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The Influences of Markowitz Model with Different Constraints for Optimal Portfolio

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ABSTRACT
This paper aims to use the Markowitz model to find the optimal portfolios under different constraints and compare the differences. In this paper, we use ten stocks that correspond to the ten listed companies that need to be analyzed. The paper includes 20 years of historical daily return for ten stocks in NYSE, a proxy for risk-free rate (1-month federal funds rate), and the equity index (S&P 500) data for ten stocks. The optimal weights of the ten stocks have done under five constraint conditions to compare each other within the minimum variance portfolio and maximum Sharpe portfolio. Return rate, standard deviation, and Sharpe ratio are also calculated for determining the optimum portfolio.

Keywords: *Portfolio Investment, Risk, The Markowitz Model, Constraints, Comparison, Analysis.*

1. INTRODUCTION

When considering objectives and products for investment, risk and return become the indispensable influencing factors.[1] In the practice of the securities market, individual risks of stocks have the most influence for investors because they must respond to system risks of the entire market and have to experience the changes in the risk factor of investment varieties for stocks they invested. In order to decrease the risks and increase the return rate in investment, the theories and methods for investment are used regularly and widely, which are very important to people and enterprises that could help them make investment choices and decisions more effectively and efficiently.[2] Sometimes, we can consider combining the risk-free assets with the investment group of the risk assets. Specifically, in a specific portfolio, the proportion should also be different due to the difference in the risk of investment varieties.[3] Suppose the proportion of investment products with high risk is high. In that case, the risk of the entire portfolio will be improved, and if the proportion of investment products with a low degree of risk, the risk of its investment portfolio will decrease.[4] In the case of lack of funds and inadequate information, the best way for investors to deal with risks is through portfolio investment.

Portfolio investment has become one of the most popular investment methods in the world because it is endowed with significant special importance for the investors. Using portfolio investment is easier than conventional investment methods to achieve the investment objectives, and the benefits and practicability of using portfolio investment also go beyond most investment methods.[5] Firstly, In economics and finance, portfolio investment represents passive holdings of securities such as stocks, bonds, or other financial assets, none of which entails the investor's active management or control of the securities' issuer [6]. Secondly, portfolio investment emphasizes rational investment methods, rational investment concepts, state of health, and a stable investment market, which uses the expected value to represent the investment return and the standard deviation to represent the risk of return.[7] Thirdly and most importantly, increasing the variety and quantity of securities unrelated to each other in the portfolio can effectively reduce the risk of the whole portfolio. In that case, portfolio investment can easily solve the risk measurement of assets.[8]

However, analyzing the results and the influences is the most significant for portfolio investment to maintain and do the next steps. In this paper, we use the data of ten firms' stocks to make a comparison with different constraints and data analysis to get the difference about

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the portfolio investment satisfied in different conditions. The ten stocks respectively correspond to the ten listed companies which need to analyze: NVDA, CSCO, INTC, GS, USB, TD, CN, ALL, PG, JNJ, and CL, and they from four disparate sectors: Technology, Financial Services, Consumer Defensive, and Health Care. The data includes 20 years of historical daily return for ten stocks in NYSE, a proxy for risk-free rate (1-month federal funds rate), and the equity index (S&P 500). The charts describe the Variation Trend of Stock Price from the ten firms between 2001-2021, and also make a brief introduction and analysis reflects that the situations of the firms from the selected ten stocks. Then we take the Markowitz Model equation to obtain the expected return of the whole portfolio, used Microsoft Excel and Excel SolverTable to determine the risk-return opportunities and show the best risk-return combinations, and also used the slope of the capital allocation line (CAL) to represent the Sharpe ratio in the chart. Next, we used the Markowitz Model to provide a list of results data of the ten stocks with Minimum Variance Portfolio and Maximum Sharpe Portfolio under five constraints, which are Model 1: Without Constraints, Model 2: Regulation T, Model 3: Arbitrary "Box", Model 4: Zero Tolerance on Short Positions, and Model 5: Effect of Inclusion of the Broad Index, and finally make comparison and analysis.

