Pre-service Mathematics Teachers' Design on Hypothetical Learning Trajectory

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ABSTRACT
Inviting the students to actively be involved in developing knowledge is the main issue in mathematics education. The ability to deliver a good opportunity-to-learn task becomes important for mathematics teachers. Developing a hypothetical learning trajectory (HLT) has become the way for activating students' learning involvement. This study aims to analyze the pre-service mathematics teachers' design of HLT. This content analysis was carried out to analyze the quality of HLT developed by the participants who are second-year students enrolled in Realistic Mathematics Learning Course. The framework of analyzing HLT considering aspects of learning objectives, learning activities, and conjectures of the learning process, was used for analyzing the categorization of the HLT. Document analysis was used to obtain the data. The result suggests that pre-service teachers cannot consider the students thinking and their misconception to develop effective teaching strategies which are an important part of conjectured students' thinking of HLT.

Keywords: *Pre-service, Teachers' design, Hypothetical learning trajectory.*

1. INTRODUCTION

Designing actively students involved learning activities is the main issue in mathematics education. The ability for designing a good learning trajectory become important for mathematics teachers to deliver a good opportunity-to-learn task. The student-oriented learning approach asks the teachers to pay more attention to the student learning trajectory[1]. Clement and Samara [2] described that the learning trajectory can have more important impact on the students learning than the curriculum of the mathematics subject itself. The learning trajectory illustrates emerging various ideas in a series of activities that when student-centered learning support cognitive development [3-4]. The series of activities experienced by the students involves problem-solving or understanding a concept activity.

Learning trajectory provides a plan or pattern that will be used as a reference for making learning plans in each learning process that will be carried out. The use of learning trajectory enables effective development of the students' mathematical thinking competencies as well as an opportunity for the teachers to understand students' learning and thinking [5]. The term hypothetical learning trajectory (HLT) itself was first put forward and used by Simon which states that hypothetical learning trajectory consists of three components in the form of learning objectives, learning activities, and the alleged learning process - predictions about how students' thinking and understanding will develop in the context of learning activities [6].

The intended purpose is the achievement of understanding mathematical concepts. The intended learning activity is a series of tasks to find out how students think. The hypothesis of students' way of thinking is intended is the student's thinking flow in understanding the concept of learning [1]. Hypothetical learning trajectory (HLT) which includes a series of instructional assignments to understand students towards the concept of mathematics learning, is one of the important aspects that must be possessed by teachers in teaching students meaningful learning. In addition, a hypothetical learning trajectory takes the students' knowledge into account in developing suitable learning trajectories. Thus, teachers need to know learning trajectory and hypothetical learning trajectory because teachers are expected to be able to develop learning models specifically for the characteristics of students based on existing theories and the initial ability of each student effectively with the appropriate learning design.

The teacher training program from the government program theoretically brings the vital qualification for the teaching. A recent study revealed that these programs are still unable to improve the quality of the teaching by the teachers [7]. The certification program that aimed as the incentive of teachers' professional development is failed to invite the teacher in improving their knowledge of teaching. The program does succeed to make the teachers in focusing their routine on the school activities. However, developing mathematics activity using HLT is different from developing inquiry-based mathematics learning [7]. The simultaneous detailed analysis on each stage within the HLT becomes a fundamental aspect that differs developing HLT from inquiry-based learning. This HLT development is part of the idea of Realistic Mathematics Education (RME). Yilmaz found that the teachers' knowledge of the HLT is far from adequate [8]. This matter had to do with the teachers' belief that has been developed since their pre-service program [9]. Therefore, finding out to what extent the pre-service teachers can develop a good HLT is crucial. Table 1 illustrate the framework for the analyzed component based on Clements and Sarama. Pre-service teachers are students in the mathematics education study program with no formal class that they manage. Regarding the importance of finding the HLT designed by the pre-service teachers, in this study, we looked into their designed HLT as a one-semester project.

