

Analysis of Bond Investment based on Immune Strategy

Shengyuan Lu^{1,*}

¹ *Soochow Business School, Soochow University, 215031, Soochow, China*

*Corresponding author. Email: 1910503084@stu.suda.edu.cn

ABSTRACT

Interest rate risk is the essential market risk faced by bond investment, mainly the price effect and reinvestment effect. With the acceleration of interest rate liberalization, the importance and urgency of interest rate risk management of bond investors are increasingly prominent. Moreover, as an essential method of immunity against interest rate risk, the duration immunity strategy has been widely used. This paper starts with the most basic concept of bond duration, using a specific case analysis. Through the calculation to build a reasonable bond portfolio, we find the risk immunization of the duration immune strategy regarding the sudden interest risk. Moreover, during the case study, we also had limitations of the immune strategy applications in the natural bond markets. Finally, we conclude the whole finding and then decide the next research direction.

Keywords: *Macaulay duration, Modified duration, Bond investment, Immune strategy*

1. INTRODUCTION

With the stock market plummeting, systemic risk increasing, and sensible investment becoming the mainstream, bonds have become the first choice for investors to avoid market risks and obtain stable returns [1]. So the studies of how to build appropriate bond investment portfolios have been meaningful and significant research, especially after the duration was introduced by Macaulay [2]. For example, Zong Wei made an empirical analysis on the asset-liability management of commercial banks by using the modified duration-convexity model [3], the research of the measurement standard of bond price sensitivity done by Conroy [4], or like Xie et al. examined the effects of default risk, call risk, and their interactions on bond duration [5].

The paper's objective is to analyze the bond investment portfolio based on the duration immunity strategy. To make the analysis specific, we make a case study that creates a bond investment portfolio of a zero-coupon bond and a coupon bond to match a student's future payment. Firstly, it calculates bond A and bond B's Macaulay duration. Then by using the immunity strategy to match the bond portfolio duration and the payment, we get the weight of two bonds in the portfolio. Moreover, by discounting the payment value today, multiplying the weight, and dividing by the present price of the bond, we

can finally get the precise investment portfolio that how many numbers of the two bonds to need to buy respectively to offset payment. Then based on the case study, the paper discusses the advantages and several limitations of the strategy.

The importance of the research on the bond investment portfolio based on the duration immunity strategy is its relatively ease and convenience for the investors to build the bond portfolio. As there are increasingly investors entering into the bond investment market, the paper is written to make a systematic explanation of the duration and its immunization strategy, meanwhile by doing a specific case study can make the readers who are research beginners, fund investment managers or retail investors more quickly to learn and master such an immune strategy especially at the time which the bond market is increasingly active nowadays.

The remainder of the paper is organized as follows: Section 2 describes the data, which include the essential information in the case like the two bonds' face value, current price, the remaining lives, and the importance and maturity of the future payment; Section 3 performs the method of Macaulay duration, modified duration, and the immune strategy; Then as the introduction of the method and definitions about durations and the strategy needed in the case study, Section 4 introduces specifically numerical analysis and get the precise and suitable

investment portfolio of the two bonds to match the requirement of payment; Section 5 shows the discussion of the advantages and limitations of the immune strategy based on duration. The last section presents our conclusions.

2. DATA

The paper assumes a case that a student has to seek advice to elaborate a hedging strategy against interest rate risk. Three years later, the student will need to make a one-off tuition payment of £ 17,000 for a postgraduate course and wonders how to invest cash available today to fund that future outflow. Suppose there are two liquid bonds in the market:

- Bond A: Face value= £ 100, current price= £ 85.48, no coupon payments, remaining life = 4 years
- Bond B: Face value = £ 100, current price = £ 100, annual coupon of 4, remaining life =2 years

The case assumes that all debt considered has the same level of default risk. The benchmark yield curve for fixed-income investments of the same riskiness is flat at a level of 4% p.a.

