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Proportional Reasoning in Mathematics: What and How is the Process?

Maharani Izzatin[1*]

[1]*Universitas Borneo Tarakan, Tarakan, Indonesia*
Corresponding Author email: maharaniizzatin@borneo.ac.id*

ABSTRACT
Reasoning is one of the abilities that need to be developed in mathematics. Mathematical reasoning can be interpreted as the basis for constructing mathematical knowledge. One of the reasoning in mathematics is proportional reasoning. Proportional reasoning involves understanding the multiplicative relationship between two or more quantities. Materials in mathematics that involve proportional reasoning are congruence, statistics, algebra, probability, social arithmetic, and others. The purpose of this research is to examine more deeply about proportional reasoning in mathematics. This research is a literature review by examining 12 references consist of journals, proceedings, and literature books that related to proportional reasoning in mathematics. Proportional reasoning is defined as a process of thinking to conclude from the information that exists in situations involving multiplicative relationships. There are five levels in the proportional reasoning process, namely: 1) non-proportional reasoning, 2) manipulative proportional reasoning, 3) replicative proportional reasoning, 4) (premultiplicative proportional reasoning, and 5) multiplicative proportional reasoning.
Keywords: *proportional reasoning*

1. INTRODUCTION

The reasoning is a very important process for understanding mathematical material. Lohman & Lakin [1]stated that reasoning refers to a process of concluding existing information. Ball dan Bass [1]explained that reasoning is a basic skill in mathematics and is needed to achieve several objectives, for example, to understand mathematical concepts, to be able to use mathematical ideas and procedures more flexibly, and to reconstruct previously forgotten but forgotten knowledge.Therefore, the reasoning is one of the skills that need to be developed in mathematics.

There are several kinds of reasoning, among others: conservation reasoning, proportional reasoning, variables control, probabilistic reasoning, correlational reasoning, and combinatorics reasoning. [2]. Proportional reasoning is a reasoning ability that needs to be developed.According to Putra [3],proportional reasoning is related to students' sensitivity to situations involving proportional relationships. Proportional reasoning deals with situations that use additive (addition) and multiplicative (multiplication) relationships.

The National Council of Teachers of Mathematics (NCTM) lists proportional reasoning as a key concept to be understood in the middle class. According to the NCTM Curriculum and Evaluation Standards, "Reasoning skills develop proportionately in all grades 5-8. This is so important that it deserves whatever time and effort it must put into ensuring its careful development. "[4]. According toWalle[5], in daily life, there are many problems regarding proportional situations, including comparisons in pricing, use of scales in maps, solving problems about percentages, use of scale in designing a building, and many more. In the branch of mathematics, proportional reasoning plays an important role because it is the basis of various learning materials, including fractions, algebra, similarities, and opportunities.

Some of the problems faced by students in the proportional reasoning process. In the use of addition and multiplication arithmetic operations, sometimes students are still confused about when to use addition or multiplication. Changing a student's view from addition to multiplication is very difficult[5]. Students who can solve a problem that involves a proportional situation, are not necessarily able to explain the reasons for the complete procedure of what they have done [1]. The same problem was expressed by Berk and friends[1]that elementary school teacher candidate students who can solve problems involving proportional situations are not able to provide arguments why they solve problems in this way. One of the causes of the lack of ability of prospective elementary school teaching students to reason proportionally is because they are used to focusing on and memorizing the appearance of the steps to get the result of a problem-solving.

Based on these problems, it is necessary to make efforts to develop proportional reasoning skills for students and prospective teachers. An in-depth study related to proportional reasoning is needed to better understand the proportional reasoning process before carrying out further research. In this paper, we will discuss the definition and process of proportional reasoning in mathematics.

2. METHOD

This research is a literature review by examining 12 references consisting of journals, proceedings, and literature books that related to proportional reasoning in mathematics. The results of this review literature are used to understand the characteristics of proportional reasoning and to determine the proportional reasoning process in solving mathematical problems. The researchers select articles that were published from 2004 to 2020, where proportional reasoning in mathematics began to be widely researched in Indonesia and internationally.

3. RESULT DISCUSSION

3.1 What is Proportional Reasoning?

Students use proportional reasoning in early math learning, for example, when they think of 8 as two fours or four twos rather than thinking of it as one more than seven. They use proportional reasoning later in learning when they think of how a speed of 50 km/h is the same as a speed of 25 km/30 min. Students continue to use proportional reasoning when they think about slopes of lines and rates of change [6].

Proportional reasoning is being able to make comparisons between the entities in ratio and proportion situations in multiplicative terms. The development of proportional reasoning is a gradual process, underpinned by increasingly more sophisticated multiplicative thinking and the ability to compare two quantities in relative (multiplicative) rather than absolute (additive) terms [7]. The same is said by Behr and Lesh[8] argue that proportional reasoning is related to sensitivity to covariation (a sense of covariation), multiple comparisons, and the ability to remember and process multiple pieces of information.

Proportional reasoning involves the ability to understand multiplicative relationships. The multiplicative relationship in this comparison is a relationship that involves multiplication. Besides involving the ability to understand multiplicative relationships, Walle stated that the ability to understand the difference between situations using additive and multiplicative relationships is one indication of proportional reasoning [1].Based on the previously mentioned definitions, it can be concluded that proportional reasoning is a process of thinking to conclude from the information available in situations involving multiplicative relationships.

Proportional reasoning involves thinking about relationships and making comparisons of quantities or values. In the words of John Van de Walle, “Proportional reasoning is difficult to define. It is not something that you either can or cannot do but is developed over time through reasoning … It is the ability to think about and compare multiplicative relationships between quantities” (2006, p. 154).

