

# The Valuation of Basket-lookback Option

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## ABSTRACT

Exotic options have becoming more popular in the markets. We designed a new over-the-counter product by working on a combination of basket and lookback options and explained its rationale. To value the derivative, we constructed a pricing model in Python using the Monte Carlo method. For a sample simulation, we chose a portfolio of 10 stocks from the driverless technology sector. After collecting the historical data, we estimated the covariance matrix. To perform the valuation, we applied the Cholesky decomposition and moment matching and simulated correlated price paths. Finally, a sensitivity analysis was conducted to evaluate the impact of variations in the parameters on the option value and seek the practical significance of the new product intuitively. With stable and reasonable simulated pricing results, we concluded that our product could play a good role in helping investors grasp the potential value of one specific sector and providing psychological comfort by minimizing regrets. This paper effectively provides a new financial product that will be of great practical significance.

**Keywords:** Lookback options, Portfolio, Driverless car, Monte Carlo method.

## 1. INTRODUCTION

Instruments of these so-called exotic options have become ever more prevalent in the markets in the past two decades. Lookback options are among the most popular path-dependent options in the financial market. It is a path-dependent option with the payoff determined by the settlement and the extrema of the asset price within the option's life. On some specialized markets, such as foreign exchange markets, these options are already traded. However, some of these options still need to be developed and be baptized by the market.

One of four different types of lookback options is floating strike call lookback options, which pay the difference between the stock price at maturity and the lowest stock level during the term of the option. Like the fixed strike lookback, the floating strike lookback would always be priced as a Monte Carlo process, and since the strike of a floating strike lookback call resets if the share price goes down, one can see that the closest way to replicate the floating strike lookback call option is as a set of forwarding starting options[1].

To derive prices for derivative securities, the Black-Scholes model, which was invented in 1973, is still used

extensively, and early researches had already deduced the general formula for derivatives and concrete computations of the prices of put and call options, even those exotic options. Computing an option price means adding the discounted expectation of the payoff. It is widely acknowledged that the Monte-Carlo method for estimating an option price is straightforward to implement. Nowadays, reasonable random numbers generators can be found in every programming language such as Python[2].

Since the Internet and smartphone revolutions, driverless cars have been deemed one of the key disruptors in the next technology revolution and have been recognized as a critical area for future research [3]. To explore the economic value of this field in-depth, we built a portfolio with 10 stocks closely related to driverless technology. Considering its underlying and long-term value, the basket-lookback option was used to study our portfolio's option pricing strategy. In other words, we mainly focused on the simulation of the pricing lookback option with a specific portfolio that we built beforehand.

Compared with existing models, the quantity of underlying assets makes the problem more complicated. When dealing with options on a portfolio, it is significant

to realize that the simulated stock prices, which ultimately contribute to the weighted cost of the portfolio, should be correlated. One popular method is called cholesky decomposition to help us to get correlated regular random numbers in the pricing model and is convenient to program and solve. And then, simulations such as the Mont Carlo method are used to price the options. Several researchers have used this method to deal with correlated variables[4]. In previous research, period, number of periods out, and risk-free discount rate are the significant parameters researchers considered when doing sensitivity analysis[5].

Thus, apart from finding the value of our basket-lookback option with floating strike price, we also conducted a sensitivity analysis on two pricing model parameters to test whether our financial package is robust and better describes the actual situation of the economic situation market. In previous research, several periods out and risk-free discount rate are the significant parameters researchers considered when doing sensitivity analysis.

Only a few pieces of research have a focus on the pricing of the basket lookback option. Sa indicated that the Monte Carlo simulation method to price is relatively more feasible[6]. In the book Numerical Methods for Pricing, the author analyzed the European Lookback Put Basket Option on a basket of two assets using the Black-Scholes equation[2]. And earlier in 2009, MB Giles had already pointed out that the Mont Carlo simulation can be used to price the basket lookback option [7]. Liu Jingwei also priced the European basket call option with n assets whose price and bonds are driven by Exponential

Ornstein-Uhlenbeck processes and concluded a pricing formula for these kinds of options [8].

In general, this paper mainly studies the pricing simulation of a specific basket-lookback option, which means we have created a portfolio in the field of driverless and used a floating strike basket-lookback option pricing model to price this specific option. By analyzing pricing results and conducting sensitivity analysis, we finally got stable and reasonable pricing results, which were connected with reality to test its practical significance.

The rest of this paper is organized as follows. In Sect.2, the stock's basic information and the logic process of how we built the portfolio, and why we used the basket-lookback option to price the portfolio are introduced. In Sect.3, we state the methods and models we used and explain the simulation process in detail. In Sect.4 and Sect.5, pricing results and the sensitivity analysis we performed on two parameters are shown. Finally, a brief conclusion is given in Sec.6.

## 2. DATA

### 2.1. Portfolio

This paper mainly collected stock data of 10 companies to construct a portfolio. Considering the company's relevance to driverless cars, we selected five types of companies that contribute to the development of the driverless car industry. The specific content is presented in Table 1 below.