The remainder of the paper is organized as follows: Section 2 describes the brief introduction for the situations of the firms to which each stock belongs and the data description for the variation trend of each stock price between 2001-2021; Section 3 introduces how to use Markowitz model to process the data of the ten stocks; Section 4 introduces the data analysis that the Markowitz Model with different constraints and the results of Markowitz Model; Finally, we make the conclusion part to the end of this paper.

2. DATA

For the data collection, this part includes the explanation of the components of the data and the result of charts after data disposal. The data that will be analyzed is generated from finance.yahoo.com and Bloomberg Terminal, which carries out the step of data collection. This part takes an overview of the firms' circumstances. The variation trend of each stock price between 2001-2021 reflects the data that utilizes 20 years of historical daily return for ten stocks in NYSE, a proxy for risk-free rate (1-month federal funds rate), and the equity index (S&P 500) from each stock differently in the most intuitive way.

2.1. NVIDIA CORPORATION (NVDA)

NVIDIA Corporation is an American multinational technology company that designs graphics processing units for the gaming and professional markets and system on chip units for the mobile computing and automotive market.

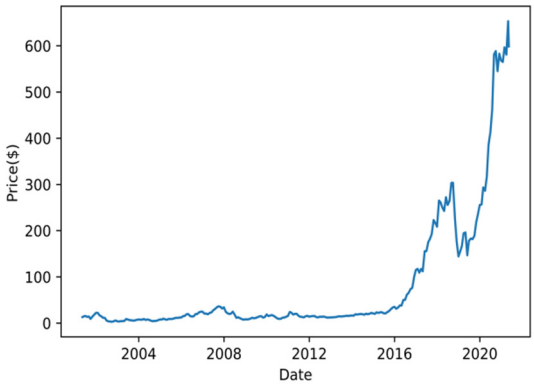

Figure 1. Variation Trend of NVDA Stock Price

The Variation Trend of NVDA Stock Price between 2001-2021 shows that it is in a stable state from May 11, 2001 to May 11, 2015, which close to USD 0.00; From May 11, 2016 to May 11, 2018 the price of the stock starts to rise and reach the price of USD 300.00; The price decreases from May 11, 2018 to May 11, 2019 and rises sharply from May 11, 2019 to May 11, 2021 about USD 500.00 in the last figure.

2.2. CISCO SYSTEM, INC. (CSCO)

Cisco Systems, Inc. is an American multinational technology conglomerate headquartered in San Jose, California, which develops, manufactures, and sells networking hardware, software, telecommunications equipment, and other high-technology services and products.

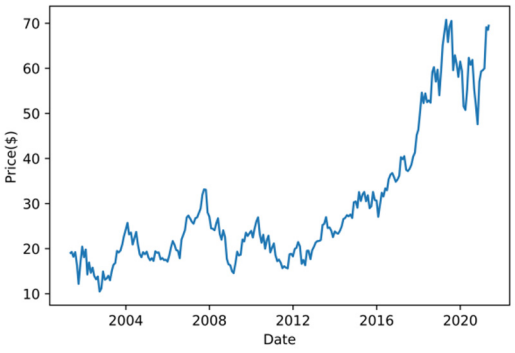

Figure 2. Variation Trend of CSCO Stock Price

The Variation Trend of CSCO Stock Price between 2001-2021 shows that from May 11, 2001 to May 11, 2016 there was a slight fluctuation in the chart, and price remained in the range of approximately USD $10.00 to USD $30.00; The stock price of CSCO has been rising from May 11, 2016 to May 11, 2019; From May 11, 2019 to May 11, 2021, the price of CSCO

stock fluctuated significantly, eventually reaching about USD $70.00 at May 11, 2021.

2.3. INTEL CORPORATION (INTC)

Intel Corporation is an American multinational corporation and technology company headquartered in Santa Clara, California, in Silicon Valley, which is the world's largest semiconductor chip manufacturer by revenue,[3][4] and is the developer of the x86 series of microprocessors, the processors found in most personal computers (PCs).

