Advances in Economics, Business and Management Research, volume 211
Proceedings of the 2022 7th International Conference on Financial Innovation and Economic Development (ICFIED 2022)

Application of Modern Portfolio Theory in Stock Market

Yumeng Liu[1,*]

[1] COLLÈGE DU LÉMAN
[*]Corresponding author. Email: yumeng_liu@cdl.ch

ABSTRACT
When investors face investing problems, the main issue is to make decisions on how to allocate resources among the variety of different assets, especially in the rapidly growing stock market.The paper aims to design an investment strategy for risk-averse investors. We use modern portfolio theory and apply mean-variance analysis, to quantify expected portfolio returns and acceptable levels of portfolio risk and provides methods to select an optimal portfolio. The daily adjusted closing stock prices between January 2010 and January 2021 of 5 companies: Facebook, Amazon, Apple, Netflix, and Google, are used. From the results we can conclude that such mean-variance optimization is applicable. Furthermore, we can obtain minimum variance at level of 23% (with expected return at level of 28%) which illustrates the risky nature of business environment in the country. Additionally, the higher return (at level of 33%) might be achieved if the investor is willing to take a risk (at level of 25%) higher than the of minimum risk. Limitations of the present research are discussed at the end.

Keywords: *Investment strategy , Diversification, Modern portfolio theory, Mean-variance analysis.*

1. INTRODUCTION

Modern portfolio theory (MPT) is an investing strategy that is widely used to optimize the trade-off between risk and return in diversified portfolios. We assume that investors are risk-averse, that is, they prefer a less risky portfolio to a riskier one at a given level of return (Faboizz et al., 1998). MPT can help investors to achieve the portfolio with higher possible long-term returns with an acceptable level of short-term market risk.

The concept of portfolio is introduced by Markowitz (1952), to give investors investment suggestions. Markowitz (1952) further proposes the modern portfolio theory which is also as known as mean-variance analysis. Mean and variance are used to measure the return and risk of the portfolio. Modern portfolio theory (MPT) aims to improve the selection and construction of investment portfolios, assuming investors tend to maximize expected return and simultaneously minimize the investment risk. It is later developed by Sharpe (1964). Portfolio can have various assets including bonds, stocks, and commodities.

However, there are some problems of the classical MPT. For example, Mahdavi-Damghani (2013) shows that there are some criticisms for mismatching between theory and the real world. The MPT is used by investors to make investment decisions based on the amount of risk that they would like to take. Using this method, investors estimate the potential risk given past market data. The real market is unpredictable, and this might cause huge loss. Moreover, this theory assumes that prices follow a normal distribution, which may not be always true. The classical MPT measures the risk of portfolios based on variance rather than downside risk, to solve this problem, post-modern portfolio theory (PMPT) is introduced (Rom and Ferguson, 1991). PMPT uses the downside risk of returns, while MPT uses the risk of returns.

With the aim to solve real-life problems, different factors and restrictions are considered. There are popular methods including:

1.Add industry factors. When choosing stocks in the investment portfolio, industry factors can be considered to the model. There are two ways to add industry factors. The first one is to compare returns of stocks in different industries and select those with highest returns in each industry. The second one is to pick industries with a large number of companies, and then choose those industries to invest in.

2. Set different risk preferences. The MPT assumes that investors are risk-averse, that is, investors prefer least risky portfolio given a certain expected return.

However, investors can customize their investment risk by choosing different type of portfolio they prefer, and this allows them to independently determine their risk/return ratio.

The modern portfolio theory is a useful tool for selecting investment. The mean-variance analysis helps investors to design an optimal portfolio based on expected return and risk. The key idea of MPT is diversification. With the aim to construct diversified portfolios, investors are suggested to choose assets that are not positively correlated. For example, stocks do not move up or down in the same directions. By doing so, the overall risk of the portfolio is less than that of the any of the individual assets.

The remainder of this paper is structured as follows. In the next section, we give a brief description of the modern portfolio theory and mean-variance model, along with its assumptions. In Section 3, we present data analysis via a specific example. In Section 4, we will give conclusion and discussion of our study.

