

Improving the Dynamic Characteristics of an Elastic Electromechanical System

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ABSTRACT

The paper presents a mathematical model of an electromechanical system in the MATLAB environment. The sensitivity of the system to changes in the parameters of the executive body and the transmission of elastic loads is studied. The question of the influence of the disturbing influence on the system is considered.

Keywords: *Optimal regulator, Elastic object, Electromechanical system.*

1. INTRODUCTION

The analysis of any automated system begins with a mathematical description of the operation of all links and mechanisms.

The mathematical model must meet several criteria:

1. Take into consideration the behaviour of an electric machine as a control object;
2. Just describe dynamic processes;
3. Allow synthesizing the obtained control laws.

When describing complex electromechanical systems, the principle of simplification is used, but the main rule must be observed: correspondence of theoretical calculations and practical results.

One of the methods that can significantly simplify the process of building a complex system is parameter normalization.

This method makes it possible to reduce the number of coefficients in the mathematical description of complex electromechanical systems and bring all the parameters of the model to the reference one. Such models are performed in relative units.

In addition to simplifying the model, this method allows you to create schemes that can later be used for studying by various methods (for example, frequency).

2. TWO-MASS ELECTROMECHANICAL SYSTEM WITH ELASTICITY

For approximate calculations of complex electromechanical automated control systems, all mechanical connections of moving elements are represented as absolutely rigid.

However, this representation does not allow us to fully assess the movement of elastic elements and their impact on the overall performance of the system.

A real projected automated object is a system with distributed parameters. The introduced simplifications allow us to consider complex mechanisms with lumped parameters.

However, for the correct formation of the system, the following criteria must be taken into consideration:

1. All forces and moments applied to concentrated masses;
2. Elastic elements are represented by links with constant coefficients of proportionality (between deformation and moment).

Figure 1 shows the most common way to simplify a complex system. In this case, the motor armature and executive body are characterized as a two-mass system with elastic transmission.

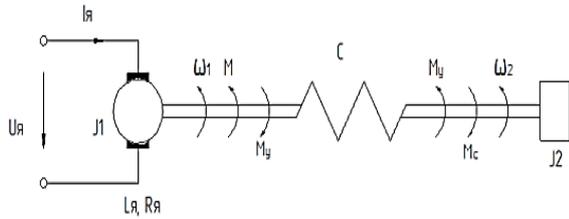


Figure 1 Functional diagram of a two-mass electromechanical system with elastic coupling.

where $J1$ is the moment of inertia of the motor and gearbox; $\omega1$ is the speed of rotation of the motor armature; M – engine torque; M_y – moment of elastic forces; C – coefficient of rigidity; M_c – moment of load resistance; $\omega2$ – speed of rotation of the executive body; $J2$ – moment of inertia of the executive body.

The elastic transmission, under the action of the torque applied to the motor armature and the load resistance moment, is deflected by a certain angle.

At the values of the operating moments, oscillations arise in the system, which are damped.

Vibration damping can occur under the action of internal frictional forces in elastic gears. The main condition for this process is to turn off the engine.

Using a method that simplifies the process of forming a mathematical model, we can conclude that the forces that describe the difference in the rotation frequencies of the masses (first and second) strictly proportional. In this case, the external viscous friction is assumed to be zero.

3. CONSTRUCTION OF AN OPTIMAL REGULATOR FOR AN ELASTIC ELECTROMECHANICAL SYSTEM

To describe the perturbed motion of a dynamical system, we use differential equation (1) in the first approximation:

$$\dot{x} = A \cdot x + B \cdot u \tag{1}$$

where A and B are matrices of numbers of size $n \times n$ and $n \times n$ m , respectively; x is a vector of state variables; u -vector and iteffects.

The initial time point is assumed to be zero: $t_0 = 0$

To determine the state of the control object, it is necessary to design an optimal controller.

For the correct construction of the regulator, it is necessary to find the matrices of numbers K (2):

$$u = K \cdot x \tag{2}$$

The finding condition is as follows: the matrix K must be of the form such that for asymptotically stable motions of the system that occur at arbitrary deviations, $x^{(0)}$ the functional (3) is minimized:

$$J = \int_0^\infty (x' Q x + u' u) dt \tag{3}$$

where Q is an arbitrary positive-definite matrix of size $n \times n$.

Matrix K is characterized as a set of gain coefficients of regulators.

