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Students' Mathematical Modeling on PISA Quantity Problems of Formulation Category: Explicit Model Vs Implicit Model

Abdul H. Rosyidi[1,*] Laili F. Fadhilah[2] Sugi Hartono[3]

[1,2]*Departement of Mathematics, Universitas Negeri Surabaya, Indonesia*
[3]*Departement of Mathematics, University of Surabaya, Indonesia*
[*]*Corresponding author. Email: abdulharis@unesa.ac.id*

ABSTRACT

The purpose of this study was to analyze students' mathematical modeling on PISA quantity problems which concerns formulating contextual situations mathematically. A total of 310 eighth grade students of a state secondary school in Sidoarjo, Indonesia, were involved in a task-based questionnaire whose responses were then analyzed using the stages of understanding the problem, making mathematical models, solving mathematical models, interpreting model solutions in real situations, and validating solutions. The results showed that 21 students made mathematical models explicitly, 29 students made models implicitly and the remaining participants did not create any mathematical model, or the model is incomplete. The explicit model made by the students is one model that represents the whole problem, while the implicit model is in the form of two models, each of which describes one part of the problem, and then they are combined. Of the five stages of mathematical modeling, constructing mathematical models, interpreting model solutions, and validating solutions are found problematics for the student participants.

Keywords: *Mathematical Modelling, Formulate, PISA questions, Quantity.*

1. INTRODUCTION

The competence of mathematics subjects at every school level is to develop a logical, critical, analytical, and thorough attitude, be responsible, responsive, and not give up easily in solving problems [1]. This implies that students are prepared to become tough problem solvers in life by using mathematics as a tool. The extent to which these competencies are achieved, one of which can be seen from the results of the assessment of Indonesian students in the Program for International Student Assessment (PISA).

PISA is a study organized by the Organization for Economic Cooperation and Development (OECD) and is held every 3 years. The purpose of PISA is not only to ascertain whether students can apply knowledge but also to examine how well students can predict and apply knowledge in the context of everyday life [2]. Indonesia has participated in the PISA study since 2000 and the results until 2019, the mathematical competence (mathematical literacy to be precise) of Indonesian students has not been encouraging, always being in the lowest 10 groups of all countries participating in the PISA study [3-8]. The results of Edo's [9] stated that in solving PISA questions, Indonesian students had difficulties in the process of formulating contextual problems into mathematical models. The most mistakes made by Indonesian students in solving PISA questions are transformation errors, students understand the problems, but they can't transform the information in the problems into mathematical models [10]. This indicates that the ability of Indonesian students in making mathematical models is still not good.

In solving problems in everyday life related to mathematics, students must have modeling skills [11] and in many cases, mathematical modeling is needed to solve them [12][13]. With modeling, students are invited to investigate a problem from a scientific discipline or everyday situation using mathematics [14].

Mathematical modeling has become one of the focuses of mathematics education, and several countries have included modeling in school curricula, for example in Australia, Brazil, Denmark, Germany, Singapore, the Netherlands, and so on [15][16]. Even since 1983, The International Community of Teachers of Mathematical

Modelling and Applications (ICTMA) has held biennial conferences to promote modeling in schools and universities [17]. However, in many cases teaching problem solving, students focus more on aspects of the application of mathematics and its results [18]. As a result, the development of students' abilities in mathematical modeling has not received much attention during problem-solving learning. Teachers need to identify students' modeling abilities so that they can develop several modeling tasks to improve students' modeling abilities [19].

Modeling is a step of changing the problem into a mathematical form [20]. According to Blum [21], the process of changing contextual problems into mathematical form to find solutions to a problem is called mathematical modeling. Mathematical modeling is the whole process of solving problems that are iterative or cyclic [22]. So, mathematical modeling is the process of making a mathematical model of a contextual problem so that the correct solution can be found. There are 5 stages in mathematical modeling used as indicators in this study, namely, (1) understanding the problem; (2) making a mathematical model of a real problem; (3) solving mathematical model; (4) interpreting model solutions into real situations; (5) validating the solution [23].

In PISA, a problem that requires mathematical modeling skills is in the formulate category. Formulation refers to how effectively students recognize and identify problems and determine the mathematical structure needed to formulate problems into the mathematical form [2]. Most of the PISA questions are presented in the form of contextual questions taken from phenomena or activities in everyday life.

