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The Application of Deferred Acceptance Mechanism in Senior High-school Admission

Qing Wang[1,*]

[1] *The College of Arts & Sciences, Boston University, Boston, Massachusetts 02215-1401, the United States*
[*] *Corresponding author. Email: wangqing@bu.edu*

ABSTRACT
In Zhucheng, Shandong Province, China, the existing mechanism in remaining admission to allocate students to public senior high-school requires each student to randomly take a place and students can exchange their places if they are unsatisfied with their random places. But exchanges must satisfy some conditions, such as requirements of both students' scores. It is hard to tell counterparties. To optimize the existing mechanism, this paper suggests modifying it by gathering preferences of all students who are unsatisfied with their random places first, treating students with different scores as different types, and then applying deferred acceptance to reallocating students by types. This research proves that this new mechanism is feasible. In this environment, DA (deferred acceptance) is a very successful mechanism, because it ensures the rational use of educational resources, individual rationality, elimination of justified envy, and strategy proof. Moreover, it can maintain the results of score diversification realized by existing mechanism, that students with different scores are randomly distributed to 4 public senior high-schools.

Keywords: *Market design, Deferred acceptance, School choice, Strategy proofness*

1. INTRODUCTION

The objective of this study is to apply a deferred acceptance mechanism to reallocate students to public senior high schools in Zhucheng, Shandong Province, China. In this market, the two sides are students and public senior high-schools. The existing mechanism includes the following steps that students take exams, each student randomly takes a public senior high school by computer program, and students who are unsatisfied with random places can exchange their places by themselves under some conditions. Although the existing mechanism has some advantages, such as realizing diversification by students randomly taking schools, it also leads to huge costs and difficulties by requiring students to exchange places by themselves. This paper aims to modify the existing mechanism by deferred acceptance in this situation while maintaining the good outcome as possible.

2. RELATED LITERATURES

For deferred acceptance, the paper of Gale and Shapley [4] first introduced the deferred acceptance mechanism, which is now very popular. And it is well known that DA satisfies non-wastefulness, elimination of justified envy, and strategy proofness [1].

For diversification, Kurata et al. [7] were the first to consider the setting in which each student has multiple types and each school imposes soft quotas on each type. Aygun and Turhan [2] focused on a model in which students are allowed to have multiple types and colleges divide the seats into groups. Fragiadakis and Troyan [3] designed dynamic quota mechanisms with fairness and incentive guarantees to discuss controlled school choice models and study the stability, efficiency, and strategic issues.

3. BACKGROUND

3.1 General Background

In Zhucheng, Shandong, China, about 10,000 students apply for public senior high-schools every year, and about 7,500 students will be admitted. Meanwhile, there are 4 public senior-high schools in this town.

According to the policy of public senior high-school applying and admitting in Zhucheng, there are two broad categories of admission: professional admission and general admission. Professional admission is for students who apply for music, art, and sports majors. In general admission, There are three types of admissions. First, recommended students' admission is for very good students who recommended by their junior high schools. Recommended students must be admitted by the school which they prefer most. In the admission plan in 2021, the percentage of students who admitted by recommended students' admission is 2.77%. Second, remaining admission is for students who take senior high-schools entrance examination to apply for public senior high-schools. In the admission plan in 2021, the percentage of students who admitted by recommended students' admission is 59.23%. Third, remaining general admission is for students who will be admitted if there are extra after recommended students' admission, remaining admission and professional admission. In the admission plan in 2021, the percentage of students who admitted by recommended students' admission is 32.95%. ("Figure 1")

Figure 1 The admission plan of 4 public senior high-schools in Zhucheng in 2021.

3.2 Current Allocating Mechanism

The current mechanism to allocate students has 4 steps. Step 1 is that students take one senior high-school entrance examination. The examination contains about 15 subjects, such as math, history, and chemistry. Step 2 is that each student gets a final score. In step 1, each student gets a grade, A, B, C, D or E in each subject, and each student get a final score according to her grades of subjects. Step 3 is that each student randomly takes one school. Students use computer program to take school randomly. This is a public process. Step 4 is that students, who are unsatisfied with the random school, can exchange their places with others under following conditions. Suppose student S_i takes school H_{s_i}, and student S_j takes school H_{s_j} in step 3. If S_i prefers H_{s_j} to H_{s_i}, and S_j prefers H_{s_i} to H_{s_j}, and S_i' final score equals to S_j's final, and both students must meet the minimum standards for exchanging. Then S_i and S_j can exchange their school by themselves.

