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On Ramsey (mK_2, P_4)-Minimal Graphs

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ABSTRACT

Let F, G, and H be simple graphs. The notation $F \to (G, H)$ means that any red-blue coloring of all edges of F will contain either a red copy of G or a blue copy of H. Graph F is a Ramsey (G, H)-minimal if $F \to (G, H)$ but for each $e \in E(F)$, $(F - e) \nrightarrow (G, H)$. The set $\mathcal{R}(G, H)$ consists of all Ramsey (G, H)-minimal graphs. Let mK_2 be matching with m edges and P_n be a path on n vertices. In this paper, we construct all disconnected Ramsey minimal graphs, and found some new connected graphs in $\mathcal{R}(3K_2, P_4)$. Furthermore, we also construct new Ramsey minimal graphs in $\mathcal{R}((m+1)K_2, P_4)$ from Ramsey minimal graphs in $\mathcal{R}(mK_2, P_4)$ for $m \geq 4$, by subdivision operation.

Keywords: *Matching, Path, Ramsey minimal graphs, Subdivision.*

1. INTRODUCTION

Let F, G, and H be simple graphs. The notation $F \to (G, H)$ means that in any red-blue coloring of F, there exists a red copy of G or a blue copy of H as a subgraph. A (G, H)-coloring of F is a red-blue coloring of F such that neither a red G nor a blue H occurs. A graph F is said to be a Ramsey (G, H)-minimal if $F \to (G, H)$ but for any $e \in E(F)$, there exists a (G, H)-coloring on graph $F - e$. The set of all Ramsey (G, H)-minimal graphs is denoted by $\mathcal{R}(G, H)$.

The determination and the characterization of all graphs F belonging to $\mathcal{R}(G, H)$ are the main problems in Ramsey (G, H)-minimal graphs. Some papers discuss the problem of determining all graphs in $\mathcal{R}(G, H)$. Burr et al. [1] proved that if H is any graph then $\mathcal{R}(mK_2, H)$ is a finite set. One of challenging problems in Ramsey Theory is to characterize all graphs in the set $\mathcal{R}(mK_2, H)$ for a given graph H.

Let K_n, C_n, and P_n be a complete graph, a cycle, and a path on n vertices, respectively. The characterization of Ramsey minimal graphs belonging to $\mathcal{R}(2K_2, K_4)$ can be seen in [2, 3]. The set $\mathcal{R}(2K_2, P_3)$ is determined by Mengersen and Oeckermann [4]. Mushi and Baskoro [5] determined all graphs in $\mathcal{R}(3K_2, P_3)$. Furthermore, the set $\mathcal{R}(4K_2, P_3)$ given by Wijaya et al. [6].

Wijaya et al. [7] showed that the cycle C_s belongs to $\mathcal{R}(mK_2, P_n)$ if and only if $s \in [mn - n + 1 \leq s \leq mn - 1]$. Recently Wijaya *et al*. [8] constructed a family of Ramsey (mK_2, P_4) minimal graphs from Ramsey $((m-1)K_2, P_4)$ minimal graph by doing 4 times subdivision on any edge belongs to a cycle in a Ramsey (mK_2, P_4)-minimal graph. Furthermore, Wijaya et al. [9] constructed a class of disconnected Ramsey (mK_2, H)-minimal graphs from a union of two or more connected graphs. Motivated by result in [9], in this paper, we focus on determining all disconnected graphs in $\mathcal{R}(3K_2, P_4)$, and found some connected graphs belonging to Ramsey $(3K_2, P_4)$-minimal. In addition, we also construct some graph in $\mathcal{R}((m+1)K_2, P_4)$ by doing subdivisions to graphs in $\mathcal{R}(mK_2, P_4)$ for m ≥ 4.

2. PRELIMINARIES

Let $G = (V, E)$ be graph. If $U \subseteq V$, then $G - U$ is a graph obtained from G by deleting vertices in U and all incident edges. If $H \subseteq G$, then $G - E(H)$ is a graph obtained from G by deleting edges in H. When $U = \{v\}$ and $E(H) = \{e\}$, for simplicity, we write $G - v$ and $G - e$, respectively.