Each PISA problem is associated with specific mathematical content. One of the contents of the PISA questions is quantity. This quantity content combines the quantification of objects and situations in real life, understands the various representations of the quantification, and judges interpretations and arguments based on quantity. So, number content applies knowledge of number operations and numbers in various representations [2]. The quantity content has correspondence with school curricula in several countries, besides that quantity content can be found on many topics including measurement, geometry, probability and statistics, and algebra [24]. The quantity content is in accordance with one of the Content Standards for mathematics subjects for junior school level which is contained in the Attachment of the Minister of Education and Culture Number 21 of 2016 namely numbers, which are related to the ability to understand the measure, number patterns, and everything related to numbers in everyday life, such as counting and measuring certain objects. Many activities in daily life are related to quantity content, for example, calculating taxes, measuring time, distance, and others. Therefore, in this study, PISA problem in the formulate category was used for quantity content to analyze students' mathematical modeling.

2. METHODS

This research is a descriptive study with data sources of 310 class VIII students at a state secondary school, Sidoarjo, Indonesia. To the subject, a PISA test was given in the formulate category for quantity content. Furthermore, the subject's answers were grouped with details of blank answers, correct answers, incorrect and complete answers, and incorrect and incomplete answers. In this study, correct and incorrect, and complete answers were taken which were then categorized based on the mathematical model used by the subject to solve the questions.

Research data were analyzed using indicators of mathematical modeling competence based on Maaß (2006). Following are the indicators of mathematical modeling.

PISA quantity problems of Formulation Category used as research instrument are as follows.

Table 1. Mathematical modelling indicators

No.	Steps	Indicator
1.	Understanding the problem	Write down the information in the problem Write down what is asked in the problem Making assumptions
2.	Making mathematical models	Converting information in mathematical form Choose the appropriate mathematical notation Choose the appropriate formula to solve the mathematical model that has been created
3.	Solving mathematical models	Using mathematical arithmetic operations in solving mathematical models that have been made
4.	Interpreting model solutions into real situations	generalize mathematical solutions in the context of the problem
5.	Validating solutions	Check the results that have been obtained Reflect on other ways to solve the problem

Jenn works in a DVD and computer game rental shop. In this shop, the annual membership fee is 10 zeds. DVD rental fees for members are cheaper than non-members, as shown in the following table:

One DVD rental fee for non-members	*One DVD rental fee for members*
3.20 zeds	*2.50 zeds*

What is the minimum number of DVDs that members need to rent so that the rental costs are cheaper than non-members? Show your work.

Figure 1 PISA "Formulate" problem.

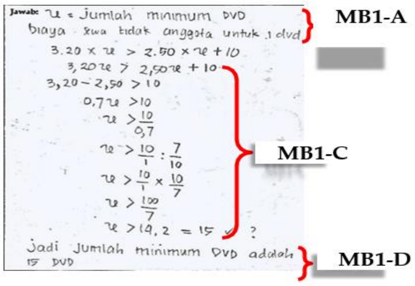

Figure 2 MB1 work result.

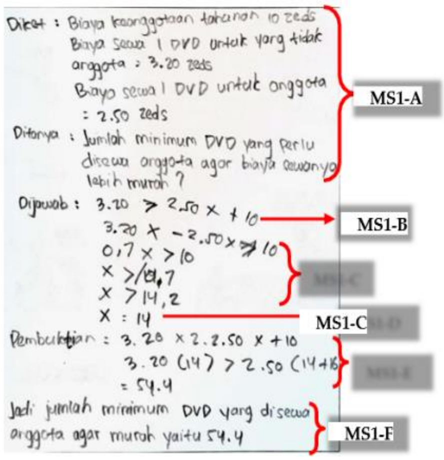

Figure 3 Work results.

3. RESULTS AND DISCUSSION

From the analysis of students' answers, it is obtained, there were 157 blank answers, 10 correct answers, 69 complete but incorrect answers, and 74 incomplete answers. Of the 69 complete but incorrect answers, there are 40 answers which indicates that they understand the question. Of the 10 correct answers and 40 students' answers which indicated that they understood the question, it could be classified into 2 forms of mathematical modeling used by students. The following are the results of the grouping of mathematical models that have been made.