Table 1. The component of HLT

HLT	Observable Component
Learning objectives	mathematical concepts
	big idea
Learning Activities	Context
	Didactical Phenomenology tasks
Conjectures of the Learning Process	Conjectures students thinking
	Teachers' Responses

2. METHODS

This study used content analysis technique to describe the pre-service teachers' HLT design from their designed mathematics lesson. The data were drawn from the mathematics lesson designed by the pre-service teacher as part of their final task for a realistic mathematics course in a form of HLT. In this study, we focus on the geometry lesson design as the content of the lesson. An instrument was developed based on the framework mentioned in Table 1 became the primary term in analyzing the designed HLT. The framework, then, was used to elicit the profile of the HLT designed by the pre-service teachers. There are 30 designed HLT by pre-service teachers who were second-year students in mathematics education study program in Indonesia enrolled in the course of Realistic Mathematics. This course discusses the RME as a domain-specific theory, the use of context, designing activities, and developing HLT. The topics of HLT can be categorized into Number, geometry, and Algebra. The dispersion of the designed HLT can be seen in **Table 2.**

Table 2. Pre-service Teacher HLT Design

Content	Quantity
Number	6
Geometry	12
Algebra	12

Content analysis was conducted involving the scaling of the designed HLT quality from 0-4. Furthermore, the findings will be elicited into 6 criteria which are extended from 3 components of HLT (see Table 1). Specific categories can be seen in Table 3.

Table 3. The framework of HLT analysis on Mathematics

HLT	Component	Code	Criteria (Level 0-4)
Learning objectives	mathematical concepts	MC	Clarity of the mathematics concepts in the HLT (0: no mathematical concepts found, 1: embedded mathematical concepts found but only as an additional, 2: mathematical concepts are irrelevant with the activities objective, 3: mathematical concepts is relevantly completing the designed activities objective, 4: unique mathematical concepts is created

HLT	Component	Code	Criteria (Level 0-4)
			functioning in the activities objective)
	Big Idea	BI	Validity of the elicited big idea relating to the mathematics concept (0: no big idea found, 1: unclear big idea found but only as terms, 2: the big idea is partly used in the design, 3: the big idea is used completely in the design, 4: the unique big idea is created functioning in the activities)
Learning Activities	Context	CT	To what extent does the context help the learning (0: no context found, 1: unclear context found with only camouflage role on the learning activities, 2: context is partly used in the design to help the learning, 3: context is used completely in the design to help the learning, 4: unique context is created functioning in helping students learning)
	Didactical Phenomenology tasks	DP	To what extent does the tasks sequence invite the students learning (0: no phenomenology tasks found, 1: unclear tasks found without using students experience to learn, 2: phenomenology tasks are partly used in the design to help the learning, 3: phenomenology tasks are used as model-of and model-for to help the learning, 4: unique phenomenology tasks is presented in an emergent model functioning in helping students learning)
Conjectures of the Learning Process	Conjectures students thinking	CS	Range of the possible students thinking from the learning tasks (0: no conjectures students thinking found, 1: unclear conjectures found but only as terms, 2: conjectures students thinking is only available as an alternative solution, 3: conjectures students thinking is described involving partially students cognitive development, 4: conjectures students thinking is described involving students cognitive)
	Teachers' Responses	TR	Proper feedback from the teachers' responses (0: no teachers' responses found, 1: unclear teachers' responses found but separated from the students' conjectures, 2: teachers' responses are partly used or unrelated to respond to the students' difficulties, 3: teachers' responses are used essentially to respond students' difficulties, 4: unique teachers' responses is created effective scaffolding in helping students difficulties)

3. RESULTS AND DISCUSSION

The data from the designed HLT will be analysed using the framework. The result will be described as an overall overview and detailed profiles for each component.

3.1 Mathematical Concepts (MC)

In this part, the data related to the Mathematical Concept presented in the HLT is presented. The document on mathematical concepts is analysed from the given learning objectives in the HLT.

The analysis of the three contents was presented in Table 4. In this part, the data related to the Mathematical Concept presented in the HLT is presented. The document on mathematical concepts is analysed from the given learning objectives in the HLT. The analysis of the three contents was presented in Table 4.

