ATLANTIS PRESS
Advances in Social Science, Education and Humanities Research, volume 610
Proceedings of the 7th Annual International Conference on Social Science and Contemporary Humanity Development (SSCHD 2021)

System Reliability Optimization Model Based on Cost Effectiveness Analysis

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ABSTRACT
System reliability optimization involves the selection of components and system architecture to maximize reliability. The fixed reliability optimization model and fixed cost optimization model are established, the search direction function is improved and simplified, and the algorithm of the optimization model is given. Based on the cost-effectiveness analysis criterion, a reliability optimization model is proposed, and an effective algorithm is given. Combined with an example, the effectiveness and practicability of the method are verified.

Keywords: *Reliability, Optimization, Cost effectiveness analysis, Heuristic method.*

1. INTRODUCTION

The research of reliability problem was developed during the Second World War because of dealing with the unreliability of electronic products. Reliability design was first applied to military, aerospace and other industrial departments, and then gradually extended to civil industry. In many aspects of industry, military and daily life, the performance of system reliability is extremely important for tasks under various conditions. For a series parallel system, redundancy is often adopted for the subsystem to increase the system reliability. Under the constraints of cost, reliability and weight, the design and selection of components has become a combinatorial optimization problem. This problem is how to select components and redundancy to maximize the system reliability under the constraints of cost, reliability and weight.

2. REDUNDANCY OPTIMIZATION MODEL

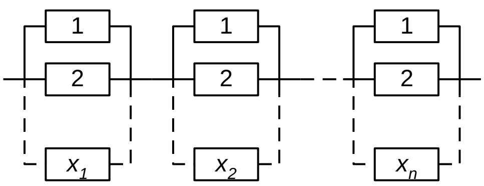

Figure 1 Series-parallel system configuration

Assuming that the system consists of independent subsystems, our goal is to optimize the cost under the condition of reliability. The schematic is in Figure 1. The reliability optimization model of constant reliability is

$$\min C_S = \sum_{i=1}^{n} c_i x_i \qquad s.t. \quad \prod_{i=1}^{n} R_i(x_i) \ge R_0 \tag{1}$$

The reliability optimization mode of constant cost is

$$\max R_S \qquad s.t. \quad \sum_{i=1}^{n} c_i x_i \le C_0 \tag{2}$$

where
C_s = system cost
R_s = system reliability
c_i = unit-price of component i
x_i =the redundancies of component i , $x_i \ge 1$
$R_i(x_i)$ = stage i reliability, $R_i(x_i) = 1-(1-p_i)^{x_i}$
R_0 =lower bound of R_s
p_i = component i reliability.

The above reliability optimization model is a NP-hard problem, and only a few methods are effective.

Therefore, many optimization methods for system redundancy reliability are proposed. In paper [1], they divided the reliability optimization models into series, parallel, series parallel, parallel series, standby and complex structure models. They also divided the optimization methods into integer programming, dynamic programming, linear programming, geometric programming, generalized Lagrange function and heuristic methods. Many algorithms are proposed, but only a few are effective in solving large-scale nonlinear programming, and no method is proved to be more effective. In paper [2], a dynamic programming method is proposed to solve the reliability optimal allocation problem. When the constraints increase, the amount of calculation increases exponentially. In order to overcome this disadvantage, the author introduces the Lagrange multiplier method to reduce the dimension of the problem. In paper [3], the Lagrange multiplier method is used to solve this problem, and compared with dynamic programming, the author turns multiple constraints into a proxy constraint. Paper [4] presents a simple and effective technique for solving integer programming, including system reliability design. In paper [5,6], Lagrange function and stepwise decreasing gradient method are used to solve the nonlinear programming problem of complex systems. They also propose a mixed integer programming method to solve the reliability problem. In paper [7-10], a competitive and stable genetic algorithm is proposed to solve this problem. The author uses the penalty function method to find a feasible solution, approximate solution or optimal solution in the region of feasible solution and infeasible solution.

3. HEURISTIC METHOD

The heuristic algorithm for solving the reliability optimization problem is as follows: From $\chi_1, \chi_2, \cdots \chi_n = (1,1,\ldots 1)$. Then add a same component at a certain position in each step until the system reliability is greater than or equal to the predetermined reliability R_0 .

The heuristic factors $D(j)\, j = 1,2,\ldots,n$ are

$$D(j) = \frac{1}{C_j}[R_s(\chi_1, \chi_2, \ldots, \chi_j + 1, \ldots, \chi_n)$$
$$-R_s(\chi_1, \chi_2, \ldots, \chi_j, \ldots, \chi_n)] \qquad (3)$$

Since the calculation of formula (3) of search direction function is complex, the following formula can be used.

$$D(i) = \frac{1}{C_i}[\ln R_s(\chi_1, \chi_2, \ldots, \chi_{i-1}, \chi_i + 1, \chi_{i+1}, \ldots, \chi_n)$$
$$-\ln R_s(\chi_1, \chi_2, \ldots, \chi_{i-1}, \chi_i, \chi_{i+1}, \ldots, \chi_n)] \qquad (4)$$

The search direction function (4) is simplified as follows:

$$D(i) = \frac{1}{C_i}[\ln R_s(\chi_1, \chi_2, \ldots, \chi_{i-1}, \chi_i + 1, \chi_{i+1}, \ldots, \chi_n)$$
$$-\ln R_s(\chi_1, \chi_2, \ldots, \chi_{i-1}, \chi_i, \chi_{i+1}, \ldots, \chi_n)]$$
$$= \frac{1}{C_i}[\ln R_i(\chi_{i+1}) - [\ln R_i(\chi_1)]$$
$$= \frac{1}{C_i}\ln[\frac{R_i(\chi_{i+1})}{R_i(\chi_1)}]$$
$$= \frac{1}{C_i}\cdot\ln\frac{1-(1-P_I)^{\chi_i+1}}{1-(1-P_I)^{\chi_i}} \qquad (5)$$

For the optimization model (1) with constant reliability, the specific algorithm using a heuristic method is as follows:

Step 1: Let $\chi_1, \chi_2, \cdots \chi_n = (1,1,\ldots 1)$ and calculate R_S .