2. METHODOLOGY

2.1. Assumptions

The mean-variance formulation of MPT involves some assumptions. These are discussed as following.

- Investors are risk averse. This means investors prefer higher return and lower risk.
- Expected return is measured by mean of returns, and risk is measured by variance of the expected returns. Our portfolio is constructed only determined by these two factors.
- The market is efficient. All investors have access to all information, so it is a fair market.
- There are no transaction costs and taxes.
- Stocks can be divided to infinitely many shares. The number of stocks is not necessary to be an integer.

2.2. Model

Suppose there are n stocks in the portfolio. Let w_i be the weight of invested in stock i, where $i = 1, \dots n,$ and we have the weights of assets satisfying

$$\sum_{i=1}^{n} w_i = 1, w_i \geq 0\,. \tag{1}$$

We assume that short sell is not allowed. Let R_i be the return of stock i. The expected return of a portfolio, denoted by $E(R_p)$, is considered as the weighted average of the return of individual stocks over a finite period, which is given by

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i\,). \tag{2}$$

Risk can be divided into two categories: systematic risk and unsystematic risk. The former refers to the risk of each individual stock in the market which can be eliminated by diversification, while the latter is the risk of the market that cannot be eliminated. We use standard deviation to measure the risk of the portfolio, which is denoted by σ_p. Suppose the portfolio has stock i and stock j ($i = 1, \dots n;\ j = 1, \dots n;\ i \neq j$). The variance is denoted by $\sigma_\rho^{\,2}$, given by

$$\sigma_\rho^{\,2} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i\, w_j \sigma_{ij}, \tag{3}$$

$$\sigma_p = \sqrt{\sigma_\rho^2}, \tag{4}$$

here w_i is the weighting of stock i and w_j is the weighting of stock j, and σ_{ij} is covariance between stock i and stock j.

There are two ways to select stocks for a portfolio. The first is to maximize the expected return for a given risk, that is

$$\max E(R_p) = \max \sum_{i=1}^{n} w_i E(R_i\,), \tag{5}$$

subject to

$$\sigma_\rho^{\,2} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i\, w_j \sigma_{ij}, \tag{6}$$

and

$$\textstyle\sum_{i=1}^{n} w_i = 1, w_i \geq 0\,. \tag{7}$$

The second way is to minimize the risk return for a given expected return, that is,

$$\min \sigma_\rho^{\,2} = \min \sum_{i=1}^{n}\sum_{j=1}^{n} w_i\, w_j \sigma_{ij}, \tag{8}$$

subject to

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i\,). \tag{9}$$

and

$$\sum_{i=1}^{n} w_i = 1, w_i \geq 0\,. \tag{10}$$

2.3. Efficient frontier

Portfolio 's volatility takes into account the weights of the volatility of each security in the portfolio with covariances. Thus, the portfolio volatility is lower than the volatility of individual stocks. We can get the feasible

region of portfolios by different weights. By solving the minimum volatility given specific expected return, we can get the minimum-variance frontier of feasible region.

The efficient frontier is a set of optimal portfolios that provide the highest expected return for a given level of risk or the lowest risk for a given level of expected return (García et al.2015). A portfolio that falls below the frontier is considered sub-optimal, because the expected return is lower at that risk. A portfolio that lies right is also sub-optimal because the risk is higher at the determinate expected return.

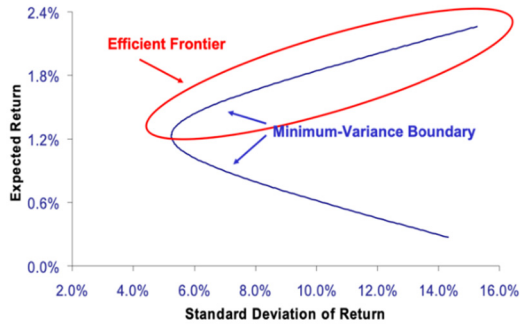

Figure 1 Efficient frontier.

Figure 1 shows the efficient frontier of all optimal portfolios. It is represented by the expected return and standard deviation of returns. The y-axis is the expected return of the portfolio, and x-axis is the standard deviation of returns, which measure the risk of the portfolio.