Table 4. Analysis of Mathematical Concept

Content	Mathematical Concept	Average Level (Min-Max)
Number	The Number concepts are both irrelevant and relevantly completing the designed activities objective	2.5 (1-4)
Geometry	The geometry concepts are irrelevant relevantly completing the designed activities objective	2.08 (1-3)
Algebra	The algebra concepts are irrelevant relevantly completing the designed activities objective	2.25 (1-3)
All	The mathematical concepts are irrelevant relevantly completing the designed activities objectives	2.23 (1-4)

The overall categories fall in level 2 which can be described as the HLT provide the mathematical concepts but mostly separated with the learning activities objectives. Regarding the spread of the HLT based on the determined level, more than 50% of the HLT falls into the second level criteria. The 30% of the designed HLT were fallen into level 3. Lastly, only 1 design that fully implements both the mathematical concepts and the learning activities objectives is intertwined.

3.2 Big Idea (*BI*)

In this part, the analysis of the big idea in the HLT will be presented. The document on the big idea can be found in the objectives of the learning activities. In detail, several designs also provided the iceberg of the topics. However, the additional learning resource related to the alternative answer towards discussed problems were described. The analysis of the three-learning design resulted in **Table 5**.

Table 5. Analysis of the Big Idea

Content	Big Idea	Average Level (Min-Max)
Number	Big idea relating to the Number concepts is partly used in the design	2.33 (1-3)
Geometry	Big idea relating to the geometry concepts is partly used in the design	2 (1-3)
Algebra	Big idea relating to the algebra concepts is partly used in the design	2.25 (1-3)
All	Big idea relating to the concepts is partly used in the design	2.17 (1-3)

The overall categories fall in level 2 which can be described as highlighted big idea only partly used in the design. Regarding the spread of the HLT based on the determined level in the use of the big idea, only 43.33% of the HLT fall into the second level criteria. The 36.67% of the designed HLT were fallen into level 3 with no HLT presented a level 4 as a unique big idea is created functioning in the activities. An example of the big idea in the proportion found in one of the HLT is the double number line (see Figure 1). This big idea appears in the topic of direct proportion with a unique context of *Mahabarata* wars.

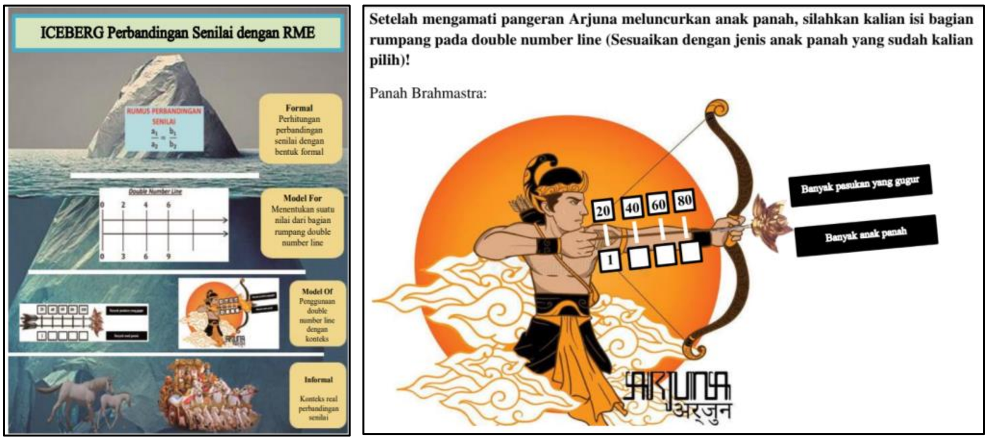


Figure 1 Example of the Iceberg in learning direct proportion with the double number line using the context of Mahabharata war.

3.3 Context (*CT*)

The use of context in the HLT is mostly presented as camouflage context without the opportunity to learn. In this part, the analysis of the Context used in the HLT will be presented. The documentation on the use of context in the HLT could be elicited from the designed activities especially in the students' worksheet. In detail, we consider whether the context presented in the HLT is camouflage or essential context. The analysis of the camouflage or essential context resulted in Table 6.