Figure 3. Variation Trend of INTC Stock Price

The Variation Trend of INTC Stock Price between 2001-2021 shows that the price fluctuated at about USD 20.00 from May 11, 2001 to May 11, 2014; The price began to rise from May 11, 2014, and fluctuated sharply from May 11, 2018 to May 11, 2021; Finally the stock price was about USD 100.00 on May 11, 2021.

2.4. THE GOLDMAN SACHS GROUP (GS)

The Goldman Sachs Group, Inc. is an American multinational investment bank and financial services company headquartered in New York City, which offers services in investment management, securities, asset management, prime brokerage, and securities underwriting.

Figure 4. Variation Trend of GS Stock Price

The Variation Trend of GS Stock Price between 2001-2021 shows that during the period of May 11, 2001 to May 11, 2007 the stock price($) has been rising from about USD 100.00 to about USD 250.00; From May 11, 2007 to May 11, 2009, the stock price has decreased significantly then increased, and then fluctuated but increased from May 11, 2009 to May 11, 2020; Finally, from May 11, 2020 to May 11, 2021the price($) quickly increased to about USD 450.00.

2.5. U.S. BANCORP (USB)

U.S. Bancorp is an American bank holding company based in Minneapolis, Minnesota, and incorporated in Delaware. It is the parent company of the U.S. Bank National Association and is the fifth largest banking institution in the United States.

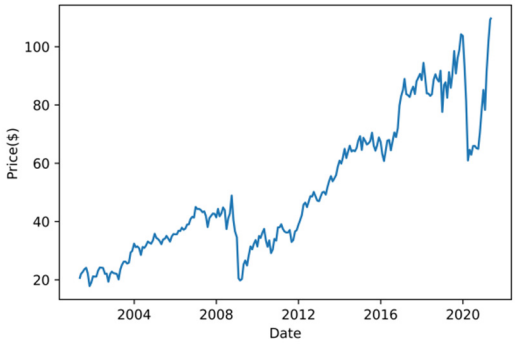

Figure 5. Variation Trend of USB Stock Price

The Variation Trend of USB Stock Price between 2001-2021 shows that from May 11, 2001 to May 11, 2008 the stock price increased from about USD 20.00 to about USD 40.00; From May 11, 2008 to May 11, 2009 the stock price decreased to under the price of USD 20.00; From May 11, 2009 to May 11, 2019 the stock price smoothly increased to about USD 100.00; From may 11, 2019 to May 11, 2020, the stock price decreased significantly to about under the price of USD 60.00; From May 11, 2020 to May 11, 2021 the stock price increased to the price above USD 100.00.

2.6.THE TORONTO-DOMINION BANK (TD CN)

The Toronto-Dominion Bank is a Canadian multinational banking and financial services corporation headquartered in Toronto, Ontario.

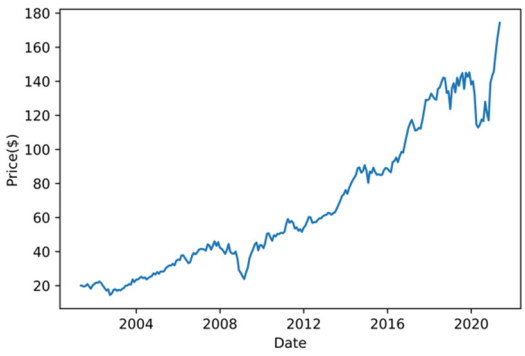

Figure 6. Variation Trend of TD CN Stock Price

The Variation Trend of TD CN Stock Price between 2001-2021 shows that the price has been rising since about USD 20.00 in May 11,2001, and the stock price decreased on May 11, 2009 and May 11, 2020, when after that it increased again. Finally, the stock price was about USD 180.00 on May 11, 2021.