The efficient frontier is the foundation of the MPT, which gives investors ideas how to construct a portfolio with the aim to maximize expected return at a determinate level of risk. It helps investors to analyze the potential risk and returns, and then choose the optimal investment strategies.

2.4. Sharpe ratio

The Sharpe ratio is a measure of a portfolio's risk-return trade-off, and it helps investors to understand the return of an investment relatively to its risk. The formula of the Sharpe ratio is the expected return of the portfolio in excess of the risk-free rate, $E[Rp] - r_f$, divided by the standard deviation of the portfolio's excess return σ_p , here r_f is the risk-free rate. The Sharpe ratio is given by

$$Sharpe\ Ratio\ = \frac{E[\mathrm{Rp}] - r_f}{\sigma_p}. \tag{11}$$

The Sharpe ratio shows the risk-adjusted performance of a portfolio. A higher Sharpe ratio is preferred because it helps investors determine the investment strategy that gives higher return or a lower risk.

3. DATA ANALYSIS

We consider five companies: Facebook, Amazon, Apple, Netflix, and Google. These five companies are chosen from different industries but all relatively successful. We use daily adjusted closing price of each stock from 01/01/2011 to 01/01/2021 (10 years' price data), obtained from Yahoo Finance. We generate 25000 random portfolios. The risk-free rate is taken from the U.S. Department of The Treasury. The rate of 0.1% is the 52-weeks treasury bill rates at the start of 2021, which is considered as the risk-free rate in our case.

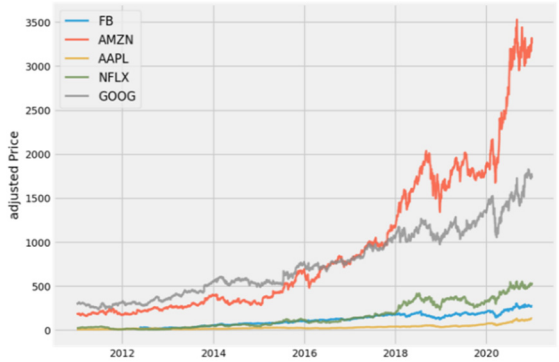

Figure 2 Adjusted closing prices of Facebook, Amazon, Apple, Netflix, and Google over 01/01/2011 to 01/01/2021.

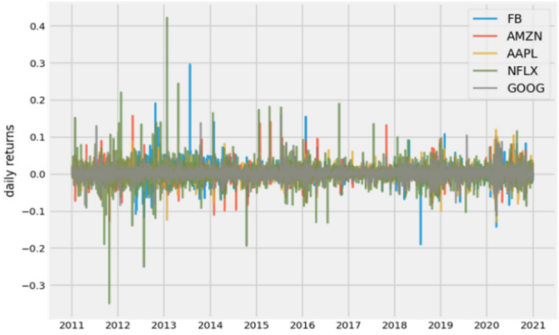

Figure 3 Daily returns of Facebook, Amazon, Apple, Netflix, and Google over 01/01/2011 to 01/01/2021.

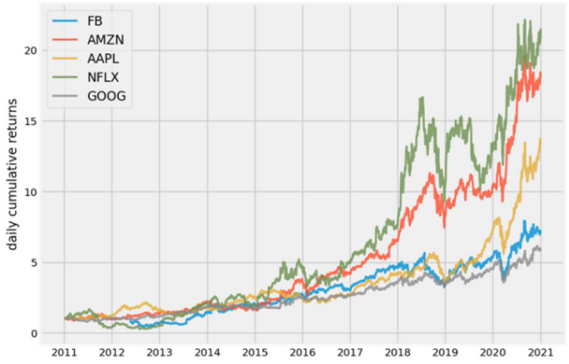

Figure 4 Daily cumulative returns of Facebook, Amazon, Apple, Netflix, and Google over 01/01/2011 to 01/01/2021.