Lemma 1 and 2 provide the necessary and sufficient conditions for any graph in $\mathcal{R}(3K_2, H)$ for any graph H.

Lemma 1. [9, 10] For any fixed graph H, the graph $F \longrightarrow (3K_2, H)$ holds if and only if the following four conditions are satisfied: (i) $F - \{u, v\} \supseteq H$ for each $u, v \in V(F)$, (ii) $F - u - E(K_3) \supseteq H$ for each $u \in V(F)$

and a triangle K_3 in F, (iii) $F - E(2K_3) \supseteq H$ for every two triangles in F, (iv) $F - E(S_5) \supseteq H$ for every induced subgraph with 5 vertices S in F.

Lemma 2. [9, 10] Let H be a simple graph. Suppose F is a Ramsey $(3K_2, H)$-graph. F is said to be minimal if for each $e \in E(F)$ satisfy $(F - e) \nrightarrow (3K_2, H)$, that is (i) $(F - e) - \{u, v\} \not\supseteq H$ for each $u, v \in V(F)$, ii) $F - u - E(K_3) \not\supseteq H$ for each $u \in V(F)$ and a triangle K_3 in F, (iii) $F - E(2K_3) \not\supseteq H$ for every two triangles in F, (iv) $F - E(S_5) \not\supseteq H$ for every induced subgraph with 5 vertices S in F.

Any graph satisfying all conditions in Lemma 1 and 2 is a Ramsey $(3K_2, H)$-minimal graph. The condition stated in Lemma 2 is called the **minimality property** of a graph in $\mathcal{R}(3K_2, H)$. In [10], Wijaya et al. defined $SF(e, t)$ as a t times subdivision of edge e in the connected graph F, and gave Theorem 3. Moreover, Baskoro and Yulianti [7] gave Theorem 4.

Theorem 3. Let F be a connected graph and $m \geq 2$ be an integer. Suppose α is one non-pendant edge of F. If $F \in \mathcal{R}(mK_2, P_4)$, then $SF(\alpha, 4) \in \mathcal{R}\big((m+1)K_2, P_4\big)$.

Theorem 4. [7] $\mathcal{R}(2K_2, P_4) = \{2P_4, C_7, C_6, C_5, C_4^+\}$, where C_4^+ is a C_4 with additional two pendant vertices as in Figure 1

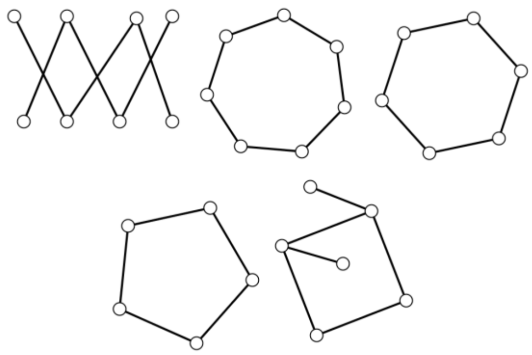

Figure 1 All graphs in $\mathcal{R}(2K_2, P_4)$.

3. MAIN RESULTS

3.1 Disconnected Graph in $\mathcal{R}(3K_2, P_4)$

In this section, we give all disconnected graphs belonging to $\mathcal{R}(3K_2, P_4)$.

Theorem 5. $G \cup P_4 \in \mathcal{R}(3K_2, P_4)$ if and only if $G \in \mathcal{R}(2K_2, P_4)$.