Table 2. General description of the mathematical modeling made by the subject

Modeling type	Correct		Incorrect		Description
	Code	n	Code	n	
Type 1	MB1	2	MS1	19	Two students were correct, did not validate the solution. Of the 19 students who got it wrong, 18 students made the wrong model, one student misinterpreted the solution, and one student validated the solution.
Type 2	MB2	8	MS2	21	Eight students were correct, did not validate the solution. Of the 21 students who got it wrong, 17 students got the model wrong and or, misinterpreted, 4 students got it wrong in completing the model, and none of the students validated the solution.

Type 1: The explicit model uses a one-variable linear inequality; Type 2: the implicit model uses a one-variable linear function; n: number of student participants

The following are the results of the analysis and discussion of student modeling in solving PISA questions in the formulate category on quantity content.

3.1. Model Type 1

In the type 1 model, there are 2 students with correct answers. Here is presented one of the results of MB1's work.

Based on the results of MB1's work in Figure 2, the following information was obtained from the modeling information used by students.

a. Understanding the problem

Based on the results of MB1's work in Figure 2, from the information presented in the problem, what MB1 did in understanding the problem was to assume the minimum number of DVDs to rent was in x (MB1-A). The minimum number of DVDs to rent is what is asked in the question. So that at the stage of understanding the problem, MB1 raises a variable to assume what is being asked in the question.

b. Making mathematical models

MB1 subjects made a mathematical model of the problem in the form of a one-variable linear inequality (MB1-B). MB1 states the fee to be paid for non-members to rent x DVDs in the form of 3.20×x, while for members 2.50×x+10. Then, MB1 combines the two algebraic forms in the form of inequality, 3.20×x>2.50×x+10. The inequality made by MB1 is correct, it fulfills the request for a question, namely that the DVD rental fee for members must be cheaper than non-members.

c. Solving mathematical models

MB1 subjects use the rules on inequalities to solve mathematical models made by involving addition, subtraction, multiplication, and division of fractions (MB1-C). At the end of the calculation, MB1 finds a solution for x that is x> 14.2.

d. Interpret mathematical results in real situations

Based on MB1's work in Figure 2, MB1 interprets the solution by finding the smallest integer that is greater than 14.2, because this corresponds to the minimum number of DVDs that must be rented. The correctness of the interpretation is also evident from the conclusion that MB1 made, "so the minimum number of DVDs is 15 DVDs." (MB1-D).

e. Validating solutions

At this stage, MB1 does not validate whether the result that is 15 DVDs is the minimum number of DVDs that can be rented or not. For the stage of validating the solution, you can substitute the results obtained, namely x = 15, into the system of linear inequalities that have been created.

In the type 1 model, there are also 19 students with wrong answers. Figure 3 indicates an example of responses showing the steps of modeling used by a student (MS1).

a. Understanding the problem

In contrast to what was written by MB1, based on the results of MS1's work in Figure 3, based on the information presented in the questions, MS1 wrote down important information on the questions and what was asked in the questions (MS1-A).

b. Making mathematical models

The model made by MS1 is the same as the MB1 model, namely making a mathematical model in the form of a one-variable linear inequality (MS1-B). MS1 also returns x as a variable, but MS1 doesn't specify what the symbol x stands for.

c. Solving mathematical models

In the stage of completing the mathematical model, MS1 uses the rules in inequalities that involve arithmetic operations of addition, subtraction, multiplication, and division of fractions (MS1-C). MS1 solve the model until a solution for x is obtained, namely x> 14.2

d. Interpret mathematical results in real situations

MS1 misinterprets the results of the model solution that he gets into real situations. MS1 understands that multiple DVDs cannot be decimal (MS1-D), but it is wrong in determining the replacement number for x that satisfies the inequality. MS1 chose 14 as the solution (MS1-D). Misinterpretation also appears when MS1 makes conclusions to answer the problem, it should be related to the number of DVDs that must be rented, not regarding the cost of renting members (MS1-F).

e. Validating solutions

In this stage, MS1 writes proof to validate the solution that has been obtained. MS1 substitutes the solution that has been obtained, namely 14 into the model that has been written (MS1-E).

3.2. Model Type 2

Based on the results of MB2's work in Figure 4, information on the modeling steps carried out by students is obtained as follows.

