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Forecast on S&P 500 Index Barrier Option with BSM Model

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ABSTRACT

Instead of buying the stocks or shares directly, the call option gives a chance for owners to buy it at a specific predetermined price, which prominently decreases the risk for the customers. Thus, it plays a significant role in the economic market and be widely used in a considerable number of financial activities. This paper mainly focuses on forecasting the price of underlying assets and the payoff at maturity for the up-and-in call barrier option. The overall market is the S&P 500 index stock. In this paper, the BSM model is used to simulate the price of the asset in a certain period. After all, the sensitivity tests are applied to identify how many variations in those basic input parameters will impact the results for this model. The test on strike price shows it has an exponential positive relation with payoff; spot price shows a linear negative relation, and barrier price shows no connection with payoff when it's lower than the spot price. Otherwise, it represents a negative relation. Some discussions are also shown about the deficiencies and future studies based on this model in the end.

Keywords: *Barrier option, S&P 500, Pricing model*

1. INTRODUCTION

The financial market is always difficult to monitor since a significant number of stocks here influence it. The S&P 500 stock index was subsequently introduced in 1957 to track the value of 500 corporations' stock listed on the New York Stock Exchange (NYSE) and the NASDAQ Composite [1]. Since it's established to represent the overall status of the U.S economy, some stocks are added or removed each year for calculating this index. S&P 500 is widely considered an essential benchmark of the U.S stock market. The composition of those 500 corporations across a breadth of industry sectors as well. Thus, it is one of the most significant stock indexes to measure how the current market changes and why most researchers forecast the future based on the parameters of the S&P 500 index. S&P 500 funds also offer a considerable return over time. In this paper, we focused on S&P 500 options.

An option is a security that gives its owner the right to trade in a fixed number of shares of a specified common stock at a fixed price at any time on or before a given date. To forecast future option prices, volatility derived from historical option prices is commonly employed with mathematical models [2]. Thus, the choice of volatility becomes significant. It needs to be both representative and reliable. In other words, the valuation of options and other financial derivatives typically changes depending on various markets [3].

Based on previous research, my paper aimed to forecast the price of the S&P 500 index up-and-in call option with four parts, for which a call option gives the right to buy the shares for the owners. Since the payoff of this option is based on the underlying asset, there still exist some elements that will increase the risks for the owners, such as the characteristics of the underlying asset and the option itself. In this case, establishing a pricing model will decrease the risks for investors, provide valuable suggestions for them, and help them to make a better investment decision.

First, basic parameters are derived from historical data from November 1, 2020, to November 1, 2021. The graph of the stock price across this period is shown as well. Second, the BSM model is utilized to simulate future prices in the following 22 days. Third, the average payoff is \$57.7660, with a present value of \$57.7603 at maturity—the underlying asset's price with those 22 days as stated and presented in a graph. In the end, sensitivity tests are built for the strike price, spot price, and barrier price to make sure this model works well.

This paper is organized as follows. Section 2 presents the data and basic parameters derived from it; Section 3 describes the essential BSM model and pricing methods for the underlying asset and option; Section 4 performs the results of the simulations; Section 5 states the further sensitivity test of the strike price, spot price, and barrier price. The last section discusses the deficiencies of this pricing model and what can be explored for future studies.

2. DATA

One year (from November 1, 2020, to November 1, 2021) ranged historical price data, average interest rate, and dividend rate are collected from the Google financial website. The prices for S&P 500 index within this year are shown in Figure 1.

Figure 1 S&P 500 index prices from November 1, 2020, to November 1, 2021.

The standard deviation of annual returns (12.8%) or what we call the stock's volatility is derived through those historical data.

Because of the considerable volatility, the barrier option will be more profitable [4-6]. A barrier option is a type of derivative which has a predetermined price placed on the underlying asset. The payoff thus depends on whether or not the underlying asset's price has reached or exceeded that barrier before or at maturity [4]. In this case, the huge volatility of the S&P 500 index will make it more likely to break the barrier and activate the option. Thus, investors can get noticeable returns by a small amount of premium cost. This paper will focus on pricing future S&P 500 index price and call barrier-option payoff based on those historical data and parameters above.

3. METHOD

There are four types of barrier options [7-9]. My paper employs the up-and-in barrier option model because of the enormous volatility and an increasing trend of the price for the S&P 500 index. In other words, there is more chance to break the barrier and result in a higher payoff for investors at maturity [10].