The current mechanism has some advantages. First, it is good for schools, because it is easy to implement for schools. Second, schools can realize diversification easily because those students take schools randomly. And school's most concerned thing, the number of students with different scores will be constant in this mechanism.

Therefore, the current mechanism can alleviate the following problem. In the past few years, some measures were taken to balance educational resources between 4 schools, e.g., rotating the

headmaster or teachers between 4 public senior-high schools. However, from most students' and parents' perspectives, differences always exist. Hence, the convergence of students' preferences is common. If schools admit students only by grades, because most students prefer $H1$ and $H2$ to $H3$ and $H4$, then the quality of the students admitted by $H1$ and $H2$ will be better than $H3$ and $H4$. That will lead to larger differences between college entrance examination results of 4 schools. Thus, students' preferences will tend to greater convergence. Then, the outcome of education will show huge gap between 4 public senior high-schools. It is intuitive that students in $H3$ and $H4$ and students living near to $H3$ and $H4$ who is plan to apply for senior high-schools will be concerned about that unfairly allocating of educational resources. However, the existing mechanism is good because that it alleviates this problem, because students take schools randomly, so students with different scores will be evenly distributed in 4 schools. And more important, randomly taking schools make sure the mechanism is fair.

However, from the perspective of students who are unsatisfied, it is difficult to successfully exchange places in step 4. Firstly, it is hard to tell their counterparties in thousands of students. There are $A_4^2 = 12$ kinds of students who want to change their schools. What's more, more than 10,000 students apply for senior high-school every year. It's very difficult for students to find a partner and both of them prefer each other's school than themselves and their final scores are exactly equal. Secondly, students don't know who can math with them, unless they ask thousands of students one by one. Compared to a platform for all students to exchange schools, changing by themselves can waste a lot of efforts.

4. MODEL

4.1 Assumptions

The followings are some assumptions to clarify the studying subjects. For the side of students, only focus on students applying for senior-high school by remaining admission. For the sides of schools, only focus on 4 public senior-high schools. Each student has a strict preference over schools and her/his outside option. Each student only care about her/his own school. Schools don't have preference. Schools have capacity constraints. Classify students according to students' scores, and students with the same score belong to one type. All students find all schools acceptable in randomly taking schools, and all students find their random schools and schools which they prefer to their random school acceptable in reallocating steps.

4.2 Deferred Acceptance

There is a finite set of students $S = \{s_1, \dots, s_n\}$ and a set of 4 public senior-high schools to which they can be assigned, $H = \{h_1, h_2, h_3, h_4\}$.

Treating students with different scores as different types. There is a finite set of types of students $T = \{t_1, \cdots, t_m\}$.

Each school has a capacity of each type of students Q_{h_i,t_j}.

$h_r(s) = random\ high\ school\ of\ student\ s\ in\ step\ 3$.

$t(s) = the\ type\ of\ student\ s$.

$Q_{h_i,t_j} = the\ number\ of\ students\ who\ satisfy\ following\ conditions, [t(s) = t_j] \cap [h_r(s) = h_i] \cap [s\ is\ not\ satisfied\ with\ h_r(s)]$.

Given that the existing mechanism can realize diversification by students randomly taking schools using computer program, and existing mechanism sets exchange requirements according to scores, the new mechanism try to make changes while maintaining this result. This paper suggests splitting step 4 into two steps.

Compared to students exchanging by themselves, establishing a platform that gathers the preferences of students who are unsatisfied and reallocates them is much better. Hence, it is better to divide the fourth step into two steps. $Step\ 4.1$ is that for students who are unsatisfied with their random school, gather their preferences of public senior-high schools which have higher ranks than their random school on their preference list. $Step\ 4.2$ is that treat students with different scores as different types, and use deferred acceptance (DA) to reallocate students to 4 schools by types. Student s_i will not be reallocated to a school which has a lower rank than H_{s_i} in their own preferences in this step.