Table 1. Maximum Sharpe Ratio Portfolio Allocation and Minimum Volatility Portfolio Allocation

Maximum Sharpe Ratio Portfolio Allocation					
Annualised Return: 0.33					
Annualised Volatility: 0.25					
Companies	Facebook	Amazon	Apple	Netflix	Google
Allocation (weights%)	10.55	35.35	40.16	13.46	0.48

Minimum Volatility Portfolio Allocation					
Annualized Return: 0.28					
Annualized Volatility: 0.23					
Companies	Facebook	Amazon	Apple	Netflix	Google
Allocation (weights%)	7.83	15.09	35.09	3.04	38.95

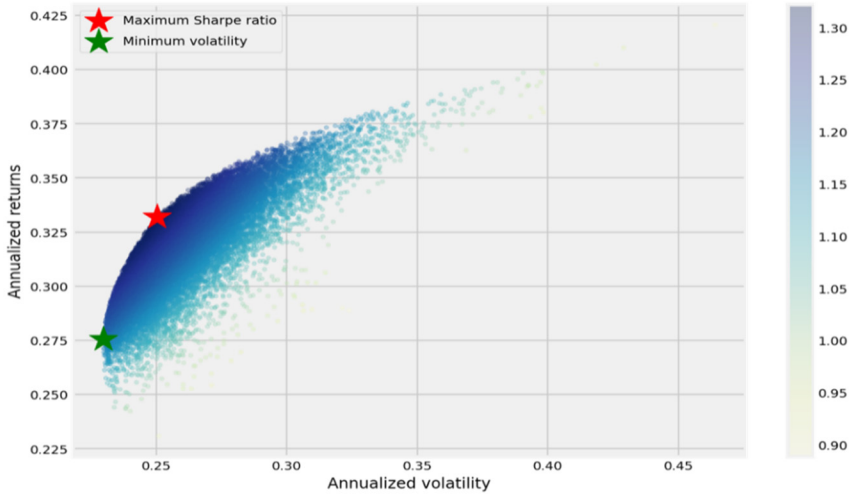

Figure 5 Portfolio Optimization and Efficient Frontier.

We can see from Figure 2 that Amazon and Google's stock price is relatively higher than those of Netflix, Facebook, and Apple. Amazon has a dramatic growth and has been the most expensive one since around 2017. Google also experiences an increase but slightly less than Amazon. However, the other three companies are squashed at the bottom, Netflix is the highest among these three, followed by Facebook, and Apple.

Figure 3 shows the daily returns of these five stocks. We can see that the daily returns are all fluctuated around zero. In addition, Netflix has the highest positive spike and largest negative spike. Facebook and Amazon also have several large distinctive positive spikes and a couple of negative ones. Apple also has some spikes stand out from the plot. The volatility of Netflix and Amazon are quite large, which implies that they are the riskiest stocks, and Google is the most stable one among these 5 stocks. It can be seen from Figure 4 that these five companies increase at the similar speed from 2011 to 2015. While after 2015, daily cumulative returns of Netflix increase most rapidly, followed by Amazon and Apple. Facebook and Google experience gentle growth.

Table 1 displays the maximum Sharpe ratio allocation and minimum volatility portfolio allocation and the weights for the corresponding results. When 10.55%, 35.35%, 40.16%, 13.46%, 0.48% of money invested in Facebook, Amazon, Apple, Netflix and Google, respectively, and then we can obtain the maximum Sharpe Ratio with annualized return of 0.25 and volatility 0.33. The portfolio with minimum volatility can be obtained given 7.83%, 15.09%, 35.09%, 3.04%, 38.95% invested in Facebook, Amazon, Apple, Netflix, and Google, respectively, and the annualized return is 0.28 and volatility is 0.23.

Figure 5 shows all feasible portfolios that are randomly generated given portfolio returns and portfolio volatility. We plot these portfolios with colour map based on Sharp ratio. The higher the Sharp ratio, the bluer. The top line of the area of blue dots is called the efficient frontier. All points along the efficient frontier give highest return for a given risk or give the lowest risk for a given return. The risk averse investors prefer the portfolios on the efficient frontier. There are two best choices: one with minimum risk and another one with maximum risk-adjusted return. The minimum volatility portfolio is represented by the green star and the maximum Sharp ratio portfolio is represented by the red star. For the minimum risk portfolio, more than 60% of our money is allocated to Apple and Google, and around 3% is allocated to Netflix. According to Figure 3, Apple and Google are least volatile stocks among 5 stocks, and Netflix is the most volatile one. In the minimum risk portfolio, more than 2/3 of the money is invested in the stable stocks, thus the overall volatility is minimum. For the maximum Sharp ratio portfolio, the return is higher for the higher risk. Compared to the minimum volatility portfolio, the larger portion of money is allocated to Amazon and Apple, where Amazon has quite high volatility and high returns (as shown in Figure 3 and 4). However, Google has less than 0.5% allocated to it in the maximum Sharp ratio portfolio, but around 39% in the case of minimum volatility portfolio.