Before pricing the option, I utilized the BSM model to predict future stock prices, which Fischer Black and Myron Scholes published in 1973. By assuming a risk-free environment, the BSM model calculates the stock price through the formula

$$S\_T = S_0 e^{\left(\alpha - \frac{1}{2}\sigma^2\right)T + z\sigma\sqrt{T}} \quad (1)$$

$$\alpha = r - \delta \quad (2)$$

for which r is the risk-free interest rate; δ is the dividend rate; S_0 is the spot price; σ is the underlying volatility; T is the time period, and z is a random number that is normally distributed. First, I simulated five hundred stock samples for 22 days. For generating each simulated day's price, I used the previous day's price as S_0 and the spot price as the first day's S_0. The normally distributed random number z is generated by $norms\ in(rand())$ in excel. Then, according to the definition of the up-and-in barrier option and call option, the final call option price is established from $IF(MAX(Day1{:}Day22) > barrier\ price, MAX(ST - X, 0), 0)$ [11-12]. Through this formula, if the stock price exceeds the barrier price between those 22 days, this call option will be activated to bring a payoff for the investors. Otherwise, it will be worthless. $MAX(ST - X, 0)$ represents the final payoff of a call option for the investors where ST is the stock price at maturity and X means the strike price for this option. After generating the average payoff D, the present value of it can be expressed as

$$PV_D = De^{-rT} \quad (3)$$

What's more, after generating the standard deviation of D, the standard error and maximum error can be expressed as

$$StE = \frac{SD}{\sqrt{number\ of\ samples}} \quad (4)$$

$$MaxE = Standard\ error \times 2 \quad (5)$$

In addition, the present value of maximum error can be established as follows:

$$PV_{MaxE} = MaxE \times e^{-rT} \quad (6)$$

Therefore, the simulated payoff will be the $PV_D \pm PV_{MaxE}$.

4. RESULT

My pricing model ends with a $57.7660 average payoff with a present value at $57.7603, a $102.2 standard deviation, and a $21.789 standard error with a current value at $21.787. The maturity date stock price and the final call option payoff for those five hundred samples are shown in figure 2.

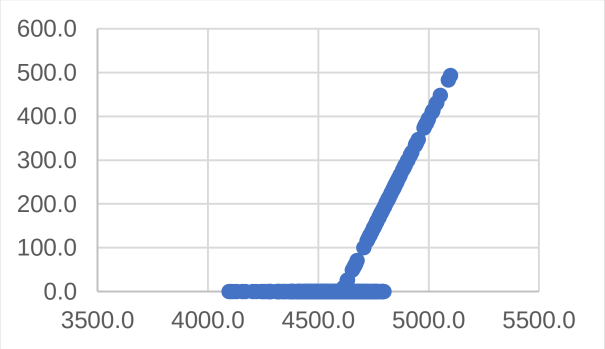

Figure 2 Stock price on maturity day and the call option payoff for investors.

However, some elements affect the final call option payoff and this pricing model, such as the strike price, spot price, and barrier price. Thus, sensitivity tests are followed to investigate how the target payoff is influenced based on those input parameters individually. However, some elements affect the final call option payoff and this pricing model, such as the strike price, spot price, and barrier price. Thus, sensitivity tests are followed to investigate how the target payoff is influenced based on those input parameters individually [13].

5. SENSITIVITY TEST

Sensitivity tests below help me to check whether this pricing model goes in the right way, how those parameters mentioned above relate to the option payoff, and which are essential for appraising this option.

5.1. Strike price

The sensitivity test between the strike price and the option payoff shows a negative correlation in Figure 3.

This graph illustrates an expected result since a higher strike price means a relatively small difference between barrier price and a large difference between spot price. In other words, even if the underlying asset breaks the barrier, investors will have less probability to get a higher payoff since we got it from subtracting stock price by strike price. Thus, the estimated payoff will decrease during the strike price of the stock rise.

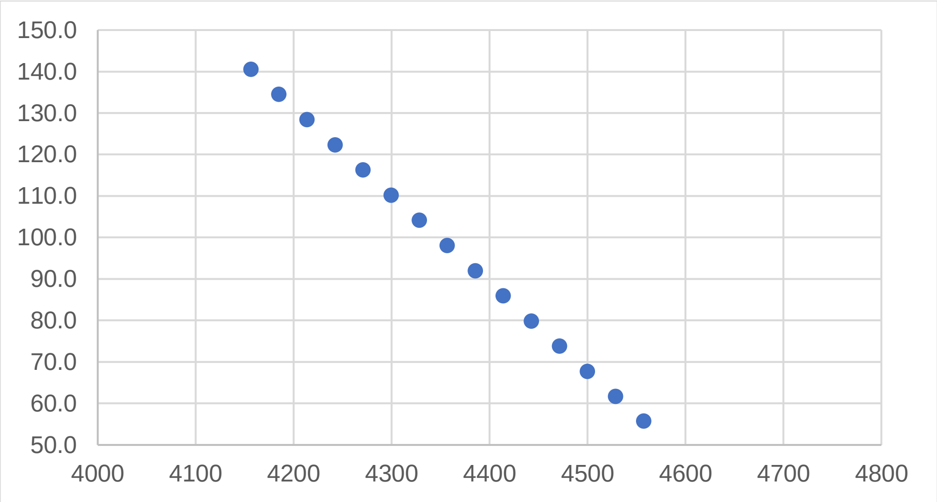

Figure 3 Strike price and option payoff

5.2. Spot price

The sensitivity test between the spot price and the option payoff shows a positive correlation in Figure 4.

