*Advances in Social Science, Education and Humanities Research, volume 610*
**Proceedings of the 7th Annual International Conference on Social Science and Contemporary Humanity Development (SSCHD 2021)**

# Characteristics of the Mathematics Textbook Language ——Takes the PEP Version Primary School Mathematics Textbook as an Example

Zezhi Zheng[1] and Bowen Song[1, 2,*]

[1]*College of Humanities, Xiamen University, Xiamen, Fujian 361000, China*
[2]*Institute of international college, Qinghai Minzu University, Xining, Qinghai 810007, China*
[*]*Corresponding author. Email: chenxing0219@126.com*

**ABSTRACT**
Due to it being a specialized language, which has both scientific and universal applications, the language of mathematics has received general attention within the field of linguistics and within educational circles. The language used within math textbooks is at the "forefront" of learning the language of mathematics. Paying close attention to the language used within math textbooks will not only help to improve the construction of said textbook sand lead to a greater level of standardization of textbook language, but it will also lead to the greater expansion of the breadth and depth of linguistic research regarding this topic. Mathematical language and the language used within math textbooks are two different concepts, establishing a clear delineation between the connotation and category of these two distinct languages is helpful when conducting in-depth research. In accordance with the idea of Synergistic linguistics, this paper believes that the language of math textbooks is a self-organized and self-regulated dynamic system. At the macro level, it presents the dynamic characteristics from the concrete to the abstract. And at the micro level, both specialized descriptive language and life-oriented language coexist, linguistic precision and colloquial language coexist, unified expression and ideation are also able to coexist. The systematic research into mathematics textbook language is great of great importance when discussing the improvement of the scientific and theoretical nature of textbook research.

***Keywords****: Mathematical language, Language of mathematical textbooks, Dynamic system*

## 1. INTRODUCTION

Bloomfield (1933) [1] once argued that: "Mathematics is just the highest level that language can achieve." This reflects that the mathematical language is"precision, rigorous and concise" Yuming Li (1986) [2] In addition to the aforementioned, it is also highly abstraction. This is because mathematical language has a strongly defined purpose. As Mr. Yuming Li once said: “Language has two aspects by which it conveys feeling and ideas, and mathematical language focuses on meaning." By studying the problems encountered by college students when answering mathematical questions, Godino, Batanero, Roa put forward the structural model of mathematical knowledge, They divided mathematical knowledge into 6 distinct categories:language, situation, method, concept, nature and argument, They believes that mathematical language is the core of mathematical knowledge and is a determining factor affecting mathematics learning. [3]This reminds us that whether students can master mathematical language directly affects whether they will be able to learn mathematics well, and mathematical textbooks are at the "forefront" of the process of mastering mathematical language.since the 1980’s,Mr. Yuming Li has paid close attention to the problems with the mathematics language used within math textbooks.He has analyzed the differences between words, sentences, grammar and natural language from the perspective of linguistics, and put forth several examples of language problems that exist in the textbooks. Some problems have since been explored and addressed in the new version of the textbooks. For example: Yuming Li (1986)[2] once proposed that: " The placement of the',' in 10,000 does not match the Chinese habit. In the traditional Chinese counting method, the phrase 'ten thousand'is not used, so it is unnatural to use that phrasing in Chinese, the same is

 

ture for the mathematical figure 1,000,000. Of course, this way of writing is very convenient for the Western language ". The language of today's primary school textbook has changed, and in the textbook" Do you know "popular science column explained:" we oftentimes see the figure"10,000" being used. This is due to the method of three-digit writing system utilized by countries whose primary language is English (e.g. UK, USA, etc)."With this, we can see that the study of the textbook language can constantly promote the development of the textbook itself and improve its scientific nature.

This paper mainly discusses of synergetic linguistics, that mathematical language used within textbook is a self-organized and self-regulated dynamic system, which readjusts and balances between natural language and artificial language. Within the theory of Synergetic Linguistics, the phrase "self-organization" refers to rule that, in the absence of external instructions; its internal subsystems can automatically form a certain structure or function according to certain rules, which are intrinsic and self-regulatory." [4]The language of the mathematics textbook is an open system. Because it has the function of teaching knowledge and transmitting ideas, its internal language system must maintain close contact with its external society and environment. This allows its language to be in line with the characteristics of the times and to match the knowledge level of different grades.

