

# Implementation of Maxwell's Equation Using Z Transforms Method to Analyze Soliton Waves on Tin Oxide

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## ABSTRACT

This research aims to model electromagnetic waves propagation in SnO<sub>2</sub> materials related to their dispersive and nonlinear properties. The FDTD method was used to describe electromagnetic waves from Maxwell's equations. The signal that formed electromagnetic waves was analyzed using time, position, and frequency domains. This electromagnetic wave on the frequency function applied the Z transforms method. Z transforms is a method that converts a sequence into a series. The research method used the basic principles of numerical approximation on differential equations by considering boundary conditions, stability, and absorption boundary conditions. The simulation results showed that the permittivity changes affected the dispersive and nonlinear properties of SnO<sub>2</sub> material. On higher amplitude of frequency, SnO<sub>2</sub> material can generate soliton waves. Furthermore, the simulation can help select nonlinear materials and identify physical phenomena related to electromagnetic waves on the material. The results of the analysis can be beneficial for many optical applications, especially for optical telecommunication development.

**Keywords:** Nonlinear, FDTD, Tin oxide, Fiber optic.

## 1. INTRODUCTION

The electromagnetic theory explains the relationship between electric and magnetic fields, which causes the propagation of electromagnetic waves by James Clark Maxwell. According to Maxwell, a changing electric field will generate a magnetic field, and meanwhile, Faraday argued that a changing magnetic field creates an electric field. Thus, the relationship between the electric field and the magnetic field can generate electromagnetic waves. In other words, electromagnetic waves are mixed waves between electric fields and magnetic fields caused by accelerated moving electric charges. However, electromagnetic waves have a waveform that is difficult to visualize. The Finite Difference Time Domain (FDTD) method can be one of the solutions. In 1966, Kane Yee introduced FDTD to analyze electromagnetic fields [1]. The simulation in FDTD uses the basic principles of numerical approximation on differential equations in the space and time domains.

Previous studies used the FDTD method to formulate Maxwell's equations numerically in a time-dependent scattering process [2]. The FDTD method was further developed in computation forms by Taflove [3]. Other studies related to FDTD mainly had been used on regional hyperthermia [4]. Meanwhile, the effect of absorption of dispersive media on the scattering process with femtosecond scale pulses also had been developed by R.M. Joseph [5]. Further research was on the effect of electromagnetic waves on biological materials [6].

Meantime, tin oxide (SnO<sub>2</sub>) material also has the potential to be studied further because it has many advantages, such as high refractive index (2.006-2.486), high transparency (43.8-75% in the visible light region), high conductivity (~ 10<sup>3</sup> cm<sup>-1</sup>), and a wide bandgap (≥ 3.6eV) [7, 8, 9]. The optical nonlinearity of SnO<sub>2</sub> had been demonstrated in a previous study by Yu through experiments [10]. Various studies obtained the dispersion properties of SnO<sub>2</sub> materials in synthesizing

SnO<sub>2</sub> thin films [11, 12]. Based on these two characteristics, SnO<sub>2</sub> has the potential to be used in optoelectronic applications. However, these two properties were obtained by complex research and needed high-quality devices. Therefore, this study proposed a faster, precise, and more straightforward simulation method to analyze the energy distribution of the material's electromagnetic waves based on the dispersive and nonlinear characteristics of the SnO<sub>2</sub> material.

Light is an electromagnetic wave, so light can be simulated using FDTD. Light is a wave at a very high frequency in the terahertz (THz) range, or 10<sup>12</sup> Hz. At optical frequencies, materials can have both dispersive and nonlinear properties. The dispersion property implies that the dielectric properties are frequency dependent.

Pulses in the time domain consist of frequencies with a specific range, in which the different frequencies can see other dielectric properties [13, 14]. In theory, the light pulses in a fibre optic cable, for example, will slowly widen due to the dispersive nature of the optical fibre. Under certain conditions, it has been proven that the nonlinearity of material can lead to the formation of solitons [15]. One-dimensional nonlinear optical pulse simulation can show soliton propagation. Using the Z transformation, the formulation of dispersive and nonlinear properties is more efficient [16]. Maxwell's normalized equation can be written as follows

$$\frac{\partial D_x}{\partial t} = \frac{\partial H_y}{\partial z} \quad (1a)$$

$$D_x = \epsilon_\infty E_x + P_L + P_{NL} \quad (1b)$$

$$\frac{\partial H_y}{\partial t} = \frac{\partial E_x}{\partial z} \quad (1c)$$

Equation (1b) contains two additional terms, linear polarization (P<sub>L</sub>) and nonlinear polarization (P<sub>NL</sub>). In the frequency domain, the linear polarization is given by

$$P_L(\omega) = \frac{(\epsilon_x - \epsilon_\infty)}{1 + 2\delta_n \left( \frac{j\omega}{\omega_L} - \left( \frac{\omega}{\omega_L} \right)^2 \right)} E(\omega) \quad (2)$$

Using z transform, equation (2) can be written

$$P_L(\omega) = \frac{\gamma_L \beta_L}{(\alpha_L^2 + \beta_L^2) + j\omega 2\alpha_L - \omega^2} E(\omega) \quad (3)$$

