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The Thinking Process of Class IX Students of Junior High School in Solving Problems Geometry

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ABSTRACT
The purpose of learning mathematics emphasizes the ability of students to think. This article describes the thinking process of class IX junior high school students in solving mathematical problems, especially the material of spatial structure. Researchers describe 2 types of thinking processes, namely numerical thinking and realistic thinking. The researcher uses a qualitative approach in describing the subject's thought process. The main instrument of this research is the researcher himself, while there are 3 (three) kinds of auxiliary instruments, namely: math problems, interview guidelines and video recording. The stages of data analysis carried out in this study were: transcribing data; reduce data; encode data; checking data validity or data triangulation; reviewing data; interpret findings; validate findings; draw conclusions. The results showed that the thinking process of the subject of numeric thinking is to assemble numbers that are according to calculations. While the thinking process, the subject of realistic thinking assembles numbers that are in accordance with calculations and the real form of a shape.

Keywords: *Thinking, Spatial Problems, Numerical Thinking, Realistic Thinking*

1. INTRODUCTION

The purpose of learning mathematics as stated in [1] emphasizes students' ability to think. Teachers must challenge and encourage students to develop their thinking skills. Learning activities carried out by students through discussing ideas, strategies, and solutions in solving problems are expected to be able to develop their thinking skills.

The thought process of a person has received a lot of attention from research experts. This can be seen from the many learning programs that offer the development of students' thinking processes such as UCLES [2]; Thinking-Based Learning (TBL) [3]; Heuristic Instruction in Mathematics [4]; Infusing Critical and Creative Thinking into Content Instruction (ICCT) [5]. In Indonesia, many experts offer programs to develop students' thinking skills, including [6] [7] [8]. Through optimizing each problem solving activity in mathematics learning classes, it can develop students' thinking skills [6]. There is a revitalization of meaningful learning developed through the Teacher Quality Improvement program (TEQIP), where one of the goals is to develop the role of teachers in triggering students to always think so that the development of student schemes will increase [7] [8].

In addition to offering learning programs to develop students' thinking processes, many experts pay attention to the development of thinking processes throughout a person's life span. Jean Piaget from Switzerland was the first to pioneer the theory of cognitive development, namely the sensory-motor, pre-operational, concrete operational and formal operational periods. Furthermore, Jean Piaget's theory was developed by Vigotsky, in Vygotsky's theory of cognitive development that children are shaped by the cultural context in which they live [9]. In addition, Dina and Pierre van Hiele who developed a theory of the development of individual thought processes in geometric thinking [10], [11] [12] developed a theory of thinking known as the three worlds of mathematics (conceptual embodied, proceptual symbolic, formal axiomatic) which describes a person traveling through different worlds when constructing a concept.

Thinking ability must be possessed by every student in studying mathematics. Students who have the ability to think can come up with ideas in solving mathematical

problems. Thinking activities require a problem solver at every step in solving problems. He must think in interpreting the problem [13] [14] [15], selecting the strategy to be used and implementing it, so that an appropriate solution is obtained [16].

The thinking skills that must exist in solving problems and making decisions include generating ideas, clarifying ideas and evaluating the reasonableness of ideas [17]. First, the problem solver must generate various ideas that are varied and new ideas in solving problems. Second, he must explain each idea that is generated and compare taking into account the similarities and differences. Third, he must evaluate the reasonableness of the idea by considering the accuracy of the observations and sources, so that he can decide which ideas to use in solving the problem.

Mathematical problems are divided into two kinds, namely the problem of finding (to find) and the problem of proving (to prove) [18]. Elementary school students are more appropriate to solve the problem type of problem finding. While the problem proves more appropriate for students who study advanced mathematics at university. Therefore, in this study using the problem of finding because it is in accordance with the research subject, namely the IX grade junior high school students.

Junior high school students in grade IX were chosen in this study because based on Piaget's theory of cognitive development [19] they were in the formal operational stage. Junior high school students of class IX have been able to think abstractly with menggun be certain symbols, can use rules-atauran formal logic, so that they can me ningkat k an analytical skills, ability to develop a possibility based on two or more possibilities, ability to attract generalist ation and inference of various objects.

One of the topics of learning Mathematics is Geometry. In Geometry, students study points, lines, planes, shapes and their properties, sizes, and relationships with each other [20]. Geometry studies are concerned with two-dimensional and three-dimensional shapes. Build three dimensions including blocks, cubes, spheres, tubes, prisms, and pyramids. So studying geometry requires students to create the concepts that are in their minds in determining the position and size of an object in three-dimensional shapes and the shape of the space.