Then, the control for constructing regulators will be (4):

$$P \cdot A + A' \cdot P - P \cdot B \cdot R^{-1} \cdot B' \cdot P + Q = 0 \tag{4}$$

Equation (4) is a matrix equation and is called the algebraic Riccati equation or the Lurie equation.

Based on the above matrix equations, there are several stages in the procedure for designing regulators:

1. Solution of the matrix Lurie equation (algebraic Riccati equation);
2. Expression from solutions of only a positive-definite matrix P^0 ;
3. Calculation of the gain matrix (regulator).

Steps 2 and 3 can be performed if the control object is fully controllable. If, and only if, there is a unique solution with a positive-definite matrix P^0 .

Complete object management is written as a condition:

$$rank[B, A \cdot B, A^2 \cdot B, \dots, A^{(n-1)} \cdot B] = n \tag{5}$$

4. SYNTHESIZING AN OBSERVING DEVICE

Equation (4), which characterizes the control of an object, is called the controllability condition for A and B .

State variables are usually calculated, but this is not always the case. Some good reasons for not being able to define variables are:

1. Sensors that measure the corresponding values;
2. Large error of measuring instruments;
3. Difficult implementation of sensors in the designed system.

Recently, the most common analog of measuring devices is the use of system state observers.

The observer evaluates the system at the initial time by the value, *of the output vectors* $(t)yt$. In this case, the condition that $t \gg t_0$ must be met.

One of the indicators of the observer's work is the system's recoverability. This is the ability to analyze the system by estimating $x(t_0)$ from the values $y(t) \gg t_0$.

The observability criterion is met for system (6) according to its definition.

$$\dot{x} = A \cdot x, \quad y = C \cdot x \tag{6}$$

The observability criterion is determined from the condition that the rank of the observability matrix (7) is equal to the order of the system:

$$Q = (C^T | A^T \cdot C^T | (A^T)^2 \cdot C^T | \dots | (A^T)^{n-1} \cdot C^T) \tag{7}$$

The observability depends on the matrices A and C.

In the system, the recovery device will be formed by Equations (8):

$$\hat{\dot{x}} = A \cdot \hat{x} + B \cdot u; \quad \hat{x}(t_0) = \hat{x}^0 \tag{8}$$

If $\hat{x}(t_0) = x(t_0) = (t_0)_{x(t_0)}$, then the solution of equation (6) exactly coincides with the solution of the system. If the metrics are not equal to each other ($\hat{x}(t_0) \neq x(t_0)$), then a recovery error (9) occurs:

$$e = x - \hat{x} \tag{9}$$

For an asymptotically stable design object, it is possible to reduce (9) for a certain amount of time.

The observation is described by equation (10), where the reconstructed value of $C \times \hat{x}$ is used.

$$\dot{\hat{x}} = A \cdot \hat{x} + H \cdot (y - C \cdot \hat{x}) + B \cdot u \tag{10}$$

where H is the matrix of observer gain coefficients.

Then the recovery error will look like (11):

$$\dot{e} = [A - H \cdot C] \cdot e \tag{11}$$

If the matrices of the equation are constant, then the values of the matrix H must be chosen so that the poles of the observer are located to the left of the poles of the optimal controller.

This condition is necessary for the end of transients first in the observer, and then in the system under study itself.

Also, do not make a strong bias, so as not to increase the sensitivity of the observer and not disrupt the study.

5. OPTIMAL SYSTEM SYNTHESIS

The resulting block diagram in the application package is shown in Figure 4.

The system synthesis is based on the influence of the parameters of the executive body and elastic elements (transmission) on the transition process.

Figure 2 shows the variation of the parameter 2J₂.

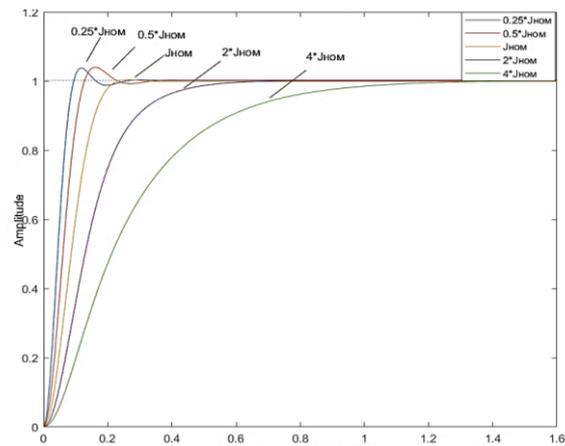


Figure 2 Variation of the parameter J₂.