2.7. THE ALLSTATE CORP. (ALL)

The Allstate Corporation is an American insurance company, headquartered in Northfield Township, Illinois, near Northbrook since 1967. Founded in 1931 as part of Sears, Roebuck, and Co., it was spun off in 1993. The company also has personal lines insurance operations in Canada.

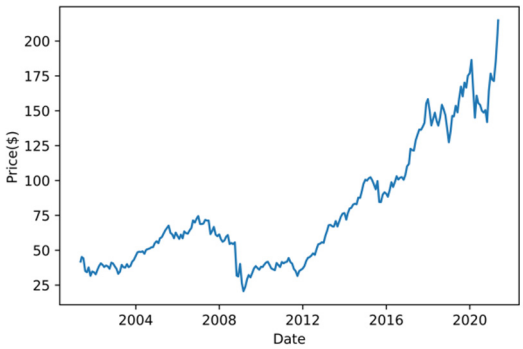

Figure 7. Variation Trend of ALL Stock Price

The Variation Trend of ALL Stock Price between 2001-2021 shows that from May 11, 2001 to May 11, 2012 the stock price has always fluctuated about the price of USD 50.00; From May 11, 2012 to the beginning of 2020 the stock price increased to about USD 280.00; During the last year of the variance trend data the stock price fell down from May 11, 2020 and increased to the price about USD 200.00 on May 11, 2021.

2.8. PROCTER & GAMBLE (PG)

The Procter & Gamble Company is an American multinational consumer goods corporation headquartered in Cincinnati, Ohio, founded in 1837 by William Procter and James Gamble.

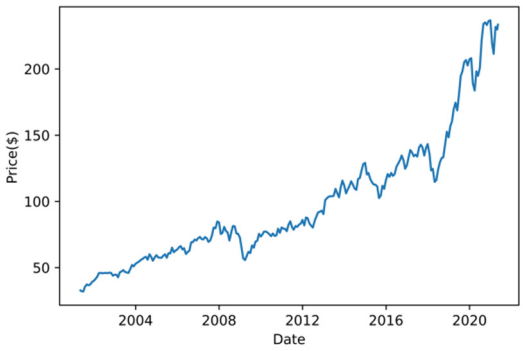

Figure 8 Variation Trend of PG Stock Price

The Variation Trend of PG Stock Price between 2001-2021 shows that from May 11, 2001 to May 11, 2018 the stock price smoothly increased from under USD 50.00 to above the price of USD100.00; From May 11, 2018 to May 11, 2021 the stock price significantly increased to the price about between USD 200.00 and USD 250.00.

2.9. JOHNSON & JOHNSON (JNJ)

Johnson & Johnson is an American multinational corporation founded in 1886 that develops medical devices, pharmaceuticals, and consumer packaged goods.

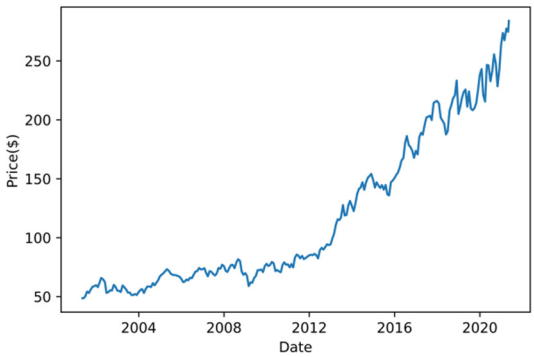

Figure 9. Variation Trend of JNJ Stock Price

The Variation Trend of JNJ Stock Price between 2001-2021 shows that from May 11, 2001 to May 11, 2021 the JNJ stock price kept increasing from about USD 50.00 to the price between USD 250.00 and USD 300.00; During the period from 2001 to 2012 the JNJ stock price increased slowly, and then the growth of the JNJ stock price accelerated from May 11, 2012 to May 11, 2021.