## 2. MATHEMATICAL LANGUAGE AND THE LANGUAGE OF MATHEMATICAL TEXTBOOKS

First of all, we should make clear the distinction between two concepts: mathematical language and the language of mathematical textbooks. Mr. Yuming Li defined mathematical language as "A mixture of natural and artificial language. A large number of formulas, symbols, and diagrams in mathematics are artificial language. However, the language used in title, narration, application questions, practice requirements, and the interpretation of symbols and arithmetic belong to natural language."[5]Based on this idea, this paper divides mathematical language into narrow mathematical language and general mathematical language. The narrow mathematical language refers only to the artificial language sub-set proposed above, including symbols, words, and graphs.It is a highly abstract system of language, where esoteric logical relationships can only be expressed with the use of letters and symbols, Bloomfield considers this to be the highest level of language. Generalized mathematical language contains not only the artificial language of mathematics, but also the natural language used within mathematics. This natural language not only contains the written language used in explanation of artificial language, but also the spoken language (e.g. the words spoken by a math teacher in class), and the mathematical language used in day-to-day life. It is also apparent that the language utilized by mathematical textbooks also belongs to the generalized mathematical language system. Xinchun Su (2007) proposed that the textbook language is divided into object language and narrative language. [6]Zezhi Zheng(2017) elaborated that the object language used in mathematical textbooks is the expression form of mathematical knowledge, including terms, formulas, calculation processes, problem solving steps, auxiliary graphics, etc. These generally do not leave room for ambiguity when it comes to comprehension; this is due to it being a part of artificial language. Narrative language is the explanatory language used to translate and interpret the language of the teaching objective, such as the narrative of applied questions in mathematics textbooks and the language description of teaching points.[7]This paper continues to refine the narrative language in mathematics textbooks into natural narrative language, dedicated narrative language, and functional textbook language. Natural narrative language refers to the non-fixed structural language used within explanation of the of a word problem, for example: "Calculate Xiao Li's two months' telephone fee and internet fee in the following table, and fill in the table completely." "Xiao Lin bought the following two books, calculate how much he spent on those two books."; The specialized narrative language is a semi-fixed structure language used to explaining the object language, which has been fixed in mathematics textbooks, for example,"every interval……", etc. Textbook functional language refers to the language of the object, which does not play an explanatory role in the object language, but instead carries the textbook teaching function and transmits the meaning of concept being taught, for example: "Family planning is a fundamental national policy in China." "The average water depth is 1.1 meters, and it is strictly prohibited to swim in the river." The systematic classification of the mathematical language is shown in Figure 1.

As a dynamic system which is self-organized and self-regulating, the language of mathematical textbooks constantly balances the two parts of "abstract" and "understanding", along with "knowledge" and "values".