Proof. ($\Leftarrow$) We will show that for any $G \in \mathcal{R}(2K_2, P_4)$, then $G \cup P_4 \in \mathcal{R}(3K_2, P_4)$. Since $G \in \mathcal{R}(2K_2, P_4)$, then $G \rightarrow (2K_2, P_4)$ and $G - e \nrightarrow (2K_2, P_4)$ for any $e \in E(G)$. Since $G \rightarrow (2K_2, P_4)$, by coloring all edges incident to any vertex in G produces a blue copy of P_4 subset of G. Thus, any red coloring of two independent edges in G produces blue copy of P_4 subset of $G \cup P_4$. Moreover, any red coloring of one edge in G and one edge in P_4 produces a blue copy of P_4 subset of $G \cup P_4$. Hence, $G \cup P_4 \rightarrow (3K_2, P_4)$. Let $e_1 \in E(G)$ and $e_2 \in E(P_4)$. Since $G - e_1 \nrightarrow (2K_2, P_4)$, there exists a red-blue coloring of $G - e_1$ where a red K_2 occurs and blue P_4 cannot be found. Therefore, there exists a red-blue coloring on $G \cup P_4 - e_1$ where neither a red $3K_2$ nor a blue P_4 occurs. Moreover, any red coloring of two independent edges in $G \subset G \cup P_4 - e_2$ produces red-blue coloring of $G \cup P_4 - e_2$ where neither a red $3K_2$ nor a blue P_4 occurs. Hence, $G \cup P_4 - e \nrightarrow (3K_2, P_4)$. Since $G \cup P_4 \rightarrow (3K_2, P_4)$ and $G \cup P_4 - e \nrightarrow (3K_2, P_4)$ for any $e \in E(G)$, then $G \cup P_4 \in \mathcal{R}(3K_2, P_4)$.

($\Rightarrow$) If $G \cup P_4 \in \mathcal{R}(3K_2, P_4)$, then $G \in \mathcal{R}(2K_2, P_4)$. For a contradiction, suppose that $G \notin \mathcal{R}(2K_2, P_4)$. Then, we have two cases.

Case 1. Suppose $G \nrightarrow (2K_2, P_4)$. Then there exist a $(2K_2, P_4)$-coloring of G. Extend the coloring to color $G \cup P_4$ and color the edges of P_4 by red. Thus, there exist a $(3K_2, P_4)$-coloring of $G \cup P_4$, which contradicts the fact that $G \cup P_4 \in \mathcal{R}(3K_2, P_4)$.

Case 2. Suppose $G \rightarrow (2K_2, P_4)$, but G is not minimal. It means there exists a graph $H \in \mathcal{R}(2K_2, P_4)$ where $G \supset H$. Thus $G \cup P_4 \supset H \cup P_4$. Since $H \in \mathcal{R}(2K_2, P_4)$, then $H \cup P_4 \in \mathcal{R}(3K_2, P_4)$ by the first case, which contradicts to the minimality of $G \cup P_4$.

Therefore, from two cases above, we conclude that $G \cup P_4 \in \mathcal{R}(3K_2, P_4)$ if and only if $G \in \mathcal{R}(2K_2, P_4)$. ■

Theorem 6. Let H be a disconnected graph in $\mathcal{R}(3K_2, P_4)$. Therefore, one component of H must be isomorphic to P_4.

Proof. Suppose to the contrary that $H = H_1 \cup H_2$ and none of H_1 or H_2 is isomorphic to P_4. Since there is no component in H isomorphic to P_4, there is no component P_4 in either H_1 and H_2. Every vertex in H is in a connected subgraph containing a P_4. Then, both H_1 and H_2 contain P_4. Therefore, there will be edges $e_1 \in E(H_1)$ and $e_2 \in E(H_2)$ such that $P_4 \subseteq H_1 - e_1$ and $P_4 \subseteq H_2 - e_2$. Since $H \in \mathcal{R}(3K_2, P_4)$, there exist a $(3K_2, P_4)$-coloring of $H - e_1$ and $H - e_2$, say J_1 and J_2, respectively. Under J_1, $H_1 - e_1$ must contain at least one red edge and H_2 must have a $(2K_2, P_4)$-coloring. Since if it is not the case, $H - e_1$ would contain a red $3K_2$ or blue P_4, a contradiction to the minimality of H. Moreover, under J_2, $H_2 - e_2$ must contain at least one red edge and H_1 must have a $(2K_2, P_4)$-coloring. We conclude that we will obtain a $(3K_2, P_4)$-coloring of H if we color H by using J_1 on H_2 and J_2 on H_1, which contradicts to the minimality of H.