This graph illustrates an expected result since a higher spot price represents a more significant possibility to break the barrier. By starting with a higher spot price, even a tiny fluctuation will drive the stock price to exceed the barrier. Therefore, the higher the spot price, the higher the payoff.

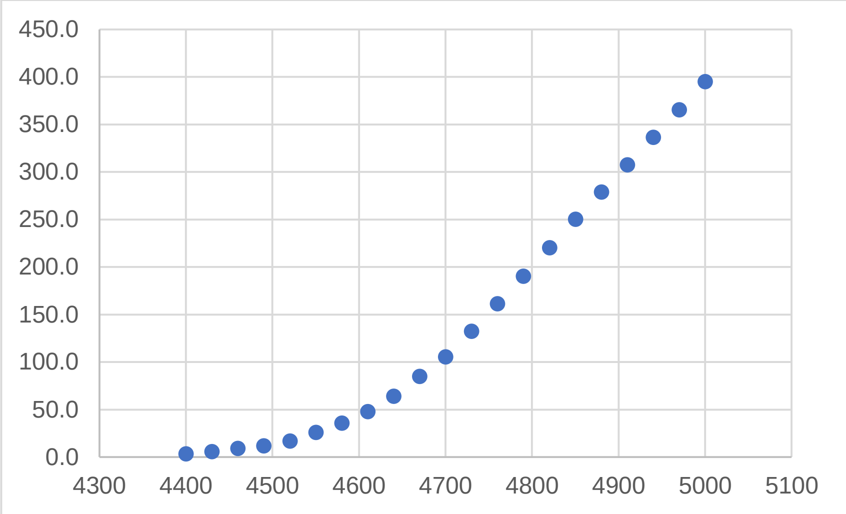

Figure 4 Spot price and option payoff

5.3. Barrier price

The sensitivity test between barrier price and the option payoff shows a negative correlation in Figure 5. This graph illustrates an expected result since we can see two different parts – a flat line and a negative gradient line. When the barrier price is below the spot price, the option is activated automatically before the starting day. In this case, the barrier will not affect the payoff at all. Thus, no matter how much it is, the option payoff will be a constant number. The flat line in the graph shows this relationship. After the barrier price passes the spot price, it will make a difference in the payoff. A higher barrier price leads to a lower possibility for the stock price to break the barrier. Therefore, the investor will have a gradually declined payoff while the barrier price increase.

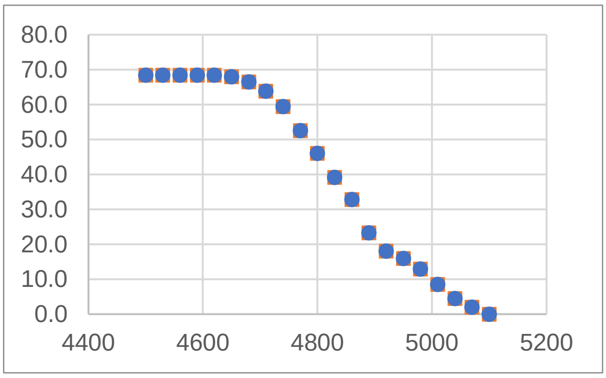

Figure 5 Spot price and option payoff

6. CONCLUSION AND DISCUSSION

This paper aims to forecast the price and payoff of the S&P 500 barrier up-and-in call option by using the BSM model. The high volatility causes the cost of the underlying asset to break the barrier more likely. The sensitivity test shows that the higher the strike price, the lower the payoff, the higher the spot price, the higher the payoff, and the higher the barrier price, the lower the payoff. Those strictly illustrate what we expect for this model and the overall trend.

Based on this model, customers can make a forecast for any payoff of up-and-in call option in any market, just by changing the basic parameters. Thus, it helps them to make a better determination of buying this option or not. What's more, according to the sensitivity tests, to make a higher payoff, investors should choose the underlying asset with a higher spot price and lower strike price but the up-and-in call option with a lower barrier price.

However, there still exist some deficiencies. First, I use the history data, like average volatility and return rate, by pricing the option as essential parameters. In other words, this model assumes those values will be constant over the option duration. But history cannot represent the future. The ignoring of future changes will result in an inaccuracy of this model. Second, this model didn't count the cost of transactions or services. Therefore, the predicted payoff may be higher than the realistic one. Third, the lognormal pattern for asset price is employed for this model, which causes the neglecting of huge fluctuations, always appearing in the real world, in the market.

With the progress of the financial market and modeling techniques, this model is too outdated to make an accurate predicted price. More influential factors should be included for modeling in further studies, such as government policies and community culture. Instead of using limited historical data, the future advanced model should collect a more comprehensive range of data to analyze further.

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