In step 4.1, the number of students who are unsatisfied will be smaller than students taking the examination in step 1 because some students will be satisfied with $h_r(s)$. Moreover, there are only 4 public senior-high schools. Hence, it is feasible to get all students' preferences.

In this new mechanism, each school reserves Q_{h_i,t_j} seats exclusively for students of type t_j; there are no remaining seats. Moreover, the numbers of students of different types after DA equal to the numbers of before DA in each school. Hence, this new mechanism can maintain the results achieved by existing mechanism, that students with different scores are randomly distributed to 4 public senior high-schools. This leads to the problem similar to capacity constraints, but there are some differences, because the capacity constrains is also the hard lower bound. Therefore, in step 4.2, the problems include familiar constrained assignment problems, such as job assignments under regional "ceiling" and "floor" quotas [6], and controlled school choice due to considerations of gender or demographic balance [8]. But in this environment, the "ceiling" and "floor" are equal. When agents have cardinal preferences and institutions do not have priorities, market-based mechanisms, such as the mechanism of Hylland and Zeckhauser [5] are desirable. In this environment, regarding students of one type, schools have no preferences, but this new mechanism set an artificial priority of time, which gives higher priority to students who apply to this school earlier to others who apply later.

$Step\ 4.2$ is the implement of DA. It contains following steps:

$Step\ 4.2.1$ is that for students who are unsatisfied with $h_r(s)$, each one applies to the fist school on her preference list. Each school consider all students who have applied to it, and accepts students as follows: for each type t, school h accepts $Q_{h,t}$. For each school h, if the number of t type students applying school h larger than $Q_{h,t}$, set a priority relation $\succ_h$. Students who applying earlier take higher priority in $\succ_h$ (i.e., if there only one seat remaining for school h of type t, and s_1 and s_2 both apply for h, and s_1 apply earlier than s_2, h will admit s_1 and reject s_2).

$Step\ 4.2.k$ is that for each student who was rejected in step $4.2.k-1$ applies to her most preferred school that has not yet rejected her. Each school considers its new applicants in step k.

It is well known that DA satisfies non-wastefulness, elimination of justified envy, and strategy proofness [1][2][3][4]. And in this environment, DA is a very successful mechanism because it is strategy proof, non-wasteful, eliminates justified envy, and individual rational.

DA is strategy proof in this case. No student can ever gain by misreporting her preferences, no matter what the other students report. Moreover, students have the incentives to report their preferences as early as they can, which will speed up students' preferences gathering.

DA is non-wasteful in this case. Whenever a student prefers a school h to her current assignment after DA, it is impossible to move her to h without violating feasibility.

DA eliminates justified envy in this case. Student s justifiably envies s' if she prefers the school of student s', has higher priority than s' at this school, and s and s' can be reassigned without violating any distributional constraints (and without altering the allocation of any other student).

DA is individual rational in this case. DA is individual rational because we assume that all students find all schools acceptable.

However, this new mechanism also has some shortcomings. For example, the new mechanism gives students priority according to how earlier they apply, so this may lead to students rush to apply, which asking platform has good system maintenance. And the maintenance of the platform also requires capital investment. Moreover, this step also needs students to be equipped with good quality of network and equipment to make sure they can express their preference as soon as they make decisions.

5. CONCLUSION

The current mechanism of public senior high schools in Zhucheng, Shandong Province, China alleviated the problem of the distribution of educational resources. However, during the step 4 of the mechanism, students are required to exchange places under conditions with a high cost. This paper suggests splitting the step of exchanging by students into the following steps: gathering students' preferences, treating students with

different scores as different types and applying deferred acceptance to reallocate students by types. It is feasible because collecting students' preference is realizable, and deferred acceptance is a successful mechanism in reallocating students to 4 public senior high-schools. In this environment, DA is a very successful mechanism because it is non-wasteful, individual rational, eliminates justified envy, and strategy proof. Moreover, this mechanism can maintain the results realized by existing mechanism that students are randomly distributed between 4 public senior high-schools according to their scores in senior high-school entrance examination, and the number of students with different scores will also be constant in the new mechanism.

AUTHORS' CONTRIBUTIONS

This paper is independently completed by Qing Wang.

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