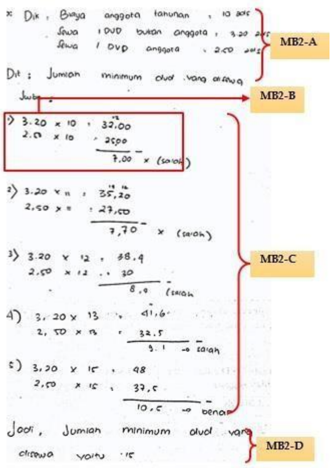

Figure 4 MB2 work results.

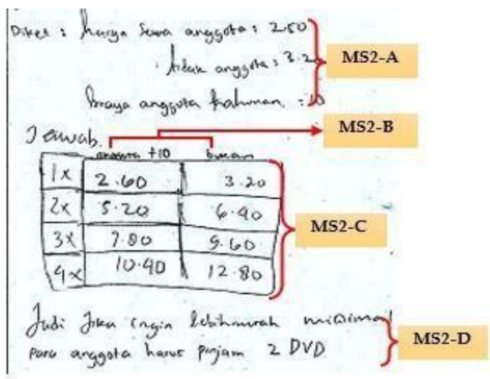

Figure 5 MS2 work results.

a. Understanding the problem

Based on the results of MB2's work in Figure 4, from the information presented in the questions, MB2 understood the questions by writing down important information on the questions and writing down what was asked in the questions (MB2-A).

b. Making mathematical models

MB2 subjects made a mathematical model of the problem implicitly in the form of a one-variable linear function (MB2-B). MB2 implicitly states the fees that must be paid for non-members to rent n DVDs in the form of $3.20 \times n = f(n)$ and states the fees that members must pay to rent n DVDs in the form of $2.50 \times n = g(n)$. MB2 uses the difference between the values of the two functions to find the annual membership fee that members must pay each year. In order, to get the minimum number of DVDs to rent, the result of the difference between the DVD rental fees for members and non-members must exceed 10 zeds which is the membership fee. The difference in the value of the linear function made by MB2 is correct because the difference obtained can cover the annual membership fee so that the rental fee for members is cheaper than non-members.

c. Solving mathematical models

MB2 subjects use decimal multiplication and subtraction operations to solve mathematical models (MB2-C). MB2 searches for many DVDs rented by members and non-members until the difference in rental fees between members and non-members is more than 10 zeds. At the end of the calculation, MB2 found 15 DVDs with a rental difference of 10.5 zeds.

d. Interpret mathematical results in real situations

Based on MB2's work in Figure 4, MB2 interprets the solution by finding the number of DVDs with a difference that can meet the membership fee that members must pay each year. The minimum number of DVDs that members need to rent is 15. The correctness of that interpretation is also evident from the conclusion that MB1 made, "so the minimum number of DVDs is 15 DVDs." (MB2-D).

e. Validating solutions

At this stage, MB2 does not validate whether the result obtained which is 15 DVDs is the minimum number of DVDs that can be rented or not. For the stage of validating the solution, MB2 can calculate the rental costs incurred by members and non-members each year when renting 15 DVDs.

In the type 2 model, there are also 21 students with wrong answers. Here is presented one of them, namely the work of MS2. Based on the results of the MS2 work in Figure 5, the following information is obtained from the modeling competence used by students.

a. Understanding the problem

Based on the results of MS2's work in Figure 4, he showed his understanding of MS2 by writing down important information on the questions, namely the cost of renting members; rental fee is not a member; and annual membership fee (MS2-A).

b. Making mathematical models

MS2 subjects implicitly create a mathematical model in the form of a one-variable linear function (MS2-B). MS2 determines the DVD rental fee for members in the form $f(n) = (2.50 \times n) + 10$ and the DVD rental fee for non-members in the form $g(n) = 3.20 \times n$, where n is the number of DVDs rented (MS2- 1). The results of these calculations are presented in tabular form (MS2-C). The table aims to compare the price of DVD rental fees for members and non-members. The solution is obtained if the rental fee for members is less than for non-members.

c. Solving mathematical models

In the stage of completing the mathematical model, MS2 compares the cost of renting members and non-members from renting 1 DVD onwards. However, there was an error in completing the mathematical model created by MS2. This is known based on the results of interviews with researchers with MS2. The following is an excerpt from an interview with MS2.