Table 6. Analysis of context

Content	Context	Average Level (Min-Max)
Number	the context in number topics is used completely in the design to help the learning	2.67 (2-4)
Geometry	the context in geometry is partly used in the design to help the learning	2.17 (1-3)
Algebra	the context in algebra is partly used in the design to help the learning	2.33 (2-3)
All	context is partly used in the design to help the learning	2.33 (1-4)

The overall categories fall in level 2 which can be described as the HLT provide the mathematical concepts but mostly separated with the learning activities objectives. Regarding the spread of the HLT based on the determined level in the use of the Context, more than 60% of the HLT fall into the second level criteria. The 30% of the designed HLT were fallen into level 3. Lastly, only one design that implements the unique and original idea for essential context in the learning activities. An example of the essential context in the proportion found in one of the HLT involves the idea of an arrow as a model-of before going to double number line model-for (see Figure 1). This big idea appears in the topic of direct proportion with a unique context of *Mahabarata* wars.

3.4 *Didactical Phenomenology Tasks* (*DP*)

The didactical phenomenology tasks related to the activities in the learning trajectory that invite students cognitive learning. This category can be elicited from the learning activities and conjectured students' thinking. However, the additional learning resource related to the alternative answer towards discussed problems were described. The analysis of the didactical phenomenology tasks resulted in Table **7**.

Table 7. Analysis of the Didactical Phenomenology Tasks

Content	Didactical Phenomenology Tasks	Average Level (Min-Max)
Number	Phenomenology tasks are partly used in the design to help the number topic learning	1.83 (1-3)
Geometry	Phenomenology tasks are partly used in the design to help the geometry topic learning	1.83 (1-3)
Algebra	Phenomenology tasks are partly used in the design to help the algebra topic learning	1.67 (1-3)
All	Phenomenology tasks are partly used in the design to help the mathematics learning	1.77 (1-3)

The overall categories mostly fall in level 2 with proportionally the remaining fall into level 1 which can be described as the tasks in the learning activities that almost have no impact in inviting the students to experience phenomenologically the mathematical idea. Regarding the spread of the HLT based on the determined level in the use of the big idea, most of them around 43.33% of the HLT fall into the level 1 criteria. The 36.67% of the designed HLT were fallen into level 2 with no HLT presented a level 4 as a unique big idea is created functioning in the activities. An example of the big idea in the proportion found in one of the HLT is the double number line (see Figure 2). Most of the tasks and their phenomena are described separately with the objectives of learning or having unclear relations.

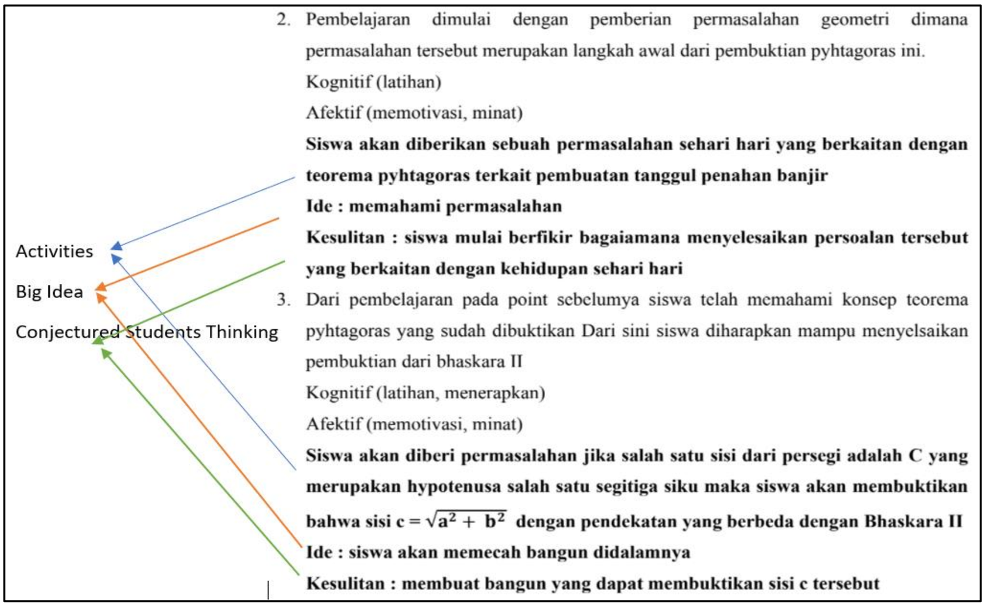
3.5 Conjectures students' thinking (*CS*)