2.10. COLGATE-PALMOLIVE COMPANY (CL)

Colgate-Palmolive Company is an American multinational consumer products company headquartered on Park Avenue in Midtown Manhattan, New York City. It specializes in the production, distribution, and provision of household, health care, personal care, and veterinary products.

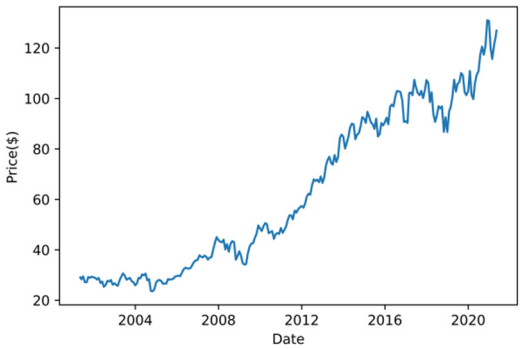

Figure 10 Variation Trend of CL Stock Price

The Variation Trend of CL Stock Price between 2001-2021 shows that between May 11, 2001 and May 11, 2005, the price of CL stock was stable at the price of about USD 30.00; From May 11, 2005 to May 11, 2018, the price of CL stock increased with fluctuating from about USD 40.00 to USD 100.00; During the period of May 11, 2018 to May 11, 2021, the price of CL stock increased sharply after a slight decline, and the price was about USD 120.00 at data ends.

3. THE MARKOWITZ MODEL

The Markowitz Model is one of the most efficient portfolio optimization models, created by Harry Markowitz in 1952. Given many risky and risk-free assets, the Markowitz portfolio selection can be made according to the following approaches.

First, from the raw data of the stocks, the monthly return mean and standard deviation value of each stock should be calculated to generate further the return and risky condition of stocks [9]. Then, obtaining the expected return of the whole portfolio by applying the equation:

$$E_{R_p} = \sum_{i=1}^{n} X_i E(R_i) \quad (1)$$

Here, X_i represents the proportion of investment in stock i, and $E(R_i)$ represents the expected rate of return on each stock i.

Next, Microsoft Excel and Excel SolverTable are used to determine the risk-return opportunities available to investors by creating the graph of the minimum-variance frontier. The graph, can directly show the best risk-return combinations.

Thirdly, the Sharpe ratio is essential in asset allocation since it is defined as the unit return on the unit of risk. Moreover, Sharpe ratio, as the slope of the capital allocation line (CAL) (CAL equation is shown below, which represents the Sharpe ratio),

$$E(R_C) = R_f + S_p \sigma(R_C) \quad (2)$$

it helps determine whether each mutual fund's CAL line is tangent to the efficient frontier. Knowing the tangency leads to the finding of the optimal portfolio.

Before continuing the last part, collecting the risk-aversion of the investors should be completed. Then computing the fraction of the complete portfolio allocated to optimal risky portfolio and to T-Bills, and the shares of the complete portfolio invested into each asset and in T-Bills renders the individuals with a better vision of how to split the portfolios between risky and risk-free assets.

4. DATA ANALYSIS

4.1. ADDITIONAL CONSTRAINTS ON THE MARKOWITZ MODEL

There are 5 different Markowitz models in total during the data analysis process, four of them with constraints and one without constraint.

Model 1: Without Constraints

This is a free model without any constraints. The aim is to find what do the general portfolios and the efficient frontier, in particular, look like.

Model 2: Regulation T

This additional optimization constraint is originated from the rules of FINRA. The constraint of Regulation T tends to simulate the condition that broker-dealers allow the consumers to take the position that 50% or more of which are funded by the customer's account equity.

$$\sum_{i=1}^{7} |w_i| \leq 2 \quad (3)$$

Model 3: Arbitrary "Box"

The arbitrary "box" has a pre-assigned weight constraint that maybe provided by future clients.