The researcher conducted a preliminary study to see the thinking processes of class IX junior high school students in solving spatial problems related to the volume of building materials. Where students are asked to make a design with a volume of 250 ml. The results of this preliminary study indicate that there are two patterns in solving the problem. The first pattern, the student determines the size of a shape whose volume is 250 ml, but he does not consider the existence of a shape with that size. The second pattern, students determine the size of a building whose volume is 250 ml and consider the rationality of its size by drawing a shape according to its size. Henceforth, the first pattern is called numerical thinking, while the second pattern is called realistic thinking.

In this articel will be studied further truth of the two patterns together with a description of the Junior Class IX student thinking in solving mathematical problems. Researchers used the theory of [17] in describing students' thinking processes. It aims to get a description of the stages or phases that students go through in carrying out a mental activity when solving problems. The description of the thinking process can be used by educators as a discourse in planning the learning process to develop students' thinking skills.

2. METHOD

2.1. Subject

The subject of the research was the IX grade junior high school students. The reason for choosing the subject is that students have learned the concept of spatial structure and are at the formal operational stage. So that they have the ability to solve the given problem.

2.2. Procedure

First, 20 students were given a math problem and asked to as much as possible express everything that was thought by voicing (think aloud) during the problem solving process. In the second stage, the researcher randomly assigned 2 students with numeric thinking and realistic thinking to become research subjects. Next, the researcher conducted task-based interviews. Broadly speaking, interviews were conducted to find out what the subject was thinking when concluding something and taking a step. Questions on the subject of "How do you Memikirkan this?" or “what are you thinking right now?” Questions are also raised for me nggetahui reason the subject when using the step thinking.

2.3. Instrument

There are two kinds of instruments in this study, namely the main instrument and the auxiliary instrument. The main instrument is the researcher himself, while there are 3 (three) kinds of auxiliary instruments, namely: math problems, interview guidelines and video *recording*. This math problem consists of 1 *essay* question (description). The mathematical problem a is adapted from [21]. The mathematical problem can be seen in Figure 1 below.

Uncle has been running the "Sari Soybean" beverage business for 1 year. Uncle innovated by changing the shape of the packaging to increase sales figures. You are asked to help uncle in designing a new, more attractive packaging.

a. Make a more attractive packaging design than the previous packaging with the same volume
b. Explain how and why you chose these designs
c. From the designs you made, which design did you choose? Give the reason!

Figure 1. Math problem

2.4. Data analysis

The data obtained from the think aloud process and interviews were transcribed and analyzed. Researchers conducted data analysis by following three stages of qualitative data analysis activities from [22] and 6 stages of qualitative data analysis and interpretation from [23]. The stages of data analysis carried out in this study were: (1) transcribing the data; (2) me the reduction of the data; (3) coding the data; (4) checking data validity or data triangulation; (5) reviewing the data; (6) interpret the findings; (7) validate the findings; (8) draw conclusions

3. RESULTS AND DISCUSSION

The researcher involved 20 junior high school students, but only 15 students were able to design a soy juice drink. Students are grouped into 2, based on the type of thinking, namely realistic thinking and numerical thinking. Numeric thinking is that students show a series of geometric sizes without thinking about the logic of these sizes. While realistic thinking is that students show a series of sizes of shapes by thinking about their logic and the real form of the shapes. Table 1 shows all 20 students involved.

Table 1 Types of thinking of middle school students class IX

No	Name	Thinking Type
1	AL	Realistic thinking
2	AN	Numerical thinking
3	BA G	-
4	BAM	-
5	CHU	Numerical thinking
6	DE	Numerical thinking
7	ERL	Realistic thinking
8	ERN	Numerical thinking
9	EV	-
10	FI	Numerical thinking
11	FR	Numerical thinking
12	GI	-
13	HA	Realistic thinking
14	HE	Numerical thinking
15	RU	Numerical thinking
16	SA	-
17	SE	Realistic thinking
18	TO	Numerical thinking
19	TOP	-
20	WA	-

Note: the sign "-" states that students are not completing math problems right

The following are the results of the research and discussion

3.1 SE (Numeric Thinking)

SE generates various ideas in solving mathematical problems. The idea that SE generated in the packaging design was made in the form of shapes other than blocks, namely balls, tubes and prisms. SE revealed that each of these shapes (except circles) can be made in various sizes, but the volume is the same, namely 200 ml. Figure 2 shows Sendy's written results.

