

# Validity and Practicality of Problem-Based Integral Calculus Teaching Materials Assisted with Mathematical Software

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## ABSTRACT

This study aims to develop Integral Calculus teaching materials to improve students' communication skills and positive character. The teaching materials developed are problem-based with the help of mathematical software. The model used refers to the Plomp model. Data on the validity of teaching materials were collected using expert evaluation questionnaires, practicality data on teaching materials were collected using student response questionnaires given to students for the 2020/2021 academic year and lecturer responses. Data were analyzed by descriptive statistics. The results showed that the teaching materials developed had met the valid and practical criteria. Based on the results of the questionnaire, the validity and practicality of teaching materials need to be optimized again in the content, assessment and answer key sections so that they can develop student creativity, especially in communication skills and positive character.

**Keywords:** *Communication Skills, Positive Character, & Problem-Based*

## 1. INTRODUCTION

In the learning process, the important component is the availability of teaching materials besides the lecturer. Teaching materials are a collection of learning resources that support lecturers and students in the learning process [1], and research [9]; [10] showed that the optimal use of teaching materials in improving problem solving skills, reasoning abilities, and mathematical communication.

The interview with students about Integral Calculus course obtained the information regarding the lack of available teaching materials in accordance with the syllabus, and that could facilitate them to explore their knowledge maximally so that they understand the concepts being studied. The availability of problem-based teaching materials is not yet available even assisted by mathematical software, even though it is known that learning mathematics really needs visualization and representation of mathematical objects so that students can easily understand a concept. Technology is an important tool and component to support visualization, so it can help representation, explore and solve problems [31]. In this regard, problem-based teaching materials assisted by mathematics software need to be provided, in

order to assist students in learning Integral Calculus independently.

The teaching materials developed are problem-based, because problem-based learning (PBM) is one of the learning models that focuses on developing communication skills [2]; [3], and as strategy that provides opportunities for students to present their creative ideas and communicate in their own language [4]. Research results [5]; [6] showed that the use of PBM that may improve students' mathematical communication skills.

The interview with one of the Lecturers who support the Integral Calculus course stated that the lack of success of students in this course is caused by the limited teaching materials according to the syllabus, and the lack of students' ability to write answers to the problems given, although when interviewed some students were able to answer correctly the results, but they find it difficult to provide written answers. The results of research [7]; [8] revealed that students considered Integral Calculus as a difficult subject, it caused students' mathematical communication skills to be weak [9]. Furthermore, [8] revealed that the results of document recording obtained 16.7% of students getting grades A and B. This is also

not much different from the condition of students in the Mathematics Education Study Program, FMIPA Universitas Pendidikan Ganesha.

As known, some concepts of Integral Calculus need to be visualized so that students may understand them more easily, therefore Integral Calculus teaching materials are developed based on problems and equipped with mathematical software so the students can also check their work. It is suspected that it can trigger students to try to answer existing problems, and indirectly students practice communicating them.

Mathematical communication ability is defined as the ability to express mathematical ideas in their own language [10]. The same thing was expressed [11] that mathematical communication is the ability to express mathematical ideas coherently through spoken and written language. The difficulty of students in understanding concepts, of course, can affect their ability to communicate mathematical ideas [14]. It means that one of the important aspects that affect the success of learning is the form of communication used by the Lecturer and students when interacting [15]. Student communication skills can be revealed through discussion of problems that exist at the beginning of each topic and making good written mathematical expressions with their own language, pictures, models and symbols [12]. According to Pugalee in [13] to improve communication skills, students are trained to answer questions accompanied by appropriate reasons, and comment on statements expressed by students, so that they understand the concepts and arguments meaningfully.

Communication in learning mathematics is very important and not only for other subjects [17], but it is also used in learning mathematics [19]. It is emphasized by [16] that mathematical communication that is a very important aspect that should be owned by students, so that mathematical communication needs to be developed among students. Therefore, Integral Calculus material must be presented as a meaningful language.

To develop students mathematical communication skills, it can be done by (1) providing adequate assignments, (2) creating a conducive environment so that they are free to express their ideas, (3) directing students to explain and provide arguments for the results given [18]. Therefore, it is necessary to prepare adequate learning tools to develop students' mathematical communication skills through the development of problem-based teaching materials assisted by mathematical software. The purpose of this research is to develop problem-based teaching materials assisted by valid, practical and effective mathematical software.