Here, we simulate the following:

$$|w_i| \leq 1, \forall i \quad (4)$$

Model 4: Zero Tolerance on Short Positions

The U.S. mutual fund industry has a typical restriction: any U.S. open-ended mutual fund should not have short positions. Therefore, this additional constraint related to this restriction is implemented.

$$w_i \geq 0, \forall i \quad (5)$$

Model 5: Effect of Inclusion of the Broad Index

Whether the inclusion of the broad index has a positive or negative effect on the portfolio is taken into consideration as a constraint. It can be concluded as the following equation:

$$w_1 = 0 \quad (6)$$

4.2 RESULT AND ANALYSIS OF THE MARKOWITZ MODEL

Table 1. Results of Markowitz model with no constraint

Markowitz	Minimum Variance Portfolio	Maximum Sharpe Portfolio
SPX	0.383665415	-1.099739477
NVDA	-0.029681386	0.224572759
CSCO	-0.028926698	0.008930463
INTC	0.013271284	-0.08185761
GS	-0.058992434	0.127252454
USB	-0.00304858	0.132126644
TD CN	0.194148224	0.464596813
ALL	-0.114813331	0.078968971
PG	0.259307239	0.534974628
JNJ	0.188331874	0.42716718
CL	0.19673839	0.18300717
Return	7.508%	16.988%
STD	10.953%	16.476%
Sharpe Ratio	0.685444564	1.0310609

Table 1 presents the result of the optimum portfolio without any constraint. The portfolio consists of all ten stocks in total. The largest investment in the minimum variance portfolio is in Proctor & Gamble, and the smallest is in the Allstate Corporation. There are five stocks that have negative weights, suggesting that in the portfolio, half of the securities should be sold in short, and the other half should be bought in corresponding weight. Meanwhile, in the maximum Sharpe portfolio, the largest investment is also in Proctor & Gamble, but the smallest is in Intel Corporation. Only Stock of Intel Corporation should be sold in short. According to figure 3, the stock price of Intel Corporation has fluctuated frequently during the recent four years. The volatility is the culprit for its selling option in the optimum portfolio. The return rate for the minimum variance portfolio is only half of that of the maximum Sharpe portfolio, indicating the latter portfolio is a better choice. The Sharpe ratio, which approximately equals one, is higher in the maximum Sharpe portfolio. The greater Sharpe ratio offers an excess return, comparing to the volatility of the portfolio [10]. Thus it can be taken as another criterion for deciding the best portfolio.

Table 2. Results of Markowitz model with the constraint of regulation T

Markowitz	Minimum Variance Portfolio	Maximum Sharpe Portfolio
SPX	0.38367	-0.4275
NVDA	-0.0297	0.15745
CSCO	-0.0289	-0.0114
INTC	0.01327	-0.0611
GS	-0.059	0.03254
USB	-0.003	0.06484
TD CN	0.19415	0.35285
ALL	-0.1148	0.01059
PG	0.25931	0.45712
JNJ	0.18833	0.3001
CL	0.1967381	0.1245069
Return	7.508%	14.008%
STD	10.953%	13.950%
Sharpe Ratio	0.685444555	1.004177477

Table 2 presents the result of the optimum portfolio with the constraint of regulation T. Both minimum variance portfolio and maximum Sharpe portfolio consist of ten stocks. The constraint of the regulation T does not change the optimum portfolio much, comparing to the previous Markowitz Model with no constraint. The largest and smallest investment in both portfolios stays the same. The maximum portfolio under this constraint has 2 stocks to sell in short, Intel Corporation and Cisco System, Inc, respectively. Due to the constraint, the return rate for the most efficient portfolio is less than the return rate under the free condition. Nevertheless, the maximum Sharpe portfolio still shares the higher return and Sharpe ratio.