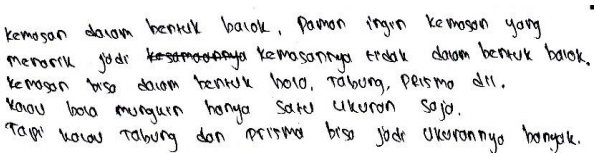
kemasan dalam bentuk balok, Dimana ingin kemasan yang menarik jadi ~~kesamaannya~~ kemasannya tidak dalam bentuk balok. kemasan bisa dalam bentuk bola, tabung, prisma dll. kalau bola mungkin hanya satu ukuran saja. Tapi kalau tabung dan prisma bisa jadi ukurannya banyak.

Figure 2 SE Written Results when Generating Ideas

SE describes each idea that is generated. Table 2 shows his explanation.

Table 2 Summary Explanation of Ideas Generated by SE

Generated Idea	How to Determine Design	Size
Ball shaped design	Using the formula for the volume of a sphere and substituting the volume into the formula	r = 6.91 cm
Tubular design	Using the formula for the volume of a cylinder, suppose the value of r	r = 7cm, t = 1.3 cm
		r = 3.5 cm, t = 5.19 cm
	Using the formula for the volume of a cylinder, suppose the value of t	r = 3 cm, t = 7 cm

		r = 2.13 cm, h = 14 cm
Triangular Prism Shape Design	Using the volume formula for a triangular prism, assume the value of t and the area of the base. Adjusting the values of *a* and *l* with the area of a triangular base	t = 10 cm, p = 5 cm, l = 8 cm
		t = 10 cm, p = 10 cm, l = 4 cm
		t = 8 cm, p = 5 cm, l = 10 cm
		t = 8 cm, p = 25 cm, l = 2 cm
pentagonal prism shaped design	Using the volume formula for a triangular prism, assume the value of t and the area of the base. Adjusting the values of *a* and *l* with the area of a triangular base	t = 10 cm, p = 5 cm, l = 8 cm
		t = 10 cm, p = 10 cm, l = 4 cm
		t = 8 cm, p = 5 cm, l = 10 cm
		t = 8 cm, p = 25 cm, l = 2 cm

SE evaluates the reasonableness of the idea by considering the accuracy of the size of each design. he substituted each of these measurements into the formula for the volume of a space object. So he can classify designs whose volume is exactly 200ml, designs whose volume is less than 200 ml and designs whose volume is more than 200 ml. Figure 3 shows SE written answer.

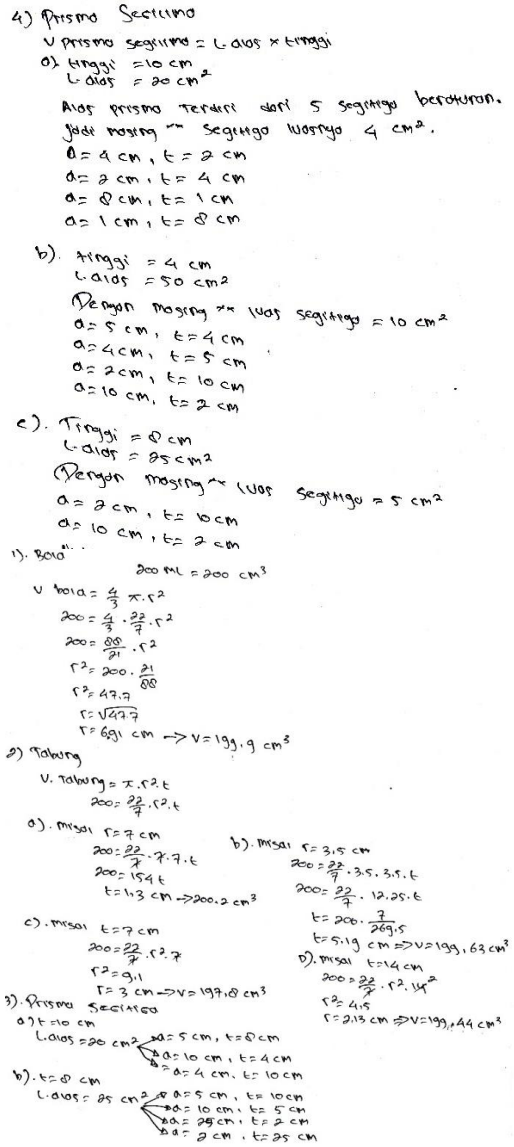

Figure 3 SE Written Results when Explaining and Evaluating the Fairness of Ideas

SE chose a cylindrical design with a size of r = 3 cm, t = 7 cm, all designs are triangular prisms, pentagons and hexagons. The reason SE chose this design is because the volume is exactly 200 ml. Figure 4 shows Se's written answer.