According to equation (3) in time domain, the linear polarization is calculated by

$$P_L^n = S_L^{n-1} \quad (4a)$$

$$S_L^n = c_1 \cdot S_L^{n-2} + c_3 E^n \quad (4b)$$

As for the non-linear polarization is divided into two parts, namely

$$P_{NL} = P_K + P_R \quad (5)$$

The PK part of equation (5) is known as the Kerr effect, given in the time domain by

$$P_K(t) = \chi^{(3)} \alpha E^3(t) \quad (6)$$

The PR part of equation (5) is known as Raman scattering, which in the time domain is given by

$$P_R(t) = \chi^{(3)} (1 - \alpha) E(t) \cdot \int_0^t g_R(t - \tau) E^2(\tau) d\tau \quad (7)$$

## 2. METHODS

### 2.1. Program design and research variables setting

The modelling was designed by applying the FDTD method in the Visual Basic 6.0 software program to analyze the optical properties of SnO<sub>2</sub> material. Variables of the study contain manipulation, response, and control variables. The manipulation variables consist of space and time domain, while the response variables consist of dispersion and nonlinearity properties. The last variable, the control variable, is a nonlinearity constant

### 2.2. Data Collection Techniques

The initial stage was collecting data by literature study. Hereafter, an FDTD software program was created to analyze EM wave propagation related to the dispersive and nonlinear properties of SnO<sub>2</sub> materials. At this stage, some quantities were measured, such as pulse propagation time, dielectric properties of the material, and some constants of SnO<sub>2</sub> materials.

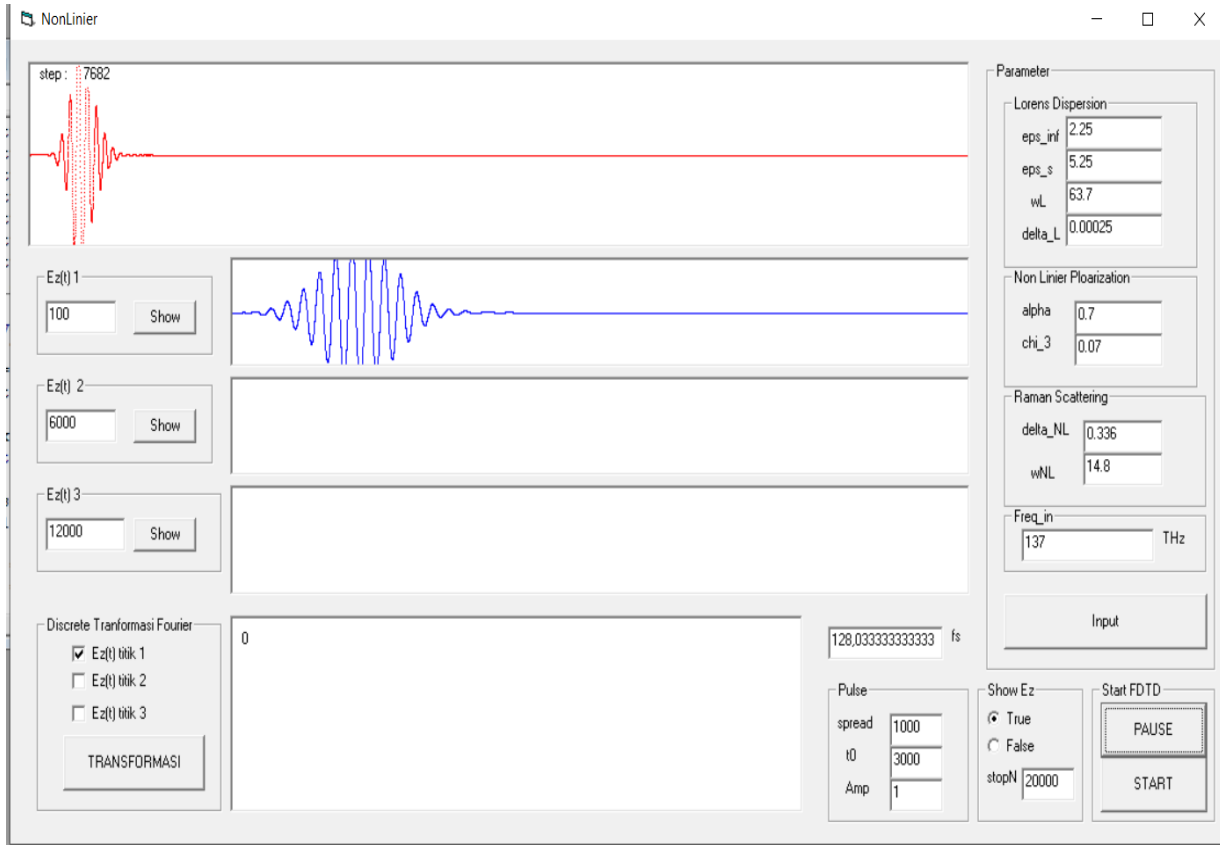


Figure 1 Nonlinear FDTD program display.