Overshoot occurs when J₂ (moment of inertia of the second mass) decreases, which leads to an increase in the overall performance of the system.

If the J₂ parameter is increased, then the performance decreases, but the system also remains functional (the type of transition process does not change significantly).

The Table 1 show the numerical values of the variable parameters.

Table 1. Variation of the studied parameters

	t _{reg} , with	σ, %
0.25*J ₂	0.0842	3.71
0.5*J ₂	0.114	3.99
J ₂	0.181	0.373
2*J ₂	0.365	0
4*J ₂	0.745	0

Figure 3 shows a study of the effect of changes in the internal friction coefficient of an elastic transmission on the system.

When the parameter Kc decreases, fluctuations occur in the system.

With an increase in the coefficient of internal friction, there is a slight increase in the control time.

The figure shows the main changes in the simulated system.

In Figure 3 shows the variation of the Kc parameter.

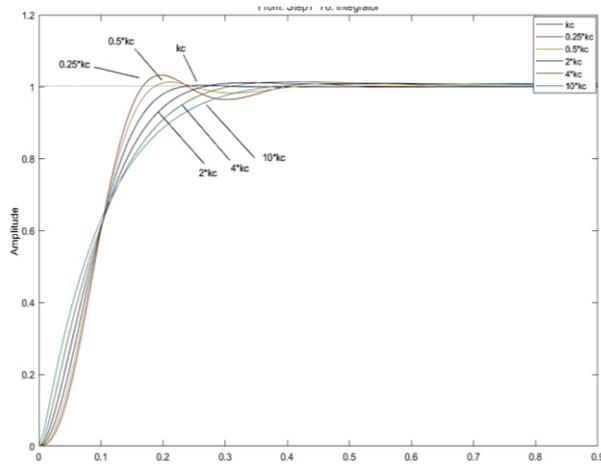


Figure 3 Variation of the K_c parameter.

The figure shows the main changes in the simulated system.

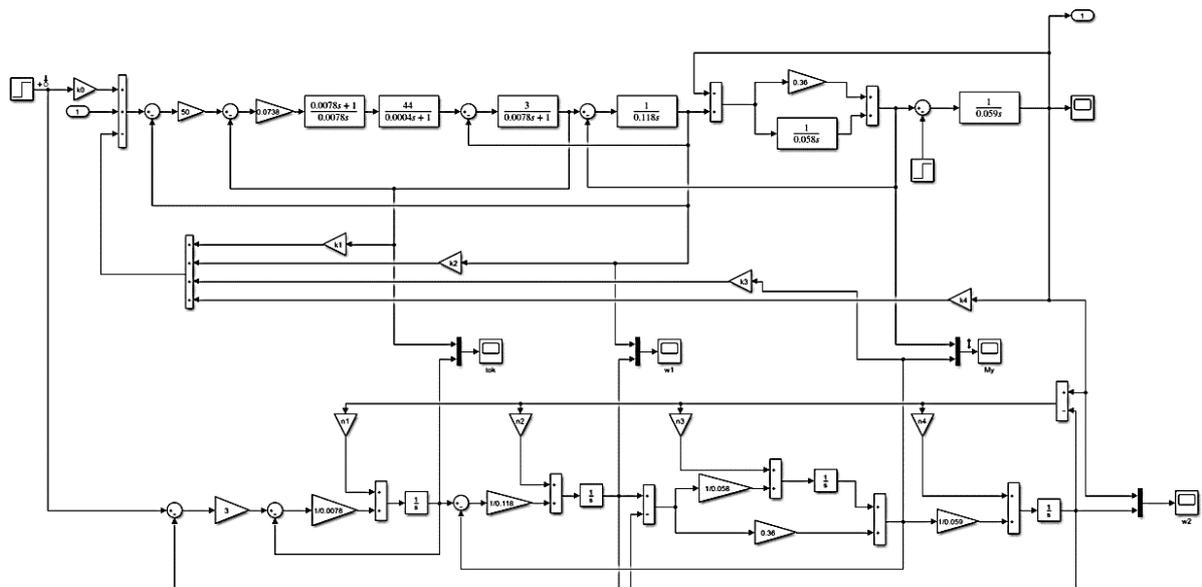


Figure 4 The resulting structural scheme in the MATLAB package.

Figure 5 shows the variation of the parameter T_c .