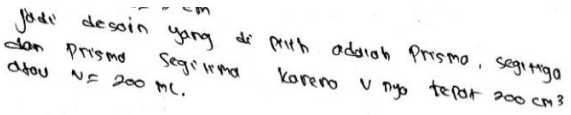

Figure 4 SE Written Results when Deciding which Idea to Use

2. AL (*Realistic Thinking*)

AL generates various ideas in solving mathematical problems. The idea he brought up in the packaging design was made in the form of a shape other than a block but the volume was the same, namely 200 ml. Figure 5 shows the answers written AL.

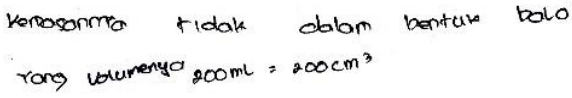

Figure 5 AL Written Results when Generating Ideas

AL explains every idea raised. Table 3 shows an explanation of the idea.

When explaining the idea, AL also considered the existence of the design. So AL also excludes ideas that are not appropriate even though the volume is 200 ml. The design with AL included is a cylindrical design with a size of r = 7 cm, t= 1.3 cm. Figure 6 shows the results written AL

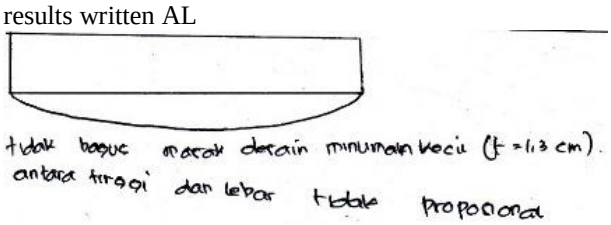

Figure 6 AL Written Results when Explaining ideas

AL evaluates the reasonableness of the idea by considering the accuracy of the size of each design. I a substituting each of these measurements into the formula for the volume of a space object. So it can classify designs whose volume is exactly 200 ml, designs whose volume is less than 200 ml and designs whose volume is more than 200 ml. Figure 7 shows the result written AL

Table 3 Summary Explanation of the Ideas Generated by AL

Generated Idea	How to Determine Design	Size
Tubular design	Using the formula for the volume of a cylinder, suppose the value of *r*, so that the value of *t is* obtained. Make a sketch of the tube according to a predetermined size.	*r* = 7 cm,*t* = 1.3 cm
		r = 3.5 cm,*t* = 5.2 cm
Triangular Prism Shape Design	Using the volume formula for a triangular prism, assume the value of t and the area of the base. Adjust the values of *a* and *l* to the area of the triangular base. Sketch a triangular prism according to a predetermined size.	*t* = 10 cm,*p* = 5 cm,*l* = 8 cm
pentagonal prism shaped design	Using the volume formula for a triangular prism, assume the value of t and the area of the base. Adjust the values of *a* and *l* with the area of the base that is triangular. Make a sketch of a pentagon prism according to a predetermined size.	*t* = 10 cm,*p* = 5 cm,*l* = 8 cm
		t = 8 cm,*p* = 5 cm,*l* = 10 cm
Hexagon prism shaped design	Suppose the size of the sides in an equilateral triangle and use the formula for the volume of a hexagon prism in determining the height of the prism. Sketch a hexagon prism according to a predetermined size.	*t* = 19.61 cm,*p* = 2 cm,*l* = 2 cm
		t = 8.55cm*p* = 3 cm,*l* = 3 cm
		h = 4.8 cm,*p* = 4 cm,*l* = 4 cm