Teaching materials were developed using several software including maple, and Geogebra. With Geogebra software abstract objects can be visualized and manipulated quickly, accurately, and efficiently [21].

The results of this study as an alternative to meet the needs of teaching materials with proper qualifications based on the results of expert validation and student responses. The variety of existing teaching materials is expected to increase student interest in learning.

## 2. METHOD

The development model used in this study referred to the Plomp model [20] that consists of five stages, namely: (1) initial investigation, (2) design, (3) realization/construction, (4) test, evaluation, and revision, and (5) implementation.

At the initial stage of investigation, an analysis of the need for teaching materials was carried out, analyzing the curriculum, analyzing the situation of students, determining the achievements of the courses used in the manufacture of teaching materials. At the design stage, an attempt was made to design a possible solution to the problem that had been defined at the initial investigation stage, and to design teaching materials according to the syllabus and characteristics of the PBM model. In the realization stage, the solution designed was realized so that an initial prototype is produced. The resulting prototype is still in the form of prototype 1, namely problem-based Integral Calculus teaching materials assisted by mathematical software which then needs to be tested for validity, practicality, and effectiveness.

At the Test, Evaluation and Revision Phase, it was carried out to the validity and practicality of the teaching materials developed, through limited trials. The instruments used in this study were: (1) validation sheets of teaching materials, (2) observation sheets of the practicality of teaching materials in the form of student and lecturer response questionnaires, (3) observation sheets of practicality of teaching materials in the form of sheets of observing the implementation of teaching materials by observers.

Evaluation activities, final revisions to improve the teaching materials developed so that prototype 2 is obtained expected to be maximally applied in the real class. The validity of this teaching material is based on the opinions of three experts involved as validators, namely the Mathematics Education Study Program Undiksha, from the Bali State Polytechnic and from the Muhammadiyah University of Surakarta. Data obtained from limited trials then analyzed and compared with the criteria in Table 1.

The teaching materials developed were said to be suitable if met the valid, practical and effective criteria. This first year research was only based on the validity and practicality test.

**Table 1.** Validity criteria and practicality of teaching materials

Aspect	Range	Category (Description)
Validity	$3,4 < S_r \leq 4,0$	Very valid (very feasible)
	$2,5 \leq S_r \leq 3,4$	Valid (feasible)
	$1,5 \leq S_r \leq 2,5$	Invalid (not feasible)
	$1,00 \leq S_r \leq 1,5$	Very invalid (Strongly not feasible)
Practicality	$3,5 < S_r \leq 4,0$	Very practice
	$2,5 < S_r \leq 3,5$	Practice
	$1,5 < S_r \leq 2,5$	Impractical
	$1,0 < S_r \leq 1,5$	Very impractical

### 3. RESULTS AND DISCUSSION

#### 3.1. Results

The results of the initial investigation related to the communication skills of second semester students for the 2020/2021 academic year on learning achievement to understand the Riemann number limit and the basic theorem of calculus was 43.65 for class B, class C was 45.55 and class D was 44.50. If converted, this average was in the less category (D).

Based on observation results, it was found that the teaching materials used by the lecturers had referred to the achievements of the Integral Calculus course, but the teaching materials used by students tended to not encourage students to engage in activities that trigger communication skills, and were not equipped with mathematical software, so it is natural that they did not understand the concept. and cannot make corrections to their answers. When students were given HOTS type problems students had difficulty in analyzing the concepts used so they were unable to communicate the reasons, because mathematics is interrelated with each other, so learning the concept of Integral Calculus cannot be done separately.

In solving Integral Calculus problems, the students tend to fail when given HOTS level questions, and even tend not to be solved. Even if there are students who answer, they are not able to write the reasons for their answers, as well as in writing down the steps. When the students interviewed, they were not able to give the correct answer. It reflects that students are not able to understand and analyze concepts, it can be assumed that their written and oral communication skills were still low. It also happens when students completed assignments individually, students tend to choose to see the results of

their friend work even just copy and paste the work. This case showed the low responsibility of controlling the progress of learning outcomes, low curiosity, and lack of honesty aspects in revealing learning obstacles. Lack of high-level learning activities include analysing, constructing and solving problems and communicating, so that students tend to be passive.