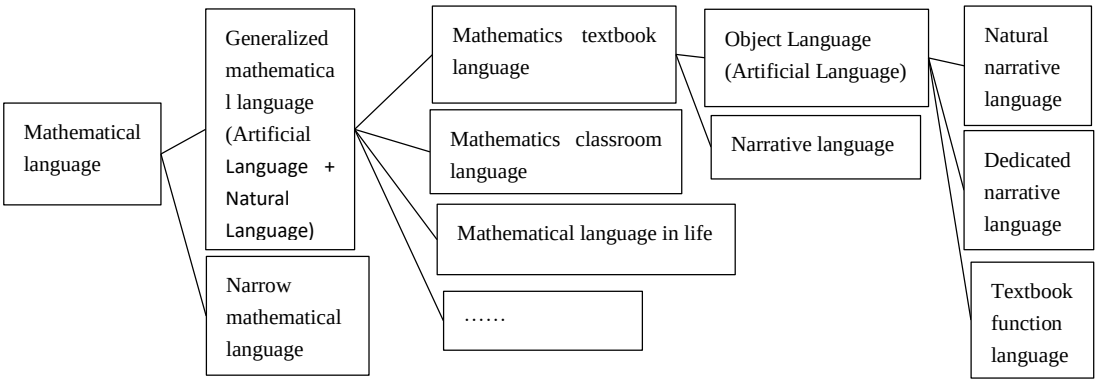


**Figure 1** Classification of mathematical language

### *2.1. "Abstraction" and "Understanding" in Mathematical Textbook Language*

Language is a tool which is able to relay information, and both the messenger and receiver of the information want the information transmitted in the most effort-saving way. The messenger wants to be able to use the simplest language to be able convey the most meaning; while the receiver wants to be able accurately understand the message in the easiest way possible. Striking this delicate balance can be summed up by"the principle of least effort" which was proposed by Zipf G K[8] This principle is particularly obvious in the language used by mathematical textbooks, for example, the language used in primary school mathematical textbooks is especially simple. This is because students just entered into the use of this specialized language system, and thus do not have a strong "decoding ability". And so a large amount of natural language is used to aid in interpretation, for example, the abstract language system is used in real life situations and in practical ways. As they continue on to higher grades, students' "decoding ability" gradually improves, and the language used in mathematical textbooks does not have to rely on as much added contextual language. Many rules which have symbolic meaning are introduced, at the same time, situational natural language starts to gradually decrease and abstract symbols and concepts gradually increase. At this stage, the characteristics of mathematical language has become more obvious and understood by the student. This process presents a dynamic system, where the textbook language constantly seeks the balance between students' abilities and the concepts which need to be taught.

### *2.2. "Knowledge" and "Values" in the Mathematics Textbook Language*

Knowledge is the most important function which is embodied by the textbook. If the knowledge of Chinese language textbooks is reflected in the requirement of students to understand language, master language and use language, then the knowledge of mathematics textbooks is more direct, reflected in their terms, formulas and internal logic, that is to say, the main knowledge of mathematics textbooks is to learn its artificial language part. The goal of Chinese textbooks can be summed up as the transmission of values, position and emotion, while mathematics textbooks seem objective and neutral. However, the textbook itself is the concentrated embodiment of national social values and national will, and mathematics textbooks naturally cannot avoid this part. For example, the second volume of the fourth grade textbook example title: "Family planning is a basic national policy in China. The national annual birth population in 1997 ~ 2003 is the figure below." In the process of conveying knowledge, mathematics textbooks also communicate national policies and national consciousness. These contextual languages are unnecessary and redundant for a narrow mathematical language, but are essential for a textbook language, so it also seeks a balance between "knowledge" (artificial language) and "values" (natural language).

## 3. THE NARROW MATHEMATICS LANGUAGE AND MATHEMATICS TEXTBOOK LANGUAGE CHARACTERISTICS

### *3.1. The Linguistic Features of Mathematics in Narrow Sense*

American mathematicians Lex and Grotte think: " Mathematics uses special non-spoken languages: some new symbols are defined, and some old characters are redefined and restricted or change their meaning. This fine, epitaxial language rarely connects to out-classroom life." [9]It can be seen that the narrow mathematical language is completely different from the natural language. First of all, the narrow mathematical language has the characteristics of symbolization, which also brings its simple and precise characteristics. Each concept, term, symbol needs to have the exact denotation, and not the ambiguous fuzzy connotation.

Although the narrow mathematical language system presents the special characteristics different from the natural language, it is still subordinate to that language

system and shows the operation law of that language system. Arbitrary and linear are two major features of language. Also in mathematical language systems the relationship between symbols is arbitrary. For example, German mathematician Michael Stifel (1544)used one or a pair of brackets indicate division sign: 8) 24 indicate 24 ÷ 8; J. Mahon (1701) by D) A +B-C indicate (A+B-C) ÷D; Michael Stifel (1545) by upper case German letter D indicate Division sign; Swiss J.H. Ryan (1659) use ÷ in an algebraic book.It can be seen that although many mathematical symbols are now fixed and become the universal mathematical language around the world, they do not" escape " language arbitrariness.

The mathematical language has the form of linear features. Saussure, the father of the linguistics, pointed out when discussing the nature of linguistic symbols: " The ability of symbols has linear characteristics.so it has two characteristics: 1. it occupies a certain space.2. It can be measured in 1 dimension."[11] It can be seen that Saussure emphasizes the linearity only for the signifier category. However, mathematical symbols have their special meaning and operational rules. Although their surface is linear, the internal structure is sometimes non-linear. For example, formula a+b×c, needs to calculate the following b multiplied by c before calculating the previous addition. Another example: when calculating a× (b+c), you need to calculate the addition inside parentheses, then the multiplication outside parentheses, or use the phrase multiplied by b plus c respectively. These formula surface forms and sound chain are linear sequences, while the internal structure is hierarchical, it's symbolic meaning changes its original one-dimensional structure to two-dimensional structure, so there are nonlinear characteristics in the linear sequence relationship on the surface of mathematical language. The key to understanding mathematical language is to understand the nonlinear relationships contained under surface linear sequences. This also fully shows the trend of the narrow mathematical language surface layer being extremely concise and infinitely complex inside.