Therefore, if H is a disconnected graph in $\mathcal{R}(3K_2, P_4)$. Then, one component of H must be isomorphic to P_4. ■

Theorem 7. The graphs $C_5 \cup P_4$, $C_6 \cup P_4$, $C_7 \cup P_4$, $C_4^+ \cup P_4$ and $3P_4$ are the only disconnected graphs in $\mathcal{R}(3K_2, P_4)$.

Proof. Using Theorem 6, if F is a disconnected graph in $\mathcal{R}(3K_2, P_4)$, then F must have a component isomorphic to P_4. Furthermore, Theorem 5 states that the other component of F must be a member of the set $\mathcal{R}(2K_2, P_4)$. Moreover, Theorem 4 determined all graphs in $\mathcal{R}(2K_2, P_4)$. Therefore, the graphs $C_5 \cup P_4$, $C_6 \cup P_4$, $C_7 \cup P_4$, $C_4^+ \cup P_4$ and $3P_4$ are the only disconnected graphs in $\mathcal{R}(3K_2, P_4)$. ■

3.2 Some Connected Graphs in $\mathcal{R}(3K_2, P_4)$

In this section, we determine some connected graphs other than the cycle belonging to $\mathcal{R}(3K_2, P_4)$. First, we show that a graph F_1, depicted in Fig. 2, is a Ramsey $(3K_2, P_4)$-minimal graph.

Proposition 8. Let F_1 be a graph as depicted in Fig. 2. The graph F_1 is a Ramsey $(3K_2, P_4)$-minimal graph.

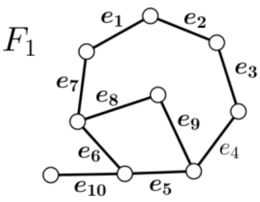

Figure 2 The graph $F_1 \in (3K_2, P_4)$.

Proof. First, we show that for any red-blue coloring of F_1 contains a red $3K_2$ or a blue P_4. We can see that $F_1 - \{u, v\}$ always contains a path P_4 for any $u, v \in V(F_1)$. It can be verified that $F_1 - E(S_5) \supseteq H$ for every induced subgraph with 5 vertices S in F_1. Since F_1 has no triangle, then by Lemma 1 we have that $F_1 \to (3K_2, P_4)$. Next, we prove the minimality property. For any edge e we will show that $(F_1 - e) \nrightarrow (3K_2, P_4)$. If e is one of the dashed edges in Fig. 3, then each red-blue coloring in Fig. 3 is the $(3K_2, P_4)$-coloring on $F_1 - e$. Therefore $F_1 \in \mathcal{R}(3K_2, P_4)$. ■

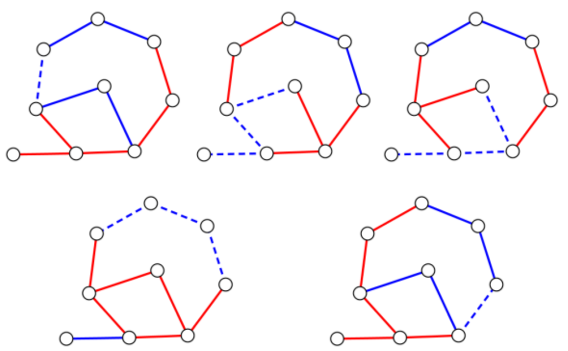

Figure 3 The $(3K_2, P_4)$-colorings on $F_1 - e$ if e is one of the dashed edges.