Based on the interview of students with low initial test scores, it showed that students have difficulty expressing/communicating both in writing and verbally when it comes to HOTS type questions, as Example when students are asked to rate a statement as true or false, then write down the reasons. For questions like this, 75% of students were wrong in giving reasons, it is because researchers suspect that when they were in high school to solve multiple choice questions, as a result, students were not trained to communicate mathematics both in writing and orally.

The results of documents recording in the even semester of the 2019/2020 academic year from 4 existing classes, as many as % of students got an A, 56% got a B, and 27% got a C score (the lectures were online), and the form of the test during UTS and UAS was in the form of a test. objectively through e-learning. However, the results obtained were different from the test results in the form of descriptions, the results of showed that as many as 20% of students did not have a good understanding and ability in communicating Integral Calculus concepts in writing.

At the design stage, it was made to design based on problems based on initial investigations, the lack of students' writing and speaking learning activities. Therefore, the design offered is problem-based teaching materials assisted by mathematical software (at the beginning of each topic starting with discussion material, so students are required to communicate the results of their discussions before discussing the next material, so here we optimize the role of students in communicating. From the affective aspect, [22] stated that the use of mathematical software may increase the confidence in solving mathematical problems, provide a positive attitude to mathematical proof [23], and increase student interest [24]. Therefore, problem-based teaching materials assisted by mathematics software was able to provide opportunities for students to explore mathematical concepts and increase their activities, because the use of Geogebra may increase conceptual and procedural knowledge [25].

Furthermore, the solution designed was realized in the form of teaching materials that are still in the form of prototype 1 that needs to be seen from the validity aspect. In each topic, it starts with learning achievements, and indicators of achievement, and further discussion material, a description of the material, examples and practice questions. The order of material in this teaching material includes 1) indeterminate integrals and definite

integrals, 2) transcendent functions, 3) integral applications, 4) integration techniques, and 5) improper integrals. The presentation of the material in this teaching material with manual answers is also equipped with answers with mathematical software.

**Table 2.** Teaching Material Validity Test Results

No.	Aspect	Average Score			
		E1	E2	E3	Ave
1	Content Eligibility	3,50	4,00	3,50	3,67
2.	Language	3,60	4,00	3,40	3,67
3.	Presentation	2,67	3,67	2,67	3,00
4	Graphics	3,50	3,00	3,50	3,33
	Average	3,32	3,67	3,28	3,44

**Table 3.** The Results of Materials Practicality

Practicality	LI	L II	Stds
Average Score	3,4	3,33	3.12
Description	Practice	Practice	Practice

The realized Prototype 1 was then tested for validity by 3 experts from the Undiksha mathematics study program, with the consideration that they had observed the development of Calculus teaching materials with Maple and he is in charge of Geometry and statistics courses, an expert from the Bali Polytechnic Mathematics lecturer with the consideration that he had researched about character education and he is in charge of the Calculus course, and an expert from the Muhammadiyah University of Surakarta, with the consideration that he is an expert in mathematics, especially Calculus.

The quality of the Integral Calculus teaching materials developed in this study only came to two aspects of assessment that was valid/feasible, and practical. Assessment scores and score conversions for each aspect as presented in Table 2. While the practicality test is in Table 3.

Based on Tables 2 and 3, the average validity/feasibility score in the content, language, presentation, and graphic feasibility was 3.44 so it was in valid category. The same with the average practicality score was in the practical category. Thus, this teaching material was met the appropriate criteria (valid, and practical).

### 3.2. Discussion

This prototype 2 Integral Calculus teaching material is a completion of the prototype 1 teaching material based on the recommendation of three validators. Based on the validator suggestion, this teaching material was revised in a consistent font, each image was numbered, completed with a summary and concept map as well as references.

The test results of presentation showed that the Integral Calculus teaching material developed had an average score of 3.58 with very valid/feasible criteria. Material delivery of teaching materials was also equipped with discussion materials for students to discuss. Another characteristic was the answers of manual calculation and mathematical software calculation.

Based on the linguistic showed that the teaching materials developed obtained an average content feasibility score of 3.67 with very valid criteria. In terms of language, it made the attention to the use of simple language so that it is easy to understand with an average score of 3.67 in the very valid category. In terms of presentation, the material had been presented sequentially so that the concept of Integral Calculus was taught sequentially from basic material to more complex material, with an average score of 3.00 in the valid category. The test results related to graphics showed that the teaching materials developed got an average score of 3.33 with valid criteria. In terms of graphics, it can be said that this teaching material was able to accommodate the needs of students for the abstract concept of Integral Calculus.