Step 2: Calculate the search direction function according to formula (5).

Step 3: If the largest one is $D(i^*) = \max_i[D(i)]$, then replace x_{i*} by $x_{i*} + 1$.

Step 4: If $R_s = \prod_{i=1}^{n}[1-(1-p_i)^{x_i}] \geq R_0$, return. Otherwise go back step 2.

The algorithm for solving the cost reliability optimization model (2) is similar to the above algorithm.

4. COST-EFFECTIVENESS ANALYSIS OF RELIABILITY

For the fixed reliability optimization model and the fixed cost optimization model, the predetermined reliability and given funds are generally given by experience without scientific basis. According to the cost-effectiveness analysis criteria, the reliability optimization model can be established with the ratio of reliability and cost as the objective function:

$$\max R_S / C_S$$
$$s.t.\quad C_S = \sum_{i=1}^{n} c_i x_i \tag{6}$$
$$R_S = \prod_{i=1}^{n} [1-(1-p_i)^{x_i}]$$

For Programming (6), (R_S^*, C_S^*) is the best solution. However, R_S^* may be relatively small, or C_S^* may be too large. In this case, the result is not satisfactory. Therefore, reliability or cost factors can also be considered in the model. The model is as follows. If only reliability is considered, the model is as follows:

$$\max R_S / C_S$$
$$s.t.\quad C_S = \sum_{i=1}^{n} c_i x_i \tag{7}$$
$$R_S = \prod_{i=1}^{n} [1-(1-p_i)^{x_i}]$$
$$R_S \geq R_0$$

The solution is as follows:

Step 1: Let $x_1, x_2, \ldots x_n = (1, 1, \ldots 1)$ and calculate R_S.

Step 2: Calculate the search direction function according to formula (5).

Step 3: If the largest one is $D(i^*) = \max_i [D(i)]$, then replace x_{i*} by $x_{i*}+1$. Calculate R_S and R_S / C_S.

Step 4: If $R_S \geq R_0$ go to step 5, otherwise go step 2.

Step 5: If R_S / C_S is decreasing, stop. Otherwise go to step 2.

5. COMPARISON OF THE EXAMPLES

Consider a system that is composed of 5 components. The reliability of components are $p_1 = 0.96$, $p_2 = 0.93$, $p_3 = 0.85$, $p_4 = 0.80$, $p_5 = 0.75$, $c_1 = 3$, $c_2 = 12$, $c_3 = 8$, $c_4 = 5$, $c_5 = 10$, $R_0 = 0.9$. The results are as shown in Figure 2 and Figure 3 with different R_0.

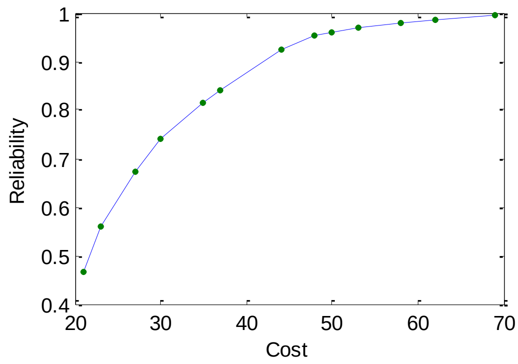

Figure 2 Relationship between cost and reliability

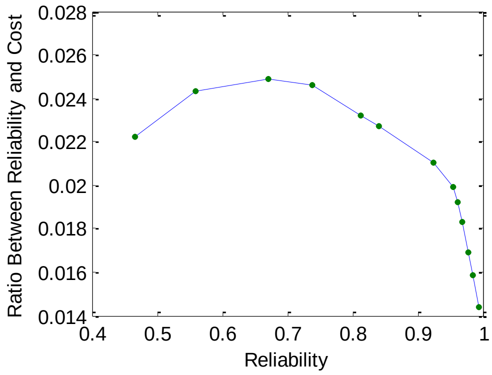

Figure 3 Relationship between reliability cost ratio and reliability

From figure 2, with the increase of cost, the reliability tends to saturation. The relationship between reliability cost ratio and reliability is shown in figure 3. From figure 3, when the reliability $R_S = 0.6718$, the ratio of reliability and cost is maximum, but the reliability 0.6718 is not high, so the ratio of reliability and cost is acted as objective function. When $R_0 = 0.95$, the redundant number are 2, 2, 3, 2, and 3, $C_S = 48$ and $R_S = 0.9548$.

6. CONCLUSION

Redundancy optimization is an important means to ensure high mission reliability of the system. In this paper, the fixed reliability optimization model and the fixed cost optimization model are established, and the optimization model of cost-effectiveness analysis is proposed, which provides a new method for reliability optimization decision-making, effectively solves the complex problem of reliability optimization, and provides an effective way for system reliability optimization decision-making. In this paper, the reliability optimization problem with only one constraint is considered, and the optimization problem with multiple constraints can be treated similarly.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grant and the National Natural Science Foundation of China under Grant 62176107.

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