The purpose of the case study is to make an investment portfolio in order to match the payment need after three years of the student by using the immunity strategy. The first step should do to calculate the Macaulay duration of the bond A and B, with the face value, current value, whether it has coupon or no-coupon, and the mature time respectively by the Macaulay duration equation:

$$D = \frac{\sum_{t=1}^T PV(c_t) \times t}{P_B} = \sum_{t=1}^T \frac{PV(c_t)}{P_B} \times t \tag{1}$$

- D—Macaulay duration
- P_B—the current price of bond
- T—the maturity time of the bond
- PV(c_t)—the present value of the future cash flows (interest or capital) of the bond at maturity time
- t—the duration of cash flow from the present to time t

As the given information, since Bond A has no coupon payments, it's a zero-coupon bond with no intermediary payments. Bodie et al. found that the zero-coupon bond, by contrast to the bond's effective maturity, is some sort of average of the maturities of all the cash flows paid out by the bond, making only one payment at maturity [6]. Its time to maturity is, therefore, a well-defined concept. So the Macaulay duration of the bond A equals maturity, which is four years.

Then calculate the bond B with the given data. By using the equation (1) of Macaulay duration, it can get a data table below:

Table 1. The Macaulay duration of bond B

t	CF	PV (CF/1+yield)	t×PV
1	4	3.85 (4/1.04)	3.85 (3.85*1)
2	104 (100+4)	96.15 (104/1.04 ²)	192.31 (96.15*2)
total		100	196.15

Table 1 can get the Macaulay duration of bond B, which equals 1.9615 (196.15/100) years. The Macaulay duration of the zero-coupon bond of the same maturity (2 years) will be the same. Because there' s no intermediate payment, and the duration will equal the maturity.

As the calculation has got the Macaulay duration of bond A (4 years) and bond B (1.9615 years), the student' s payment after three years which because of it being paid at that time, its duration equals to 3.

3. METHOD

3.1. Macaulay duration

Macaulay duration is a well-known interest rate risk measure for a portfolio of regular bonds (options-free and default-free). It measures the percentage change in portfolio value due to the instantaneous change in interest rates. It also defines a portfolio immunization horizon over which the portfolio value remains immunized from an instantaneous shock in interest rates [7].

A bond's Macaulay duration D is the weighted average of the maturity of individual cash flows, with the weights being proportional to their present values, so the formula of Macaulay Duration is:

$$D = \sum_{t=1}^T t \frac{CF_t / (1+y)^t}{P} = \sum_{t=1}^T t \frac{PV(CF_t)}{P} \tag{2}$$

- D—Macaulay duration
- y—the bond's yield to maturity
- T—the maturity time of the bond
- t—the duration of cash flow from the present to time t
- CF_t—the cash flow made at time t
- PV(CF_t)—the present value of the future cash flows (interest or capital) of the bond at maturity time
- P—the current price of the bond

The Macaulay duration reflects the average length of time that would elapse before the value of a bond is compensated by its future known stream of fixed payments [8].

For this, when it comes to the calculation of Macaulay duration, there are always two situations: zero-coupon

bond and coupon bond. The Macaulay duration of a zero-coupon bond equals its time to maturity in most cases because the zero-coupon bond is a bond that pays no interest which trades at a discount to its face value, so with only one payment, the average time until payment must be the bond's maturity [9].

In contrast, the coupon bonds have a lower duration than its time to maturity because the coupon in the bond's early life lowers the weight of the average time until the payments [10]. Notes that the factors affect duration: a bond's price, maturity, coupon, and yield to maturity. All these elements should be considered in the calculation of duration. According to these factors and the formula above, there are several other roles:

Rule 1 Holding maturity constant, when a bond's coupon rate is higher, its duration will be lower. With the coupon rate rising, the yield to maturity will be relatively high so that the duration will be higher.

Rule 2 Holding the coupon rate constant, a bond's duration generally increases with its time to maturity, which the duration always increases with maturity for bonds selling at par or at a premium to par.

Rule 3 Holding other factors constant, the duration of a coupon bond is higher when the bond's yield to maturity is lower.

Rule 4 The duration of level perpetuity is:

$$\text{Duration of perpetuity} = \frac{1+y}{y} \tag{3}$$

For instance, if the yield is at 12%, the duration of perpetuity will be $1.12/0.12 = 9.33$ years, but at an 10% yield it is $1.1/0.1 = 11$ years. This is also a representation of the **Rule 3**.

Duration is a key concept in fixed-income portfolio management for at least three reasons. First, it is a simple summary statistic of the effective average maturity of the portfolio. Second, it turns out to be an essential tool in immunizing portfolios from interest rate risk. Third, duration is a measure of the interest rate sensitivity of a portfolio.