The answers of the exercise with mathematical software is an advantage of this teaching material compared to other teaching material products developed by other researchers. The practicality of these teaching materials was also strengthened by the results of a limited trial of students in grades B, C and D. The average practicality score based on the results of the student response questionnaire analysis is 3.12 in the practical category. Based on the analysis of the lecturer's response questionnaire, an average score of 3.37 was obtained which was also in the practical category. This study was supported by [26] found that the use of the Geogebra program had a good effect in terms of learning outcomes and motivation, research [27] found that learning using mathematics teaching materials in the form of tabloids assisted by Wolfram Mathematica software with a contextual approach was better than conventional learning on integral material and also supported by [30] that the use of software, especially Geogebra helps students to focus on the material, and they also admit that they can visualize graphics well using Geogebra.

Seels & Richey [28] revealed that one of the educational technology characteristics is the use of all potentials as learning resources in order to obtain

maximum learning outcomes. Technological integrated Learning is integrated with technology that becomes a requirement in this era. With the use of technology, it is expected that learning will be more effective, efficient and varied, because the use of technology such as the use of mathematics software in mathematics learning has a positive influence in increasing motivation to engage in learning activities, and students who use mathematics software have higher scores and had a positive influence on student confidence in learning Calculus [29], as well as [30] that more than 50% of students gave a positive response about the use of Geogebra in learning calculus. Geogebra is useful for helping students understand integral calculus material such as the area between two curves, the surface area, and the volume of a rotating object. In addition, learning calculus with Geogebra gave higher motivation to students and makes learning fun.

Based on the results, this study can be alternative reference in preparing teaching materials, both in terms of development procedures and processes to see the quality of teaching materials developed.

## 4. CONCLUSION

The teaching materials developed had been tested for validity and practicality. The results of the validity test showed an average score with valid criteria. The results of the practicality test showed an average score of 3.12 (student response questionnaire) and 3.37 (lecturer response questionnaire) that was practical category. Based on the results obtained, it can be concluded that the teaching materials developed met the valid and practical criteria and acceptable and feasible to use at broader level

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## APPENDIX

### Learning Outcome

Demonstrate positive attitudes and perceptions, understanding, and critical and creative thinking skills in applying the integrals to determine the area bounded by a curve, and several lines and volumes of rotating objects. Achievement indicators: students are able

1. use definite integrals to calculate the area of a region bounded by the coordinate axes and a function;
2. use definite integrals to calculate the area of an area bounded by two functions;
3. construct an area bounded by a karva, and two lines and determine its area;
4. explain the use of definite integrals to calculate the volume of a rotating body;
5. calculate the volume of a rotary object using the disc, ring or tube shell method.

### Discussion Materials

1. In everyday life, problems are often encountered such as calculating the area of an area, because in designing buildings it is often constrained for areas whose sides are not simple, such as Figure 1.



Figure 1. An example of material

If these problems are found, how do you determine the area of a region whose sides are not simple??

2. In some cases, problems are often found where the area lies between 2 or more curves. How to calculate the area?

Example 1. Help Badu to measure the land area! Badu buys a plot of land, shaped like Figure 1, If

you are a land officer, Help Badu to measure the land area!

**Clue:** if the scale on the map is 1: 1 hectare calculate the Area using the y-axis parallel partition.

**Answer:**

Manual calculation 1:

The picture of Badu Land in map has the similarity with the area restricted by  $y = 2x^2$ ,  $y = 0$  dan  $y = -x + 3$   
Picture scale

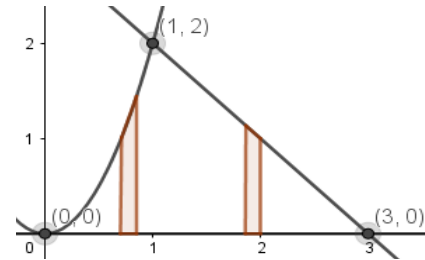


Figure 2.