Suppose $V(C_n) = \{v_1, v_2, \dots, v_{n-1}, v_n\}$ is the vertex-set of C_n. We define a graph C_n^a as a graph obtained from C_n by adding a pendant vertex, say v_{n+1}, adjacent to v_a for $a \in [1, n]$. A graph $C_n^{a,b}$ is obtained from C_n by adding two pendant vertices, say v_{n+1} and v_{n+2}, adjacent to v_a and v_b, respectively, for $a, b \in [1, n]$. Moreover, following Wijaya et al. in [10], we define special graphs with certain circumference. Let a, b, c, d, e, f, g and h be eight integers. Graph $C_n[(a, b), (c, d)]$ is obtained from C_n by adding two new edges $v_a v_b$ and $v_c v_d$. Graph $C_n[(a, b), (c, d), (e, f)]$ is obtained from C_n by adding three new edges $v_a v_b$, $v_c v_d$, and $v_e v_f$. Graph $C_n[(a, b), (c, d), (e, f), (g, h)]$ is obtained from C_n by adding four new edges $v_a v_b$, $v_c v_d$, $v_e v_f$, and $v_g v_h$. Now, consider graphs $C_6[(1,4), (2,5), (2,6), (3,5)]$, $C_7^5[(1,3), (2,6), (5,7)]$ $C_7[(1,5), (3,7)]$, $C_7^7[(2,6), (3,7)]$, $C_8[(2,7), (4,7), (6,8)]$ $C_6^{3,4}[(1,4), (3,6)]$ as depicted in Fig. 4. We will show that those graphs are Ramsey $(3K_2, P_4)$-minimal.

Theorem 9. All graphs in Fig. 4 are Ramsey $(3K_2, P_4)$-minimal graphs.

Proof. Let F be any graph in Fig. 4. It is easy to see that F satisfies all the conditions in Lemma 1. Then, $F \longrightarrow (3K_2, P_4)$ holds. Now, we will show the minimality property of F. Let e be any edge in F. If e is one of the dashed edges, then a $(3K_2, P_4)$-coloring on $F - e$ is provided in Figures 5, 6, 7. 8, 9 and 10 respectively for all cases. ■

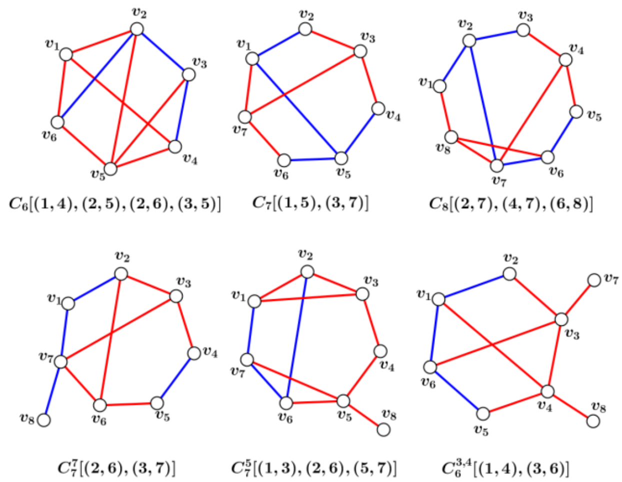

Figure 4 Six non-isomorphic graphs belonging to $\mathcal{R}(3K_2, P_4)$ which is obtained from C_n with some cords or pendant vertices or combination both.

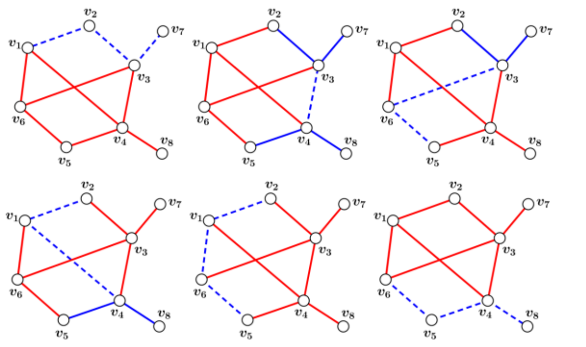

Figure 5 The $(3K_2, P_4)$-colorings on $C_6^{3,4}[(1,4),(3,6)] - e$ if e is one of the dashed edges.