### 3. RESULTS AND DISCUSSION

#### 3.1 Wave propagation at amplitude of 0.7 V/m

Figure 2 (a) illustrates the propagation of electromagnetic waves on an SnO<sub>2</sub> material, forming a wave packet at an amplitude of 0.7 V/m. The wave pulse forms the spatial-dependent electric field  $E_z(x)$  on the  $N_{step}$  7271 at an interval of 120 fs. While Figure 2 (b) shows the pulse propagation on the  $N_{step}$  2536 at an interval of 422 fs. The simulation results show that the pulse wave spreads or be dispersed due to the relatively low and insufficient applied voltage amplitude to generate the nonlinearity of the material.

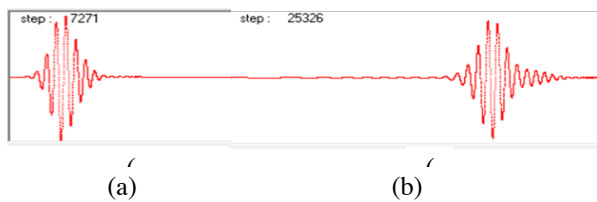


Figure 2 Simulation results of wave pulses (a) on  $N_{step}$  7271 (b) on  $N_{step}$  2536.

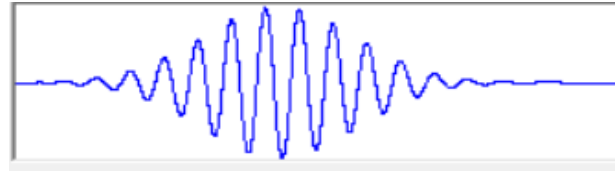


Figure 3 Result of plot  $E_z(t)$  when  $t=100$  fs.

Figure 3 describes the time-dependent propagation of electric field pulses at 100 fs. Based on the simulation results, an electric pulse of 0.7 V/m is dispersed due to the dependence on the material permittivity.

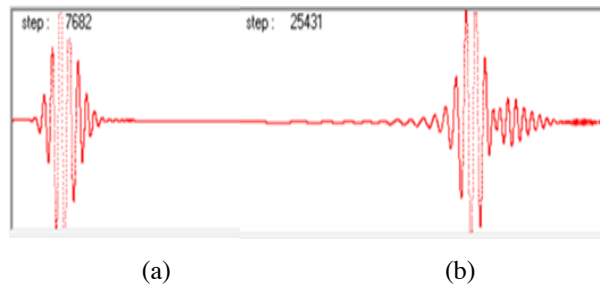
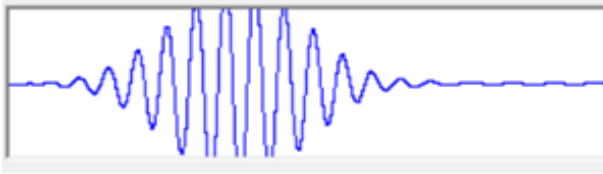


Figure 4 The simulation results of wave pulses (a) on  $N_{step}$  7682 (b) on  $N_{step}$  25431.



**Figure 5** Result of plot  $E_z(t)$  when  $t=100$  fs.

Figure 5 describes the time-dependent propagation of electric field pulses at 100 fs. The simulation results show that the electric pulse of 1 V/m is not dispersed or form a soliton wave due to the nonlinearity of the material. Thus, the voltage value can be obtained when the wave remains dispersed or when the nonlinearity effect has been generated and causes a soliton wave. The information can be helpful for optical communications development.

### 3.2 Wave propagation at an amplitude of 1 V/m

Figure 4 (a) illustrates a wave pulse when the higher voltage amplitude is 1 V/m. The wave pulse forms spatial-dependent of electric field  $E_z(x)$  on  $N_{\text{step}}$  7682 at an interval of 129 fs. Meanwhile, Figure 4 (b) shows the pulse propagation on  $N_{\text{step}}$  25431 at an interval of 423 fs. The simulation results show that the two pulses are relatively constant. In other words, the wave pulse is not dispersed or forms a soliton wave due to the nonlinearity of the material that begins to be generated and balances the dispersion properties of the  $\text{SnO}_2$  material.

## 4. CONCLUSION

The study was able to simulate the propagation of wave pulses due to the dispersion and nonlinear properties of  $\text{SnO}_2$  material. Both of these properties resulted from the material permittivity changes when voltage pulse was applied. Pulse signal in  $\text{SnO}_2$  material indicated dispersion effect at lower amplitudes and formed soliton when increasing the amplitude. In the future, this simulation can select nonlinear materials and identifying physical phenomena related to electromagnetic waves in the medium. Researchers can get a voltage value where the signal in the material is dispersed or forms soliton waves due to its nonlinear properties begins to be generated. This information is beneficial for various optical applications, especially for optical telecommunications development.

## AUTHORS' CONTRIBUTIONS

Asnawi: conceptualization, method and preparation of the manuscript; Madlazim: Review and editing of manuscripts; T Prastowo: validation and analysis of manuscripts; M Anggaryani: proofreading; R A Firdaus: FDTD programmer and M Khoiro: data visualization and editing.

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