Table 3. Results of Markowitz model with the constraint of arbitrary "box"

Markowitz	Minimum Variance Portfolio	Maximum Sharpe Portfolio
SPX	0.383666	-1
NVDA	-0.02968	0.215033
CSCO	-0.02893	0.00309
INTC	0.0013271	-0.08148
GS	-0.05899	0.114582
USB	-0.00305	0.122532
TD CN	0.194149	0.44924
ALL	-0.11481	0.068741
PG	0.259307	0.523281
JNJ	0.188331	0.410176
CL	0.196738	0.174802
Return	7.508%	16.557%
STD	10.953%	16.065%
Sharpe Ratio	0.685444686	1.03064727

Table 3 presents the result of the optimum portfolio with the constraint of an arbitrary "box". Again, the constraint does not affect the two types of portfolios to a great extent. All the ten stocks are included in both portfolios, with the biggest investment in Proctor & Gamble. The smallest in Allstate Corporation for minimum variance portfolio and in Intel Corporation for maximum Sharpe portfolio.

The maximum Sharpe portfolio is chosen due to the distinctly higher return rate and Sharpe ratio.

Table 4. Results of Markowitz model with constraint of zero tolerance of short positions

Markowitz	Minimum Variance Portfolio	Maximum Sharpe Portfolio
SPX	0.094903896	0
NVDA	0	0.109475789
CSCO	0	0
INTC	0	0
GS	0	0
USB	0	0
TD CN	0.198484661	0.237259368
ALL	0	0
PG	0.289065272	0.425597283
JNJ	0.206230919	0.161693465
CL	0.211315252	0.065974096
Return	8.876%	12.057%
STD	11.266%	13.119%
Sharpe Ratio	0.78780906	0.919040963

Table 4 presents the result of the optimum portfolio with the constraint of zero tolerance on short positions. The minimum variance portfolio consists of four stocks, and among the four stocks, Proctor & Gamble has the biggest weight, and the Toronto-Dominion Bank has the smallest weight. The maximum Sharpe portfolio contains five stocks. In this portfolio, the biggest investment is still in Proctor & Gamble, but the least investment is in Colgate-Palmolive Company. The portfolios are less diversified while the constraint of zero tolerance on short positions is fortified, and due to the characteristic of the constraint, every stock in both portfolios has positive weights. The difference in return rate between the two portfolios is smaller than the model with no constraint, but the maximum Sharpe portfolio is slightly better than the other.

Table 5. Results of Markowitz model with the constraint of Inclusion of the Broad Index

Markowitz	Minimum Variance Portfolio	Maximum Sharpe Portfolio
SPX	0	0
NVDA	-0.00971	0.149271
CSCO	0.000842	-0.006852
INTC	0.025124	-0.10145
GS	-0.00993	-0.01303
USB	0.034982	0.024327
TD CN	0.246957	0.306465
ALL	-0.08168	-0.02272
PG	0.28914	0.433103
JNJ	0.25576	0.236122
CL	0.248513	0.056426
Return	8.706%	13.058%
STD	11.177%	13.688%
Sharpe Ratio	0.778929467	0.953967346

Table 5 presents the result of the optimum portfolio with a constraint of inclusion of the broad index. In this Markowitz model with the constraint, both portfolios have all ten stocks. The biggest and smallest investment from both portfolios is the same as a result of the model with no constraint. More of the stocks in the maximum Sharpe portfolio has negative weights, implying a short position. The return rate and Sharpe ratio keep higher in the maximum Sharpe portfolio, guaranteeing the optimal choice.

5. CONCLUSION

Optimal portfolio selection plays an important role in financial investment. Markowitz model, as one of the most traditional tools to analyze given securities, has been applied throughout this study. The study aims to use this model and the other five manipulated constraints to seek the best portfolios for the given ten stocks. Yet, the study has several limitations. From the aspect of data source, the raw stock price that is used in the paper only ranges from 2001 to 2021, 20 years in length. Such insufficient data may cause the result of the optimal portfolio to be less precise. The other deficiency is related to the analysis approach. In this paper, the Markowitz model is the only method that is taken into consideration when determining the optimum portfolio. The calculation of the portfolio is restricted, which may lead to a less efficient final analysis. In future study, data sampled within a longer duration of time can be collected. The analysis should concentrate on more diversified models, like index and constant correlation models, to further compare the result of the optimum portfolio from different models.

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