## *3.2. Characteristics of the Mathematics Textbook Language*

In the mathematics textbook, there are a lot of definitions and principles, which are concise, non-ambiguous and abstract. For example: "ray", "flat angle", "associative law of addition", etc. The textbook describes and explains the propositions that require explanation in mathematics, and finally end in the abstract as the final term. It is a process from concrete to abstraction. The definition narrative of elementary school mathematics textbooks also follows this process, for example, " a ray rotates around its endpoint for half a week, and the angle is called a flat angle." "Add the previous two numbers or the latter two numbers, and unchanged, which is called the associative law of addition." The definition in the textbook basically adopts this way of first specific description and then giving the definition and principle. This process from concrete to abstract is also suitable for students learning psychology at this stage. The presentation system of the mathematics textbook language is shown in Figure 2:

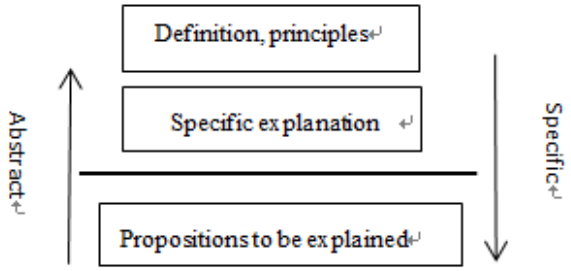


**Figure 2** The presentation system of the mathematics textbook language

The definition, principles are the artificial language part in mathematics textbooks, the specific description and interpretation are the natural language part, and the proposition to be explained is the objective existence. This constitutes the most basic content system of the mathematics textbook language. Under this macro-scale system, the mathematics textbook language has its specific characteristics.

### *3.2.1. Both Specialized Descriptive Language and Life-oriented Language Coexist*

There is much semi-fixed special narrative language used within mathematics textbooks. Lu Zhou (2019) specially studied the special narrative language format of mathematics, that the special narrative language is: "stable structure, relatively fixed collocation and certain use context, and semi-fixed and fixed structure in knowledge, such as every xyz(every unit length), in-intercept (Y on X)"[12] In addition to this fixed and semi-fixed special narrative language, the textbook also utilizes every day language which relates to general and scientific knowledge, such as"The Blue Whale is the largest animal in the world.""In daily life and production, we often use numbers larger than ten thousand"These languages are not special narrative languages, but only provide some background knowledge, but they meet the needs of primary school textbooks and are in line with the psychological expectations of students' knowledge acceptance.

### *3.2.2. Linguistic Precision and Colloquial Language Coexist*

Its exactness as manifested in the sentence "An angular bisector is the axis of the Angle symmetry"appears to create no problems. In fact, this is a false proposition in mathematics, because the axis of symmetry is a straight line, and the angular bisector may

be rays. In order to express no ambiguity, the true proposition is "the straight line where the angular bisector is the symmetry axis of the angle." It can be seen that the mathematical language must be very rigorous, and all the possibilities should be considered precisely, and with no exceptions. But at the same time, there are also very colloquial language in the textbook, for example, "我出一组算式(I give a set of calculations)"in the last volume of the fourth education edition, with "出(give)" and "算式(calculations)" collocation, the actual expression should be"出算式题(calculation question)", but the "question" is omitted in the math textbook language . In the third volume of grade: "观察黑板面和国旗的表面，说一说哪一个面比较大。黑板面比国旗面大。(Observe the surface of the blackboard and the surface of the national flag, say which one is bigger. The blackboard is bigger than the national flag.)" "国旗的表面" in its question was changed to the "国旗面"when it was answered, which is obviously inconsistent. The use of the "的" in the textbook is also more casual. From this point of view, the general usage of mathematics textbook language has rigor that is not very scientific. The artificial language part of the textbook is very exact. And the natural language part, especially the primary school mathematics textbook language, is purposefully closer to the pupil language level, close to life, so far as it does not cause misunderstandings and ambiguity to exist.