In this part, the representation of the conjectured students' thinking from the analyzed HLT will be presented. The documentation of the conjectured students' thinking can be found in conjectured part of the HLT design. However, the additional learning resource related to the alternative answer towards discussed problems were described. In the presentation of the conjectured students' thinking, the misconception is mostly neglected in the learning design. In this part, the analysis of the students' misconceptions will be presented. The document on students' misconceptions could be elicited in the same section when the lesson design was discussing the conjectured students' thinking. In detail, it appears as a further explanation of the students' conjectures. The analysis of the conjectures students' thinking resulted in Table 8.

Table 8. Analysis on the Conjectures students' thinking

Content	Conjectures students' thinking	Average Level (Min-Max)
Number	conjectures students thinking on the number topic is described involving partially students cognitive development	2.67 (2-3)
Geometry	conjectures students thinking on the geometry topic is only available as an alternative solution	2.00 (0-3)
Algebra	conjectures students thinking on the algebra topic is only available as an alternative solution	2.42 (1-3)
All	conjectures students thinking is only available as an alternative solution	2.30 (0-3)

The overall categories fall in level 2 which can be described as the HLT provide the conjectures students' thinking but mostly separated with the learning activities objectives. Regarding the spread of the HLT based on the determined level in the use of the Context, more than 53% of the HLT fall into the second level criteria. 40% of the designed HLT were fallen into level 3. Lastly, there is 1 design that falls into level 0 and no HLT categorized in level 4. An example of the conjectured students' thinking in the Pythagorean Theorem found in one of the HLT that not only provide alternative answers, but the possible students' difficulties and responses are also presented (see Figure 2).



2. Pembelajaran dimulai dengan pemberian permasalahan geometri dimana permasalahan tersebut merupakan langkah awal dari pembuktian pyhtagoras ini.
Kognitif (latihan)
Afektif (memotivasi, minat)
Siswa akan diberikan sebuah permasalahan sehari hari yang berkaitan dengan teorema pyhtagoras terkait pembuatan tanggul penahan banjir
Ide : memahami permasalahan
Kesulitan : siswa mulai berfikir bagaiamana menyelesaikan persoalan tersebut yang berkaitan dengan kehidupan sehari hari
3. Dari pembelajaran pada point sebelumya siswa telah memahami konsep teorema pyhtagoras yang sudah dibuktikan Dari sini siswa diharapkan mampu menyelesaikan pembuktian dari bhaskara II
Kognitif (latihan, menerapkan)
Afektif (memotivasi, minat)
Siswa akan diberi permasalahan jika salah satu sisi dari persegi adalah C yang merupakan hypotenusa salah satu segitiga siku maka siswa akan membuktikan bahwa sisi c = $\sqrt{a^2 + b^2}$ dengan pendekatan yang berbeda dengan Bhaskara II
Ide : siswa akan memecah bangun didalamnya
Kesulitan : membuat bangun yang dapat membuktikan sisi c tersebut

Figure 2 Activities and the tasks presented along with conjectured students thinking.

3.6 Teachers' Responses (TR)

The teachers' responses category related to the conjectured part of the HLT design. Most of the quality in this category relies on the component used in explaining the concept. Whether the HLT provides adequate information in teachers' responses related to responding to the occurred students' difficulties. The explanation can be found in the conjectured part of the HLT (see Figure 2). The analysis of the teachers' responses resulted in Table 9.

Table 9 Analysis of the teachers' responses.

Content	Teachers' Responses	Average Level (Min-Max)
Number	Teachers' responses are partly used or unrelated to respond to the student's difficulties in learning number	2.33 (2-3)
Geometry	Teachers' responses are partly used or unrelated to respond to the student's difficulties in learning geometry	2.25 (1-3)
Algebra	Teachers' responses are partly used or unrelated to respond to the students' difficulties in learning algebra	2.08 (1-4)
All	Teachers' responses are partly used or unrelated to respond to the students' difficulties	2.20 (1-4)

The overall categories fall in level 2 which can be described as the in learning. Regarding the spread of the HLT based on the determined level in the availability of the teachers' responses, 40% of the HLT falls into the second level criteria. The 30% of the designed HLT were fallen into level 3. Lastly, only one design that implements the unique and original idea for essential context in the learning activities.