The area of  $A_1: 0 \leq x \leq 1$

Length  $= l = \Delta x$

Width  $= p = y_2 - y_1 = f(x)_{\text{atas}} - f(x)_{\text{bawah}} = 2x^2 - 0 = 2x^2$

$$\Delta A = p \cdot l = 2x^2 \Delta x$$

So  $A_1 = \int_0^1 (2x^2) dx = \left[ \frac{2}{3} x^3 \right]_0^1 = \frac{2}{3}$  unit area

The area of  $A_2: 1 \leq x \leq 3$

Width  $= l = \Delta x$

Length  $= p = y_2 - y_1 = f(x)_{\text{up}} -$

$f(x)_{\text{down}} = (-x + 3) - 0 = -x + 3$

$\Delta A = p \cdot l = (-x + 3) \Delta x$

$$A_2 = \int_1^3 (-x + 3) dx = \left[ -\frac{1}{2} x^2 + 3x \right]_1^3 = \left( -\frac{9}{2} + 9 \right) - \left( -\frac{1}{2} + 3 \right) = 2$$

$$\text{Total area} = A_1 + A_2 = \frac{2}{3} + 2 = \frac{8}{3}$$

Because the scale used is 1:1 hectare, the area of land that Badu bought is hectare.

Calculation with Maple

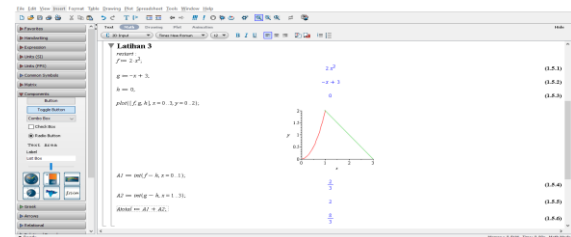


Figure 3.

**Figure 2.** Determine the volume of the object formed from the figure below, if it is rotated around an axis with partitions parallel to the axis.

Manual completion

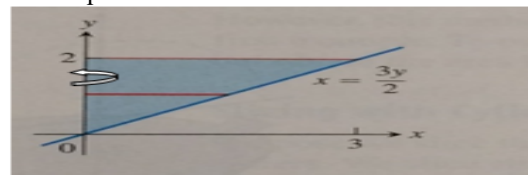


Figure 4.

Since the slices are parallel to the axis, then  $R(y) = \frac{3y}{2}$   
dan  $0 \leq y \leq 2$ .

$$\begin{aligned}\Delta V &\approx A(y)\Delta y = \pi\left(\frac{3y}{2}\right)^2\Delta y \\ V &= \pi \int_0^2 \left(\frac{3y}{2}\right)^2 dy = \frac{\pi}{4} \int_0^2 9y^2 dy = \frac{9\pi}{12} y^3 \Big|_0^2 \\ &= \frac{72\pi}{12} = 6\pi\end{aligned}$$

The volume is  $6\pi$  unit volume

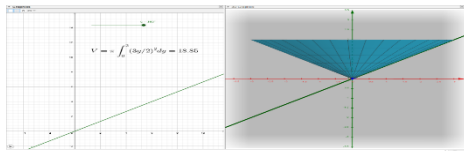
Calculation using *Maple* software obtained:

```
> with(Student[CalculusI]):
>
```

$$\text{VolumeOfRevolution}\left(\frac{3 \cdot y}{2}, y = 0..2\right)$$

$6\pi$

Calculation using with *Geogebra* obtained:



**Figure 5**

Example 3. Determine  $\int \frac{x dx}{\sqrt{4+x}}$

Manual completion

For example  $u = \sqrt{4+x} \rightarrow u^2 = 4+x$

So  $2udu = dx$

$$\begin{aligned}\int \frac{x dx}{\sqrt{4+x}} &= \int \frac{u^2 - 4}{u} \cdot 2udu = 2 \int (u^2 - 4) du \\ &= \frac{2u^3}{3} - 8u + C \\ &= \frac{2(4+x)^{\frac{3}{2}}}{3} - 8\sqrt{4+x} + C \\ &= \frac{2}{3} \sqrt{4+x}(4+x-12) \\ &\quad + C\sqrt{4+x}(x-8) + C\end{aligned}$$

The answer using *Maple*:

$$\begin{aligned}> f(x) := \frac{x}{\sqrt{x+4}} \\ &f = x \rightarrow \frac{x}{\sqrt{x+4}} \\ > \text{int}(f(x), x) \\ &\frac{2}{3} \sqrt{x+4} (-8+x)\end{aligned}$$