Proportional reasoning is sometimes perceived as only being the study of ratios, rates, and rational numbers such as fractions, decimals, and percents, but it permeates all strands of mathematics. For example, proportionality is an important aspect of measurement, including unit conversions and understanding the multiplicative relationships of dimensions in area and volume. [6]

Cramer & Post [9] mentioned three types of proportional reasoning tasks developed by the Rational Number Project, there are *missing values, numerical comparison, and qualitative prediction/comparison*. Karplus' Tall-Man-Short Man is one of the examples of *missing value problems*. The problem is in the figure, it is given that the Mr. Short height 6 paper clips when it is measured by paper clips and is 4 buttons when measured by buttons. On the other hand, the height of Mr. Tall is 6 buttons when measured by button, what is the height of Mr. Tall when measured using paper clips. One example of *numerical comparison* is the orange juice task. The students are provided with the information that the orange-juice mix in the glass is marked by a shaded area and the water is not shaded. Then, the student is required to imagine then the orange-juice mix is put into the jug followed by the water. Students are then asked to evaluate which jug has the highest level of orange juice flavor or whether the mixtures in the jug would have the same flavor. One example of *qualitative comparison* is when there are two friends hammered a line of nails into different boards.

Bill hammered more nails than Greg. Bill's board was shorter than Greg's. On which board is the nails hammered closer together?

3.2 How is the proportional reasoning process?

Though proportional reasoning is taught in a mathematics setting and can often be reduced to a simplelinear model $y = kx$, it is a psychological construct: "The essential characteristic of proportional reasoning is that it focuses on describing, predicting or evaluating the relationship between two relationships (i.e., a second-order relationship) rather than simply a relationship between two concrete objects (or two directly perceivable quantities)." Students first learning proportional reasoning rarely approach the subject from a purely mathematical standpoint. It is an important concept in middle school mathematics because it is often a student's *first* exposure to explicitly modeling these kinds of second-order relationships[10].
To master proportional reasoning, a student must be able to [10]:

1. Conceive a *multiplicative relationship* and possess a notion of *change in a relative sense*
2. Recognize that when two quantities are changing, the change in one depends on the change in the other (*covariance*)
3. Recognize that while some aspects of the situation change, the ratio relationship remains constant (*invariance*)
4. Employ an *appropriate multiplicative strategy* to solve problems;

There are five levels in the proportional reasoning process, namely: [11]

1. level 0 (non-proportional reasoning)
   students who are t this level have not shown proportional reasoning skills in solving proportional problems, usually, they solve the problem of one unknown value using difference or using a pattern less count, namely using any number or operation.
2. level 1 (manipulative proportional reasoning)
   students who are at this level show proportional reasoning skills using the help of pictures, models, or manipulations in solving a problem of an unknown value.
3. level 2 (replicative proportional reasoning)
   students who are at this level show proportional reasoning skills by using repeated addition or building both measures in solving a problem of one unknown value.
4. level 3 (pre-multiplicative proportional reasoning)
   students who are at this level show proportional reasoning skills by using unit values or using a scale factor in solving a problem with an unknown value.
5. level 4 (multiplicative proportional reasoning)
   students who are at this level show proportional reasoning skills by using cross multiplication or using fractions worth in solving a problem of an unknown value.

In solving problems involving proportional situations, students can use a variety of strategies, there are cross-product algorithms, unit rate strategy, a factor of change strategy, equivalent fractions, equivalence class, and build-up strategy[12]. Bexter and Junker [10]grouped proportional problem-solving strategies into five stages based on the theory of building proportional reasoning as in the following table:

Table 1: The proposed cognitive model for the development of proportional reasoning

Stage	Observable Performance
I) Qualitative	Young students generally possess a good deal of knowledge about the quantity that permits them to answer questions about more and less eg, which drink is sweeter?) or fairness (eg, divide pizza or cookies so everyone gets a fair share).
II) Early Attempts at Quantifying	Early attempts at quantifying often involve constant additive differences (ie, a − b = c − d) rather than multiplicative relationships
III) Recognition of Multiplicative Relationship	Students have the intuition that a ratio is two numbers that change together but the change may be additive or multiplicative. They often rely on additive strategies such as build-up when multiplicative reasoning is required. Situations involving absolute change are not always distinguishable from situations involving relative change.
IV) Accommodating Covariance and Invariance	Students begin to develop a multiplicative change model. They recognize that while some quantities may be changing, relationships amongthe quantities remain invariant. They view a ratio as a single unit towhich basic arithmetic operations may be applied. They can typicallydistinguish situations involving an absolute change from those involving relative change. Strategy use is context−specific and when the numbers are hard these students may resort to additive reasoning in multiplicative situations. Concepts of covariance fail when students are asked to scale up a figure
V) Functional and Scalar Relationships	Students recognize the invariant nature of the relationships between pairs of changing quantities. These students have a repertoire of generalizable strategies and they select the most efficient strategy for a given problem. Conceptions of covariance and invariance are well developed.

4. CONCLUSION

Proportional reasoning is defined as a process of thinking to conclude from the information available in situations involving multiplicative relationships. Three types of proportional reasoning tasks developed by the Rational Number Project are *missing value, numerical comparison*, and *qualitative prediction/comparison*.

There are five levels in the proportional reasoning process, namely: 1) non-proportional reasoning, 2) manipulative proportional reasoning, 3) replicative proportional reasoning, 4) premultiplicative proportional reasoning, and 5) multiplicative proportional reasoning.

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