3.2. modified duration

The Modified Duration(MD) measures a bond's interest rate risk by its price sensitivity, the relative price change with respect to a change in yield, which in other words, it illustrates the effect of a 100-basis point (1%) change in interest rates on the price of a bond [11].

The formula for the modified duration is as follows:

$$\text{MD} = \frac{1}{1+y} \times \sum_{t=1}^T t \frac{\text{PV}(\text{CF}_t)}{P} = \frac{D}{1+y} \tag{4}$$

- MD—Modified Duration

- D—Macaulay duration
- y—the bond's yield to maturity

From the formula of modified duration, it is actually the first derivative of the bond price with respect to the yield.

When it comes to the application of the modified duration, for individuals, when market interest rates are expected to fall, the larger the correction duration, the greater the rise in bond prices, thus benefiting investors. Investors should choose bonds with a longer maturity. In this way, when market interest rates do fall in the future, the bond can get a higher appreciation [12].

Since the modified duration measures the interest rate risk by its price sensitivity, the price change is relative to the difference in yield.

First, as the rate of return of the denominator in the modified duration formula of bond B in the case increases by 0.1%, the price will decrease by $1.9615/(1+4\%)*0.1\% = 0.188606\%$ accordingly. This shows that the relationship between the change in yield and the modified duration is negative and proportional.

Second, as the change in yield becomes large, so does the resulting change in price by calculation. For example, the price change of bond B resulting from an increase in yield of 10% will decrease $1.88606*10\%=18.8606\%$. This is also the case where we find the limitations of modified duration, which seem to exaggerate their effect on bond prices when yields rise; When yields fall, modified duration seems to underestimate their impact on bond prices.

So the change in yield is very small, and the change in price based on duration is approximately valid. So if yields vary a lot, the modified duration will not be a valid approximation.

3.3. immunity strategy

The Macaulay duration above defines a portfolio immunization horizon over which the portfolio value remains immunized from a quick interest rate risk. A fundamental immune strategy is to match the present value and the Macaulay duration of a bond investment portfolio with the liabilities. The portfolio is then immunized to rapid small parallel movements in yields (the whole yield curve is shifted up or down by the same increment) [13]. So, a dynamic change of the investment portfolio based on immune strategy is needed to keep the portfolio immunized from small parallel shifts by continuously rebalancing.

The basic immunization problem is one asset - one liability immune strategy model: If the term structure of interest rates reflected by the bond yield curve is flat, and the yield curve shifts only with interest rates, then the sufficient and necessary condition for an interest rate

immunization is that asset durations and liability durations are equal [14]. Moreover, this asset can also be regarded as a combination of assets, then this model can understand the immune strategy model of several assets and one liability. Such one asset - one liability immune strategy model will be specifically analyzed in the following—the numerical analysis of the case.

4. NUMERICAL ANALYSIS

The aim of the case is to make a portfolio that includes both bonds A and B now by using an immunity strategy based on the Macaulay duration to match the payment after three years. So the first step of building the portfolio is to get the weight of bond A and bond B. To get the weight of two bonds, match the duration with the equation (5) below:

$$D_P = w_A \times D_A + w_B \times D_B \quad (5)$$

- D_P —Macaulay duration of payment
- w_A —weight of Bond A
- D_A —Macaulay duration of Bond A
- w_B —weight of Bond B
- D_B —Macaulay duration of Bond B

And the sum of weight equals to 1:

$$w_A + w_B = 1 \quad (6)$$

Then substitute the data into the two equations and combine them:

$$3 = 4(1-B) + 1.9615B \quad (7)$$

So we get the weight of bond A and bond B, which is 50.94% and 49.06%, respectively.

The next step is to calculate the present value (PV) of the after-three-years payment discounts to today, which is also the value of the portfolio at the beginning of the immunity strategy portfolio. By using the equation of discounted cash flow:

$$P = \frac{D}{(1+i)^n} \quad (8)$$

- P —the present value
- D —the book value
- i —the market interest rate
- n —the maturity of the bond

So the discounted payment now is $17000/(1+4\%)^3$, which equals £ 15112.94. As the weight of the bond A and B have been calculated of 50.94% (bond A) and 49.06%(bond B), times with the present value of the portfolio (£ 15112.94) and divided by its current price (£ 85.48 of bond A, £ 100 of bond B) respectively, then we finally get the amounts of the two bonds in the

immunity strategy portfolio with the 86.74 units of bond A and 76.99 units of bond B.