### *3.2.3. Unified Expression and Idea Will Are Coexist*

Wenhao Chen (2018) once mentioned: The Italian astronomer Galileo Galilei said: Mathematics is the words God uses to write the universe. Because of this concept, American astronomer and popular science writer Carl Sagan is convinced that no matter how different the technological civilizations in the universe are, there is a common language—the language of mathematics. Haizhong Zhou, a Chinese mathematician and linguist, also believes that mathematical language has the advantages of clarity, single sense, compactness, universality, abstraction and logic, and is an ideal tool for interstellar communication. Therefore, mathematical language became the first choice of universal language for mankind. [13]Thus shows the artificial language of mathematical language is universal in the world and unified. However, behind the stable form of artificial language still stands different languages and cultures. Russian language scholar Privarova believes: "National cultural language consciousness is synergistic, language and culture as two forms of social consciousness, On the one hand, it has the nature of synergy, and on the other hand it has the parameters of interaction."[13]This kind of synergy also exists in the mathematical artificial language and the culture where it uses its country. Different countries, ethnic groups, are faced the same mathematical artificial language and can still understand the connotation behind it, but the linguistic expression of their thinking shows their unique characteristics. In the case of native Chinese and native English speakers, the connotation expressed by 5% is the same, and there is no difference in the way of writing with Arabic numerals and percent signs, but the linguistic thinking expressed is different. For example, in writing the same address, Chinese habits are from "big" to "small", and English habits are from "small" to "big" writing order. Chinese expressions account for five out of one hundred, and the order from total to score is called "百分之五(every 100 have 5)". This also causes the phenomenon that the number writing system of Chinese mathematics language does not match the Chinese character writing system. The native English speakers focus on "small" to "big", called five percent (5 in every 100). Therefore, no matter natural language or artificial language, no matter generalized mathematical language or narrow mathematical language, behind the same surface reflects the spirit and will of different countries and nations.

## 4. CONCLUSION

This paper introduces the Synergetic linguistics idea that mathematics textbook language is a self-organized, self-regulatory dynamic system. Throughout the whole system, it shows that abstract symbols are gradually increasing, the density of terminology is constantly increasing, and the characteristics of mathematical language are gradually becoming more obvious. When observing its interior, its subsystems can automatically form a certain structure or function according to the rules from concrete to abstract, from objective reality to its specific description and interpretation, and finally to form definitions and principles. Through the discussion in the article, we also find that the mathematical language of the mathematics textbook is not the same concept, and the two cannot be mixed into the concepts and characteristics. In fact, the mathematics textbook language is an open non-balanced system that maintains close contact with the external environment and carries the functions of the teaching materials. Only by clarifying the overall system, state and characteristics of the mathematics textbook language can we better study the textbook language, improve the construction of the teaching materials, and thus improve the quality of the subject being taught.

## ACKNOWLEDGMENTS

This Project was supported by basic scientific research business in central universities (2072021013); Fujian Provincial Social Science Project (FJ2019B158.

## REFERENCES

[1] Zhiwei Feng, Fengguo Hu, Mathematical linguistics, The Commercial Press, 2012.

[2] Yuming Li, Overall characteristics of the mathematical language. [J], Chinese Teaching & Studies (10). 1986.

[3] Juan D. Godino, et al."An Onto-Semiotic Analysis of Combinatorial Problems and the Solving Processes by University Students." Educational Studies in Mathematics 60.1(2005): doi: 10.1007/s10649-005-5893-3.

[4] Lifen Liu, New development of foreign linguistics: Synergetic linguistics [N].Chinese Journal of Social Sciences, 2016-08-22 (007).

[5] Yuming Li. Mathematical Language and Mathematical Language Education [M].Language learning and language education. Beijing: Beijing Broadcasting Academy Press, 2003.

[6] Xinchun Su, Jingjing Du, Junhong Guan, Shuhua Zheng. The nature, characteristics and research significance of the textbook language [J]. Applied Linguistics (04). 2007.

[7] Zezhi Zheng. Exploration on the Language Research of Subject Textbooks——A Case Study of Math Textbook Language Research [J]. Journal of Beihua University (Social Sciences)(06) .2017.

[8] Zipf G K. Human behavior and the principle of least effort: An introduction to human ecology Cambridge: Addison-Wes-ley.1949.

[9] Jiansheng Bao. Teaching of Mathematical Language [J] Mathematics Bulletin, (10) .1992.

[10] Liansheng Piao. The origin of the " +-×÷=". People's Education.07 (1983).

[11] Saussure, F.de.1916.Coure in General Linguistics. New Your: McGraw-Hill, 1966.

[12] Lu Zhou. Study of Language of Mathematics Textbook in Basic Education. PhD dissertation [D]. Xiamen University, 2019.

[13] WH Chen. What language to communicate with aliens [N]. Science Journal of China, 2018-07-13 (003).