy that helps us to minimize the risk of interest rate fluctuation. In this paper, we aim to help the student to hedge the interest rate risk. Here the basic information is shown to us.

From Table 1, we can know that Bond A is a zero-coupon bond, which means it has no annual coupon payment and return all the money at maturity. And we can know the current price, face value of bond A. What is more, we know that all debt considered has the same level of default risk and the benchmark yield curve for the fixed-income investments of the same riskiness is flat at a level of 4% p.a. Therefore, the opportunity cost of capital is 4%. Because the hurdle rate is 4% compared with other investments of the same risk, in this case, we can explain the calculation of the current price of Bond A by discounting the face value for 4 years:

$$\text{Current price}=\frac{face\ value}{(1+4\%)^4}= 85.48 \quad (1)$$

Furthermore, we can calculate the present value each year. And we know the exact data of $t \times PV$. Then we can figure out the duration of Bond A through the formula of Macaulay duration. Apart from that, since Bond A is a zero-coupon bond, we can directly answer its duration, which is equal to maturity (Table 1).

Table 2 shows that the fundamental information of bond B, such as time, coupon, discount factor, and the weighted value of time. To explain in detail, the annuity of bond B is known to us, which is 4. What is more, we know the discount factor, line factor (calculated by the data provided in the question) of each year. And we know the weighted value of time in year 1 and year 2, which are 3.84 and 7.36, respectively. The sum of the weighted value of time is also shown to us at the bottom of the table.

Table 1, Bond A's cash flow

t	CF (cash flow)	PV. (present value)	t PV
1	0	0	0
2	0	0	0
3	0	0	0
4	100	85．48	341.92

Table 2, Bond B's cash flow

Time t	Coupon C_t	Discount factor $1/(1+i)^t$	Line factor $C_t/(1+i)^t$	The weighted value of time. $t \times C_t/(1+i)^t$
1	4.00	0.96	3.84	3.84
2	4.00	0.92	3.68	7.36
3	100.00	0.92	92	184
Total			99.52	195.2

Above is the background information, which illustrates the basic information about bond A and bond B. And we are based on the question related to the topic of duration and immunization. The specific method of calculating duration and modified duration will be given to you in the next part of the paper.

3. METHOD

In this chapter, our group calculated the duration of bond A and the duration of bond B and the calculation of modified duration and weight. Each of the four members in our group completed the above calculation and put them together. Finally, we use the duration with immunization strategy to solve the problem: hedging the risk.

3.1.Duration

Duration, which is also called Macaulay duration, is a measure of the sensitivity of the price of a bond or other debt instrument when there is a change in the yield to maturity. And it calculates the weighted average time before a bondholder would receive the cash flows of the bond. And weight is the individual cash flow divided by the total cash flow. Here is the formula of Macaulay duration:

$$\text{Macaulay Duration} = \sum_{t=1}^{T} t \times \frac{PV(CF_t)}{P} \quad (2)$$

Where: T represents the duration of bond in the period, PV is the present value of the cash flows in each period, P means current bond price

Finally, according to formula (2), we can get the answer of bond A and bond B, which are 4 and 1.96, respectively.

3.2.Modified Duration

Macaulay duration is the weighted average result of the future cash flow period of interest-bearing bonds, and its weight is the proportion of the current cash flow in the current bond price. The formula can be expressed as formula (2).

Macaulay duration measures the weighted average time of the future cash flow period of a financial asset. It assumes that the yield curve is parallel. That is, the interest rate changes of each asset are the same, and the same discount rate is used to discount the future cash flow.

Macaulay duration model is an important tool to measure interest rate risk. Still, its simplicity and several important assumptions greatly limit its availability and accuracy as an interest rate risk management tool. We introduce the following hypothesis.

Hypothesis 1: Macaulay duration only considers the relationship between price changes and yields. In fact, the real relationship between bond prices and yields is nonlinear, which is called the convexity of the yield curve. Therefore, only when the rate of return changes very small, the linear relationship represented by Macaulay duration can be approximately established. When there is a large change in the yield rate, there will be a large error in using Macaulay duration estimate the price change.

Hypothesis 2: The term structure of interest rate is flat. The Macaulay duration uses the same discount rate for all cash flows, meaning that the term structure of interest rates is assumed to be flat. In fact, the flat term structure of interest rate is only a very special case. In general, the term structure of interest rate is not flat, and most of them are in the upward nonlinear shape.

Hypothesis 3: Future cash flows will not change when interest rates change. Macaulay's duration model assumes that bond cash flow will not change with interest rate fluctuations. However, for financial instruments with implied options, their future cash flow will generally change with interest rate fluctuations, and their prices will also change accordingly.

Hypothesis 4: The yield curve is parallel moving. The parallel movement of the yield rate is considered. In fact, due to the influence of time factors, the response of different maturity bond yields to market factors is different. That is, the changes of different maturity yields are inconsistent, resulting in changes in yields.