4. CONCLUSION

In this paper, we study the modern portfolio theory with its application in stock market. Modern portfolio theory is a widely used tool for portfolio selection, particularly for risk-averse investors. The main idea of modern portfolio theory is to use mean-variance analysis to evaluate the performance of the portfolio. We use the weighted sum of the returns of individual stocks to measure the expected return of the portfolio and use the variance of each individual stock and correlation of stocks to measure the risk of the portfolio.

We consider portfolios consist of 5 companies: Facebook, Amazon, Apple, Netflix, and Google from 01/01/2011 to 01/01/2021, and use mean-variance analysis to analyse the risk and return of portfolios. We construct 25,000 portfolios by randomly generated weights and obtain an efficient frontier of all these portfolios. The efficient frontier is a set of portfolios that maximize the expected return for a given risk. We can suggest risk-averse investors to choose the portfolios on the efficient frontier. Our results show that minimum volatility portfolio maximum risk-adjusted return portfolio can be obtained if more 75.5% money is in invested in Amazon, Apple and minimum volatility portfolio can be obtained if more 74% money is in invested in Apple and Google.

There are several advantages to MPT. For example, it is a useful tool to build diversified portfolios. The portfolio volatility can be much lower in this case by introducing stocks that have negative correlations. It is also easy to understand and implement to majority investors. We can plot the possible portfolios on a graph, with the y-axis showing portfolio risk and the y-axis showing expected return and find the optimal portfolios by the efficient frontier. However, it MPT also has its disadvantages. A big problem is that MPT evaluates portfolios based on variance rather than downside risk. This model is also unrealistic because it's based on some assumptions, for example, it does not take into consideration any costs in trading and it assumes that all investors are risk-averse and are rational, which are not always true. Next step, we would consider more assets in this portfolio. With more assets, we might obtain relative lower risks as well as higher return as the portfolio can diversify more risks. Adding risk-free bonds can also be a way to lower the risk of portfolios.

AUTHORS' CONTRIBUTIONS

AUTHOR INVESTIGATES THE APPLICATION OF MODERN PORTFOLIO THEORY IN STOCK MARKET AND SHOW THAT BY CREATING A PORTFOLIO LOWER RISK CAN BE OBTAINED.

REFERENCES

[1] B. Mahdavi-Damghani, The Non-Misleading Value of Inferred Correlation: An Introduction to the Cointelation Model Wilmott, 2013, 67, 50-61.

[2] F. J. Fabozzi, Foundations of financial markets and institutions. Upper Saddle River, N.J., Prentice Hall, 1998.

[3] F. García, J. A. González-Bueno, J. Oliver, Mean-variance investment strategy applied in emerging financial markets: Evidence from the Colombian stock market. Intellectual Economics, 2015, pp. 22–29. DOI: https://doi.org/10.1016/j.intele.2015.09.003

[4] H. M. Markowitz, Portfolio Selection, The Journal of Finance, 1952, pp. 77–91.

[5] B. M. Rom, K. Ferguson, Post-Modern Portfolio Theory Comes of Age. Journal of Investing, 1993, pp. 27-33. DOI: https://doi.org/10.3905/joi.2.4.27

[6] W. F. Sharpe, Capital Asset Prices A Theory of Market Equilibrium under Conditions of Risk. Journal of Finance, 1964, pp. 425-442.

[7] X. Zhou, D. Li, Continuous-time mean-variance portfolio selection: A stochastic LQ framework, Applied Mathematics and Optimization, 2000, pp. 19-33.