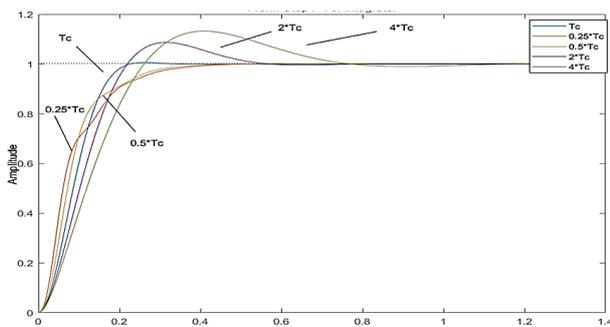


Figure 5 Varying the T_c parameter.

Based on the results of the study, it can be concluded that the type of transition process does not change significantly in the range of values of T_c [0.02;0.06] s.

Table 2 shows the numerical values of the variable parameters.

Table 2. Variation of the studied parameters

	$t_{per,c}$	$\sigma, \%$
$0,25 * K_c$	0,152	3,35
$0,5 * K_c$	0,162	1,37
K_c	0,181	0,373
$2 * K_c$	0,207	1
$4 * K_c$	0,233	1,22

Figure 5 shows a study of the effect of changing the time constant T_c .

The elastic coefficient is inversely proportional to the time constant T_c , so an increase in the value of T_c corresponds to a decrease in the value of the elastic coefficient.

With further increase in the value of T_c , overshoot appears, and then fluctuations.

Table 3 shows the numerical values of the variable parameters.

Table 3. Variation of the studied parameters

	$t_{reg, with}$	$\sigma, \%$
$0.25 * T_{with}$	0.263	0
$0.5 * T_{with}$	0.248	0
T_{with}	0.181	0.373
$2 * T_{with}$	0.413	8.45
$4 * T_{with}$	0.611	13

Based on the results of the study, it can be concluded that the system with the optimal regulator is rather rough, i.e. it is insensitive in some sense to changes in the parameters of the executive body, elastic transmission and load.

6. OPTIMAL SYSTEM SYNTHESIS СИСТЕМЫ

There are requirements for the operation of any electromechanical system. To check the correct operation, the method of working out the disturbing effect is used.

Figure 6 shows a transient process that reflects the system's response to a disturbance.

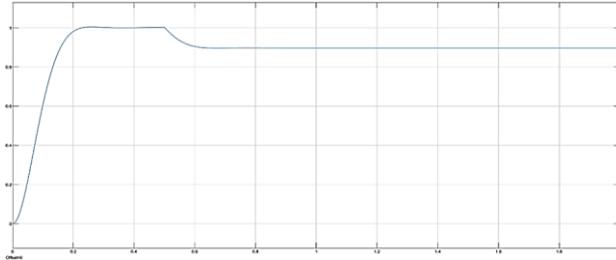


Figure 6 System response to Mc disturbance.

Based on the graph, we can conclude that the electromechanical system under study does not meet the quality requirement. To eliminate system inaccuracies, add a link (dividing line) with the transfer function (12) to the system:

$$W(p) = \frac{0.1 \cdot p^2 + 0.775 \cdot p}{0.1 \cdot p^2 + 0.775 \cdot p + 1} \quad (12)$$

The results of modeling the system response are shown in Figure 7.

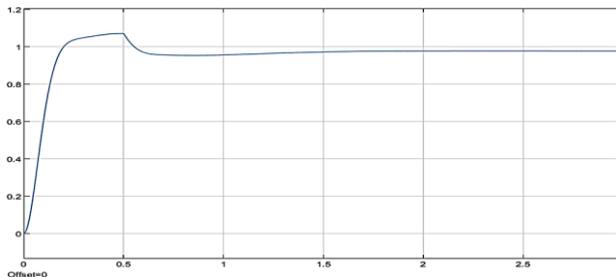


Figure 7 Response of the corrected system to the Mc perturbation.

The correction made it possible to set the statistical error at the level of $\approx 3\%$, and the compensation of the disturbing effect occurs in approximately 1 second.

7. CONCLUSION

The main task in this paper is modeling and synthesis of an elastic electromechanical system.

As a result of the work, the optimal fourth-order regulator with feedbacks along the vector of state variables was determined.

A full-order observer is developed to restore the elastic moment that is inaccessible to measurement.

A rather low sensitivity to changes in the parameters of the executive organ and elastic transmission was achieved.

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