Modified duration is an adjusted version of Macaulay duration. It measures the change in the value of a bond in response to a change in yield. Namely, for every 1% movement in interest rates, the bond would inversely move at a price by MD%. Modified duration can be calculated by dividing Macaulay duration by (1+YTM).

$$\text{Modified Duration}=\frac{Macaulay\ Duration}{1+YTM} \quad (3)$$

Where: YTM is yield to maturity

Namely, for every 1% movement in interest rates, the bond would inversely move at a price by MD%. Modified duration can be calculated by dividing Macaulay duration by (1+YTM)

Based on the Macaulay duration, we can calculate the modified duration by dividing Macaulay duration by (1+YTM)

$$MD = \frac{D}{1+Y} = -\frac{1}{P}\frac{\partial P}{\partial Y} = 1.88 \quad (4)$$

3.3.Calculation of the weight of portfolio bond

Immunization, also known as multi-period immunization, is a risk-mitigation strategy that matches the duration of assets and liabilities, protecting our portfolio against interest rates change. This strategy helps large companies and institutions protect their portfolios from exposure to interest rate fluctuations. And it is always related to the Macaulay duration because bond managers usually use the immunization strategy to manage the bond portfolio risk. In this essay, we will use this strategy to immunize the interest rate risk. And the main goal is to calculate the weight of the portfolio bond by using the immunization strategy, which we will explain to you in the following section.

We would use an immunization strategy to establish a portfolio. In the previous three sections, we have already calculated the duration of both bond A and bond B and the modified duration of bond B. And now we need to calculate how many units of each bond need to be bought or sold.

Firstly, since we have already known the duration, we just need to match the duration of two different bonds with the duration of liability, which is the most important point for using the immunization strategy. And in this question, it says, "three years from now, the student will need to make a one-off tuition payment". In this case, it is noticeable that the duration of liability is 3. And we assume that the portfolio share of bond A is w, so the weight of bond B is (1- w). Knowing that the duration is additive, we can list an equation as follows:

$$Duration\ of\ liability = w \cdot D(A) + (1-w) \cdot MD(B) \quad (5)$$

Where, D(A) is Duration of bond A; MD(B) means Modified Duration of bond B

By solving the equation, we can get the answer that the portfolio share of bond A and Bond B are approximately 0.53 and 0.47.

Secondly, we are supposed to calculate the present value of the liability. We can know that the liability is 17,000 and the opportunity cost of capital is 4% in the former questions. Next, we just need to discount the liability for 3 years.

$$Present\ value = \frac{liability}{(1+r)^3} \quad (6)$$

Where r is the discount rate

Finally, we can calculate the units of bond A and bond B that should be bought or sold:

$$PV(A) = PV \times w \quad (7)$$

$$PV(B) = PV \times (1-w) \quad (8)$$

According to the formulas (6) (7) (8), we can know that the money we should invest in bond A and bond B, are 8009.85 and 7103.08, respectively. To some extent, it immunizes the students from the risk of interest rate changes. And the so-called immunization strategy is the construction of a specific portfolio that the reinvestment return can make up any capital loss caused by interest rate fluctuation. However, there are still some limitations in this situation. On the one hand, it is unrealistic that there is a flat yield curve. The interest rate is not fixed, which may change along with the flowing of time.

On the other hand, as time passes and the interest rate changes, the portfolio must be periodically rebalanced in order to maintain an immunized position. That is because the data of duration may change as time passes by and interest rate changes. To explain it, we need to look back to the formula (2)(5).

Through the formula, we can know that the duration will change as time flows and the interest rate change, thus causing the change of portfolio share of each bond.

Therefore, what I mentioned above are the limitations of the immunization strategy.

4. CONCLUSION

In this paper, we mainly focus on the Macaulay Duration, Modified Duration, and the application of the Macaulay Duration in determining the best portfolio. Based on the Macaulay duration formula, we first calculate the Macaulay duration for bond A and bond B. We obtain the data of the modified duration for bond A and Bond B. Finally, we use this to determine the portfolio.

Macaulay duration is the weighted average term to maturity of the cash flows from a bond. The weight of each cash flow is determined by dividing the present value of the cash flow by the bond price. In this paper, we calculate the duration of two bounds. The first bond, the first bond, has a duration of four years and the second bond has a duration of 1.96 years. Our research on the duration still finds some disadvantages and shortcomings, which will greatly limit its availability and accuracy as an interest rate risk management tool.

First, Macaulay duration only considers the relationship between price changes and yields. In fact, the real relationship between bond prices and yields is nonlinear, which is called the convexity of the yield curve. Therefore, only when the rate of return changes very small, the linear relationship represented by Macaulay duration can be approximately established. When there is a large change in the yield rate, there will be a large error in using Macaulay duration estimate the price change.

Secondly, Future cash flows will not change when interest rates change. Macaulay's duration model assumes that bond cash flow will not change with interest rate fluctuations. However, for financial instruments with implied options, their future cash flow will generally change with interest rate fluctuations, and their prices will also change accordingly.

Thirdly, the yield curve is parallel moving. In fact, due to the influence of time factors, the response of different maturity bond yields to market factors is different. That is, the changes of different maturity yields are inconsistent, resulting in changes in yields.

Thus, we will try to consider the time and the bond price and yields as the important factors to determine the portfolio in the future.

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