3.7 Categorization of the HLT

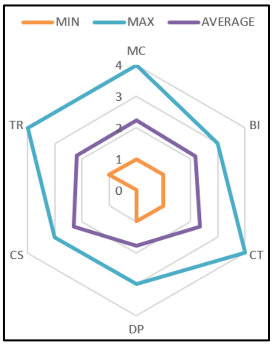


Figure 3 The summary of the analyzed HLT.

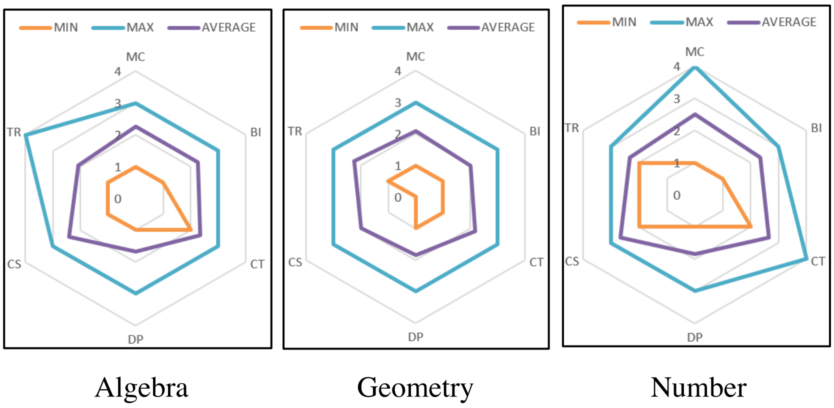


Figure 4 The analyzed HLT based on the mathematics topics.

In this part, the summary of the categorization of the HLT is presented. The level of each category will be presented in Fig 3. The category of the three different topics which are numbers, geometry, and algebra also presented separately. The lowest and the highest level on each category as well as the average level are also provided. On average, all of the categories fall in level 2, which are the mathematical concepts is irrelevant with the activities objective, the big idea is partly used in the design, the context is partly used in the design to help the learning, the phenomenology tasks is partly used in the design to help the learning, the conjectures students thinking is only available as an alternative solution, and the teachers' responses are partly used or unrelated to respond the students' difficulties. A response indicating level 4 can be found in the category of mathematical concept (MC), the use of 3.5. Conjectures students' thinking (CS), and the Teachers' responds (TR) but with only 1:30. Moreover, in the category of the conjectured students' thinking, answer solutions are usually presented regardless its relationship with the students cognitive reasoning [10-11].

The profile of the HLT based on the topics of algebra, geometry, and number, show that mostly has reached level 2 on average. However, the topic of number suggests that more HLT has already reached level 3 with which shows that the designed HLT is ready to be tested in actual learning. The HLT in the topic of geometry, on the other hand, suggest that the HLT still failed to full filled each category especially in the conjectured students' thinking category that we found some of the HLT fall into level 0, which mean the HLT does not provide the conjectured students' thinking which is the important part in the HLT (see Figure 4)

4. CONCLUSION

The result suggests the pre-service teachers are still unable to develop a good HLT because the component of the HLT is mostly presented in the design without comprehensive relation with the mathematical task and students' activities. This conjectured students' thinking was almost neglected in the HLT. Besides, relating to topics on the HLT, the HLT in the number topic is better than the HLT for geometry topics. The facts mentioned earlier suggest that the pre-service teachers still need more guidance in designing HLT in the topics of geometry especially in presenting the conjectured students' thinking.

AUTHORS' CONTRIBUTIONS

Shofan Fiangga and Evangelista L. W. Palupi devised the project, the main conceptual ideas and proof outline. Ahmad Wachidul Kohar and Shofan Fiangga worked out almost all of the technical details, and performed the qualitative analysis. Rooselyna Ekawati and Rini Setianingsih verified and validated the analysis. Rooselyna Ekawati and Rini Setianingsih suggest the discussions with Shofan Fiangga, Evangelista L. W. Palupi and Ahmad Wachidul Kohar wrote the manuscript.

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