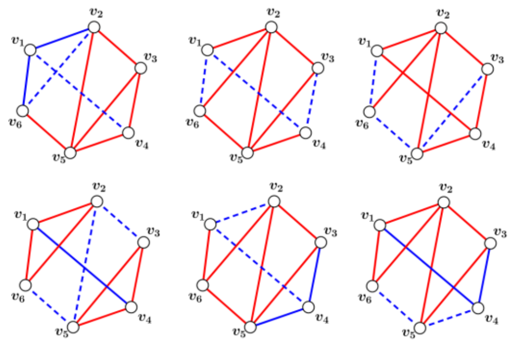

Figure 6 The $(3K_2, P_4)$ -colorings on $C_6[(1,4),(2,5),(2,6),(3,5)] - e$ if e is one of the dashed edges.

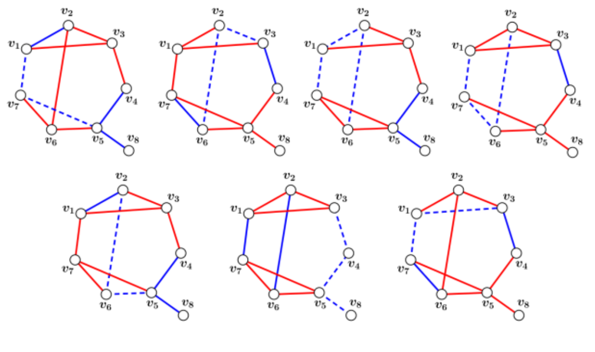

Figure 7 The $(3K_2, P_4)$ -colorings on $C_7^5[(1,3),(2,6),(5,7)] - e$ if e is one of the dashed edges.

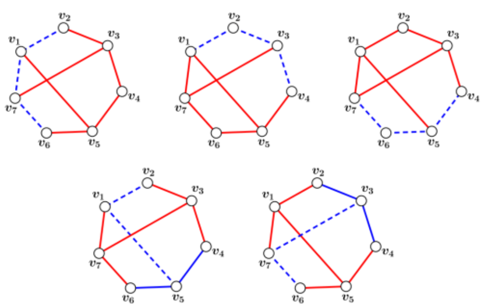

Figure 8 The $(3K_2, P_4)$ -colorings on $C_7[(1,5),(3,7)] - e$ if e is one of the dashed edges.

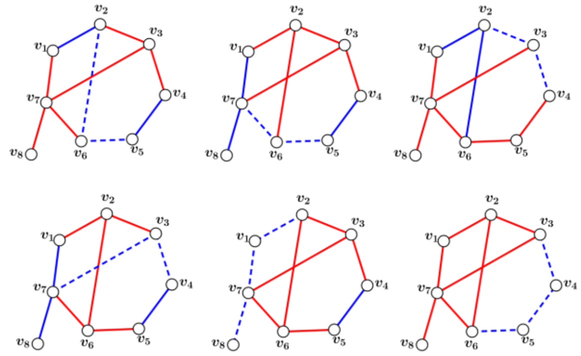

Figure 9 The $(3K_2, P_4)$ -colorings on $C_7^7[(2,6),(3,7)] - e$ if e is one of the dashed edges.

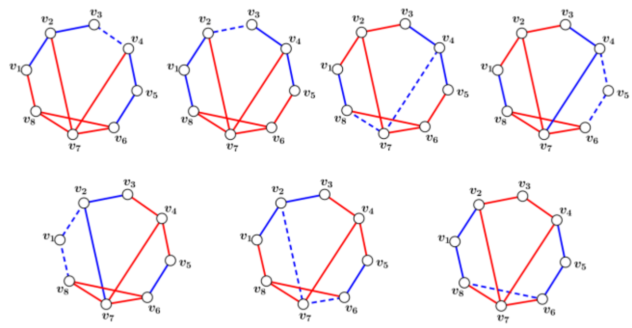

Figure 10 The $(3K_2, P_4)$ -colorings on $C_8[(2,7),(4,7),(6,8)] - e$ if e is one of the dashed edges.