In conclusion of the case, if the student wants to match the payment after three years by making a bond investment portfolio of strategy immunity based on duration, he should buy a portfolio making up of 86.74 units of bond A and 76.99 units of bond B in order to precisely offset the future expenditure theoretically.

5. DISCUSSION

The construction of a bond investment portfolio based on immune strategy makes up the calculation of the duration of the target portfolio and the calculation of the duration of the available hedging securities [15].

For the portfolio of like the case of one zero-coupon bond and another coupon bond, its interest rate risk immunity effect is that when the market interest rate rises, the asset value of a zero-coupon bond will decline, but due to the rise of interest rate, the principal interest of coupon bond and the value of reinvestment with the interest of the bond will increase, vice versa. Thus, such a portfolio would allow the price and reinvestment to cancel each other out no matter how interest rates changed.

The advantage of the immunity strategy is to reduce potential risk at the cost of some potential risk so that this strategy may suit risk aversion investors. Duration immunization is a negative investment strategy. For example, portfolio managers do not pursue excess returns through interest rate prediction but make the investment portfolios under the condition of avoiding the risk of interest rate fluctuations to achieve the established rate of return target. In the portfolio design, in addition to national debt can be selected into the portfolio, part of higher yield corporate bonds and financial bonds can also be added to the portfolio on the condition that the matching duration is well controlled.

There are also some limitations of immunity strategy that:

(1) In order to achieve the effect of interest rate risk immunity, it is necessary to find bonds with appropriate yield and maturity in the market, which may not be available. For example, the case numbers include a decimal point and know the quantity of bond is an integer. Furthermore, it is difficult to forecast the cash flow of liabilities. So it is not easy to get the portfolio in the market with the investors' different certain needs.

(2) As the modified duration above, When the market interest rate changes greatly, the current duration will be less efficient. So if the original desired effect is required to be maintained, the investment portfolio needs to be rebuilt, which is a relatively large workload.

(3) The other limitation is also the advantage of the strategy that while it refuses the risk, the immune strategy also compensates for the potential decline at the expense of the potential expected return which may be like the hedge fund. We have an increasing market interest rate when we are in a positive economy. Compared with a single bond yield, the immune portfolio will rise more slowly when the market interest rate moves in a favorable direction [16].

6. CONCLUSION

The Interest rate risk is the most crucial market risk that bond investment faces. With the acceleration of the global interest rate liberalization process, interest rate risk management's importance in bond investment is increasingly prominent. As an essential method of immunity against interest rate risk, the duration immune strategy has been widely used. So in this article, we do the case study to use a specific example that usually meets in the bond investment market. The paper first introduces the data in the case and then defines the durations and immune strategy with some of their applications in the bond investment. Moreover, in the specific study, we use one asset - one liability immune strategy model to get the final portfolio.

We finally get a precise bond investment portfolio for a specific cause for the one asset - one liability immune strategy model used in building the bond portfolio by the duration immune strategy. Furthermore, during the case study, we also find some advantages like the immunization of some potential interest risk and its limitations, such as the large workload of the dynamic change in need of building the portfolio in every stage and some loss of potential gain when the market expects steady growth.

By the case study of one asset - one liability immune strategy model, we get one of its specific application in building the bond investment portfolio and find the advantages and limitations while using the immune strategy. The outcome of the case study and findings of the paper gives a representative example of bond portfolio based on duration immunization strategy for the readers while their decide to do some investments in bond market with the strategy, and help the research beginner to have a basic and general understanding of some measuring subjects of sensitivity in the fix-income market.

The shortcomings of this paper mainly include two aspects. First, while doing the case study, we assume that all debt considered has the same level of default risk. When it comes to the real market, the default risks of different debts will not be the same. Second, in the case study of this paper, we research the fundamental one asset - one liability immune strategy model. Besides this model, there are some other more complex models of

duration immune strategy. This is also the research direction in the future that when the debt is not a term but a debt flow in the immune strategy model and then compares it with the one - debt model in the paper.

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