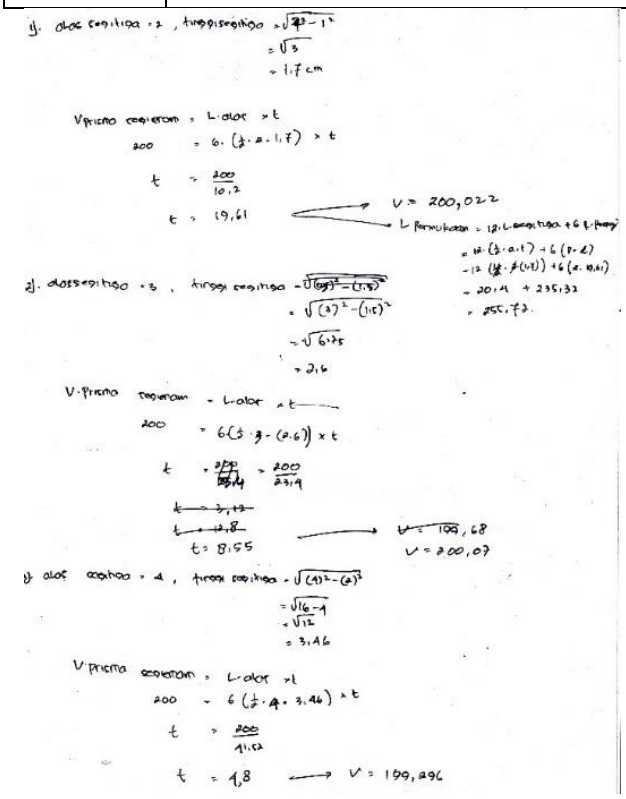

Figure 7 AL Written Results when Explaining and Evaluating the Fairness of Ideas

AL chose a design in the form of a hexagon prism with a size of *t* = 19.61 cm, *p* = 2 cm, *l* = 2 cm. The reason AL chose this design was because the volume was close to 200 ml, which is 200 0.02 ml.Figure 8 shows the result written AL

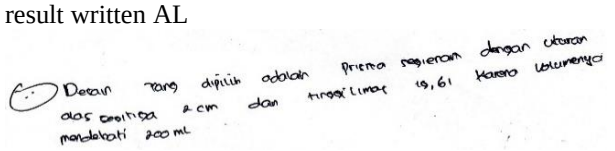

Figure 8 AL Written Results when Deciding on the Idea to Use

Based on the research data, researchers can see that there are 2 types of thinking from junior high school students. The characteristics of the two types are shown in Table 4.

Table 4 Characteristics of thinking

Thinking Type	Characteristics
Numerical thinking	The subject shows a series of geometric sizes without thinking about the logic of these sizes
Realistic thinking	The subject shows a series of sizes of shapes by thinking about their logic and the real form of the shapes

The characteristics of student thinking in Table 4 show their differences Students who numeric thinking tends to describe what he experienced, interpret the problem from memory and try to imitate the way that has been observed. He try to select the numbers for the height and area of the base of a geometrical volume of 200 ml. Therefore, according to [24], the thinking characteristics of these students are characteristics of thinking at the recall stage. In addition, he also uses his reasoning by using multiplication and addition operations. Therefore, according to [24], the thinking characteristics of these students are characteristics of thinking at the basic stage.

Students who realistic thinking tends to analyze the problem, determine the adequacy of the data to solve the problem and decide the need for additional data to solve problems and determine kevalidatan conclusion. This can be seen from the thinking of students who not only determine the size of a shape with a volume of 200 ml, but also think about the presence or absence of a shape with that size. Therefore, the thinking characteristics of these students according to [24] are characteristics of thinking at the creative stage.

Table 5 Comparative thinking

Stages of Thinking (Krulik,Rudnick, &Milou, 2003)	Thinking Type
Creative	*Realistic thinking*
Critical	
Basic	*Numerical thinking*
Recall	

The students' thinking types in table 2 can be compared with the Thinking Stages according to [24] as in Table 5

4. CONCLUSION

Thinking process of students numeric thinking at this stage is generate ideas it generates ideas that vary in solving mathematical problems. The idea that he generated in the packaging design was made in the form of shapes other than blocks, namely balls, tubes and prisms. He revealed that each of these shapes (except circles) can be made in various sizes, but the volume is the same, namely 200 ml. At the stage of clarifying ideas, he explained each idea that was generated by describing the size of each shape. At the stage of evaluating the reasonableness of the idea, he considers the accuracy of the size of each design. I a substituting each of these measurements into the formula for the volume of a space object. So it can classify designs whose volume is exactly 200 ml, designs whose volume is less than 200 ml and designs whose volume is more than 200 ml. He chose a tube-shaped design with a size of r = 3 cm, t = 7 cm, all designs are triangular prisms, pentagons and hexagons because the volume is exactly 200 ml.

The student's thinking process is realistic thinking, at the stage of generating ideas is that it generates various ideas that vary in solving mathematical problems. The idea he came up with in the packaging design was made in the form of a shape other than blocks but the volume was the same, namely 200 ml. At the stage of clarifying ideas, he explained each idea that was generated by describing the actual size and shape of each shape. At the stage of evaluating the reasonableness of the idea, he considers the accuracy of the size of each design. In addition it also determines the surface area of each shape. So it can classify designs that have an exact volume of 200 ml, designs that have a volume of less than 200 ml, designs that have a volume of more than 200 ml and designs that require minimal size packaging materials. He chose a design whose volume was close to 200 ml.

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