3.3 Some New Family of Ramsey (mK_2, P_4)-Minimal Graphs

Recall that $SF(e,t)$ is a subdivision t times of edge e. In the previous section, it has been shown that $F_1 \in \mathcal{R}(3K_2, P_4)$. According to Theorem 3, if we subdivide (4 times) any non-pendant edge of F_1, then we obtain three non-isomorphism graphs belonging to $\mathcal{R}(4K_2, P_4)$, namely $SF_1(e_1, 4)$, $SF_1(e_5, 4)$, and $SF_1(e_8, 4)$ as depicted in Fig.11 (4 vertices, green vertex). The proof of the minimality of a graph $SF_1(e_5, 4)$ can be seen in Fig.12, while the minimality of the other graphs can be represented in the same way.

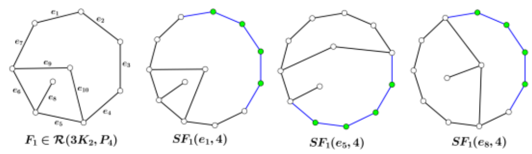

Figure 11 Three non-isomorphism graphs belonging to $(3K_2, P_4)$ are obtained by subdividing four times (4 green vertices) a non-pendant edge of F_1.

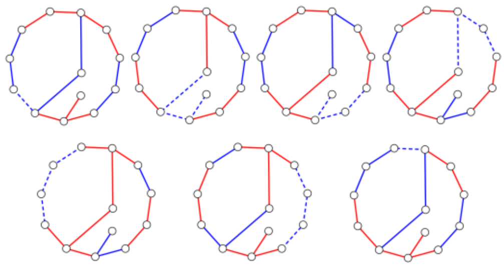

Figure 12 The $(4K_2, P_4)$-colorings on $SF_1(e_5, 4) - e$ if e is one of the dashed edges.

Now, we consider graph $C_7[(1,5),(3,7)]$. Since every edge in $C_7[(1,5),(3,7)]$ is non-pendant, then according to Theorem 3, the subdivision (4 times) on any edge of $C_7[(1,5),(3,7)]$ will produce three non-isomorphism graphs in $\mathcal{R}(4K_2, P_4)$. By repeating this process for the resulting graphs, we obtain Corollary 10.

Corollary 10. Let $m \geq 4$ be an integer. Then, the graphs $C_{4m-5}[(1,5),(3,7)]$, $C_{4m-5}[(1,4m-7),(4m-9,4m-5)]$, and $C_{4m-5}[(1,4m-7),(3,4m-5)]$ are in $\mathcal{R}(mK_2, P_4)$.

Proof. Let $\{v_1, v_2, \ldots, v_7\}$ be the vertex-set of $C_7[(1,5),(3,7)]$. The subdivision (4 vertices) on the edge $e = v_1v_2$ will result $C_{11}[(1,9),(7,11)]$. Since $C_7[(1,5),(3,7) \in \mathcal{R}(3K_2, P_4)$, then by Theorem 3, we have that $C_{11}[(1,9),(7,11)] \in \mathcal{R}(4K_2, P_4)$. Furthermore, by subdividing (4 vertices) the edge $e = v_1v_2$ of $C_{11}[(1,9),(7,11)]$, we obtain $C_{15}[(1,13),(11,15)]$. By Theorem 3, we have that $C_{15}[(1,13),(11,15)] \in \mathcal{R}(5K_2, P_4)$. By continuing this process and applying it to the resulting graph, then we obtain the graph $C_{4m-5}[(1,4m-7),(4m-9,4m-5)]$. By Theorem 3, $C_{4m-5}[(1,4m-7),(4m-9,4m-5)] \in \mathcal{R}(mK_2, P_4)$. Next, by subdivision (4 vertices) on the edge $e = v_3v_4$ of the graph $C_7[(1,5),(3,7)]$, repeatedly, and apply Theorem 3, we obtain $C_{4m-5}[(1,4m-7),(3,4m-5)] \in \mathcal{R}(mK_2, P_4)$. By doing the same way to the edge $e = v_7v_1$, we obtain $C_{4m-5}[(1,5),(3,7)] \in \mathcal{R}(mK_2, P_4)$. ■

In the same way, we can construct some other graphs in $\mathcal{R}(mK_2, P_4)$ from some graph in $\mathcal{R}(3K_2, P_4)$, namely, $C_6^{3,4}[(1,4),(3,6)]$, $C_6[(1,4),(2,5),(2,6),(3,5)]$, $C_7^5[(1,3),(2,6),(5,7)]$, $C_7^7[(2,6),(3,7)]$, and $C_8[(2,7),(4,7),(6,8)]$. Therefore, we have Corollary 11.