P: *To finish you create a table, try to explain what this table is for?*

MS2-1: *This is the cost of renting one DVD for members, it's 2.50, meaning 1×2.50 and then adding 10 to make 2.60 and the rental fee for non-members is 3.50. Continue for the next I fold.*

P: *Folded what do you mean?*

MS2-2: *So renting 2 DVDs for members means 2.60×2 for non-members 3.50×2 and so on*

P: *Why calculate it like that?*

MS2-3: *I've met renting 1 DVD which is 2.60 members who are not 3.50 members, so if you want to rent 2, just multiply by 2, and so on*

From the results of the interview, MS3 was wrong in adding decimal numbers and was wrong in calculating the DVD rental fee for members. To determine the cost of renting 2 DVDs for members, MS2 does not use the previously created mathematical model (MS2-2 and MS2-3).

d. Interpret mathematical results in real situations

Based on the results of MS3's work in Figure 5, MS2 determines the amount of DVD rental by choosing the lesser rental fee for members than non-members. So MS3 concluded that "so, if you want cheaper, at least the members have to borrow 2 DVDs." (MS2-D).

e. Validating solutions

At this stage, MS2 does not validate whether the result that is 2 DVDs is the minimum number of DVDs a member has to rent so the membership fee is useful. For the stage of validating the solution, MS2 can calculate the rental costs incurred by members and non-members each year when renting 2 DVDs.

The results of the study which showed that only 10 students were correct in solving PISA questions in the formulate category on quantity content, indicated that there were still problems in mathematical modeling. These results confirm the conclusions of research by Edo [9], Karimah [10], Jankvist & Niss [25], and Khusna & Ulfah [26]. The low ability to make mathematical models can also be seen from very few students (3 students) who are right in using variables to represent problems into mathematical models.

In the type 1 model, the explicit model, in this case, the students write down the mathematical model used to solve the problem (e.g. MB1-B and MS1-B), 21 students in this category, all of them are correct that the mathematical model of the problem involves a linear inequality of one variable with hyphen more than or less than. However, only 3 students were able to complete the components of the inequality correctly so that the mathematical model made was correct.

Students who use the type 1 model and produce the correct solution, have problems at the validation stage. They do not reflect and make sure that the model they make, the model solution, and its interpretation are in accordance with the context of the problem because there are not many model solutions. One student who was correct in making the model and its solution, but failed to interpret it, in the validation process, was wrong in choosing a model solution that was used to verify the correctness of the model. This result is relevant to the conclusion of the research by Frejd & rlebck [28] which states that middle-class students are not adept at clarifying the purpose of the model made.

In the type 2 model, the implicit model, in this case students do not write down the model used but what they write on the answer sheet implies that they use a certain mathematical model when solving problems (Figure 3, Figure 4, and MS2-1). This indicates that students have difficulty in representing the mathematical model they use. They understand it but have difficulty putting it in written form. It takes the ability to translate representations to be able to formulate problems towards a mathematical model [28], which in this study is from verbal representation to mathematical symbols.

The implicit model used by students is not a model that represents the problem in its entirety, only partially. That is, students, divide the problem into several parts, then each part is made a mathematical model. Problem-solving is done by students by comparing the values of each model they have made. They understand the problem as well as the mathematical model, but some of them make a mistake in the calculation process (MS3-1). Jankvist & Niss [25] also found that some students had problems with calculations in modeling activities. As students who use explicit models, students with implicit models also do not validate the solutions they get.

4. CONCLUSION

The mathematical modeling carried out by students in solving PISA problems in the formulate category on quantity content is problematic at the step of making models, interpreting model solutions, and validating solutions. For students who use an explicit model, only 2 people are correct in making the model as well as interpreting it, and none of them validate the model. Students who use the implicit model, only 8 people who understand the model he made in mind, and they can interpret it, and no one validates the solution. For the stages of understanding the problem and solving the model, they tend to understand it.

Given the importance and demands of including mathematical modeling skills in learning, the results of this study recommend that in mathematics learning that aims to improve mathematical modeling skills, teachers should pay attention to each stage of mathematical modeling, especially modeling, interpreting solution model and validating solutions. Then for the modeling step, especially in the context of complex problems, the teacher needs to divide it into several simpler problems, model each simple problem, and combine these simple models according to the context of complex problems. Then, further research is needed regarding students' difficulties in representing models that are understood in their minds, but they fail to represent them in written form (symbolic form).

AUTHORS CONTRIBUTION

All authors conceived and designed this study. All authors contributed to the process of revising the manuscript, and at the end all authors have approved the final version of this manuscript.

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