Corollary 11. Let $m \geq 4$ be an integer. Then the following 19 graphs are in $\mathcal{R}(mK_2, P_4)$.

1. $C_{4m-6}^{3,4}[(1,4),(3,6)]$,
2. $C_{4m-6}^{4m-9,4m-8}[(1,4m-8),(4m-9,4m-6)]$,
3. $C_{4m-6}^{3,4m-8}[(1,4m-8),(3,4m-6)]$,
4. $C_{4m-6}[(1,4m-8),(4m-10,4m-7),$
   $(4m-10,4m-6),(4m-9,4m-7)]$,
5. $C_{4m-6}[(1,4m-8),(2,4m-7),(2,4m-6),$
   $(4m-9,4m-7)]$,
6. $C_{4m-6}[(1,4),(2,5),(2,6),(3,5)]$,
7. $C_{4m-5}^7[(2,6),(3,7)]$,
8. $C_{4m-5}^{4m-5}[(2,4m-6),(4m-9,4m-5)]$,
9. $C_{4m-5}^{4m-5}[(2,4m-6),(3,4m-5)]$,
10. $C_{4m-5}^{4m-5}[(2,6),(3,4m-5)]$
11. $C_{4m-5}^5[(1,3),(2,6),(5,7)]$,
12. $C_{4m-5}^{4m-7}[(1,4m-9),(4m-10,4m-6),(4m-7,4m-5)]$,
13. $C_{4m-5}^{4m-7}[(1,4m-9),(2,4m-6),(4m-7,4m-5)]$,
14. $C_{4m-5}^{4m-7}[(1,3),(2,4m-6),(4m-7,4m-5)]$,
15. $C_{4m-5}^5[(1,3),(2,4m-6),(5,4m-5)]$,
16. $C_{4m-5}^5[(1,3),(2,6),(5,4m-5)]$,
17. $C_{4(m-1)}[(2,7),(4,7),(6,8)]$,
18. $C_{4(m-1)}[(2,4m-5),(4m-8,4m-5),$
    $(4m-6,4(m-1))]$,
19. $C_{4(m-1)}[(2,7),(4,7),(6,4(m-1))]$.

4. CONCLUSION

In this paper, we discuss the construction of a disconnected Ramsey minimal graph in $\mathcal{R}(3K_2, P_4)$ from Ramsey minimal graph ini $\mathcal{R}(2K_2, P_4)$. We show that all disconnected graphs in $\mathcal{R}(3K_2, P_4)$ are $C_5 \cup P_4$, $C_6 \cup P_4$, $C_7 \cup P_4$, $C_4^+ \cup P_4$, and $3P_4$. In addition, we give some connected graphs in $\mathcal{R}(3K_2, P_4)$, namely, F_1, $C_6[(1,4),(2,5),(2,6),(3,5)]$, $C_7^5[(1,3),(2,6),(5,7)]$ $C_7[(1,5),(3,7)]$, $C_7^7[(2,6),(3,7)]$, $C_8[(2,7),(4,7),(6,8)]$ $C_6^{3,4}[(1,4),(3,6)]$ as depicted in Fig. 4. Furthermore, we also construct nineteen new families of Ramsey (mK_2, P_4) minimal graphs for $m \geq 4$.

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AUTHORS' CONTRIBUTIONS

Asep Iqbal Taufik: Conceived and designed experiments; Conducted experiments; Wrote the paper - original draft preparation.

Denny Riama Silaban: Supervision and validation; Wrote the paper - review and editing.

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