**Table 1.** Names and stock tickers of 10 companies.

Category	Company	Ticker
Automobile enterprise	NIO Inc.	NIO
	Tesla Inc.	TSLA
	Ford Motor Company	F
	General Motors Company	GM
Algorithm research(IT)	Baidu Inc.	BIDU
	Alphabet Inc.	GOOG
	Apple Inc.	AAPL
CPU chip	NVIDIA Corporation	NVDA
Laser radar technique	Velodyne Lidar Inc.	VLDR
Car camera	Panasonic Corporation	PCRFY

Throughout the global market, the main force in the advancement of the driverless car industry is definitely the traditional automobile enterprises. Take Tesla as an example. Tesla is making a leap forward in the race to build self-driving cars with its strong research teams and relatively complete self-developed autonomous driving mode. For more specific professional research, enterprises like Google are mainly contributing to algorithm development. Thus, they are also closely related to the driverless car industry. For example, Google has been developing self-driving cars and testing them under employee supervision on public roads since 2009[9].

Furthermore, considering the supply of different components for driverless cars, we selected three types suppliers (chips, sensors, cameras). NVIDIA uses its decades of experience in the fields of high-performance computing, imaging, and AI to build a software-defined end-to-end platform for the transportation industry that can achieve continuous improvement and continuous deployment through wireless updates. The platform can meet all the needs for large-scale development of autonomous vehicles. Velodyne Lidar is a technology company located in Silicon Valley, California. Its products include real-time lidar (light detection and range finding) sensors used by companies such as Google, Uber, and Baidu. To participate in driverless R&D, Japan's Panasonic Corporation has developed an unmanned automatic valet parking system and an AR-HUD (augmented reality head-up display), which can realize fully automatic driving (SAE L4) in a limited area, and such technologies will be installed On the Toyota Motor Corporation's concept car LQ.

To collect the information of these 10 stocks, we retrieved dynamic data from Yahoo Finance (<https://finance.yahoo.com/>) and downloaded the monthly stock close price of 10 companies from October 1,2019 to October 1,2021. In addition, we visualized the stock prices in Excel to make the steadily upward trend of each stock more intuitive. Figure 1, Figure 2 and Figure 3 below are three representative stock prices graphs.

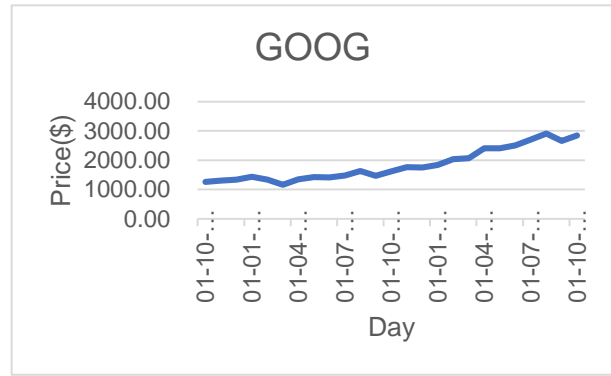


Figure 2 Stock prices trend line of GOOG

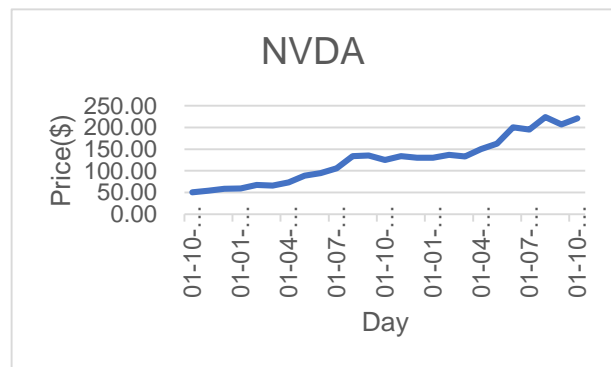


Figure 3 Stock prices trend line of NVDA

In excel, we got the correlation matrix and covariance matrix of 10 stocks price data and calculated the return and standard deviation of the portfolio to get the Sharpe ratio:

$$\text{SharpeRatio} = \frac{E(R_p - R_f)}{\sigma_p} \quad (1)$$

Using the function called “Data Analysis” in Excel, we finally got the optimal portfolio based on the maximal Sharpe ratio.

### 2.2. Basket-Lookback Option

Path-dependent options, which have become more actively traded recently, provide payoffs that may be better suited to customers' needs than ordinary options. Among these, lookback options are very attractive options because they keep track of past events. As a consequence, they allow their holder to take advantage of anticipated market movements without knowing the exact date of their occurrences and may provide psychological comfort by minimizing regrets[10].

Driverless technology is not yet mature, and the driverless car is also not yet popularized. However, the investment community highly anticipates the promising prospect of this field. Thus it is meaningful and practical to use the basket-lookback option to study option pricing strategies of our portfolio.

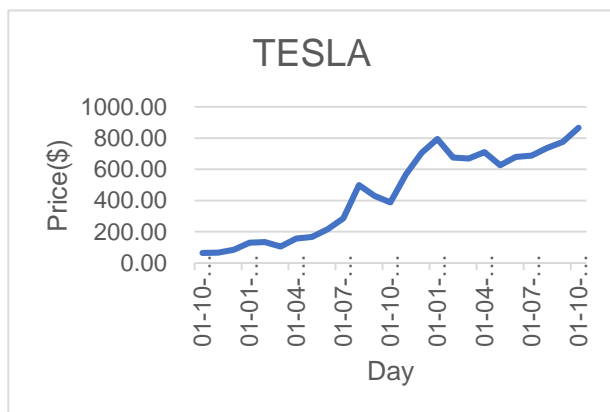


Figure 1 Stock prices trend line of TESLA

### 3. METHOD

#### 3.1. Model Structure

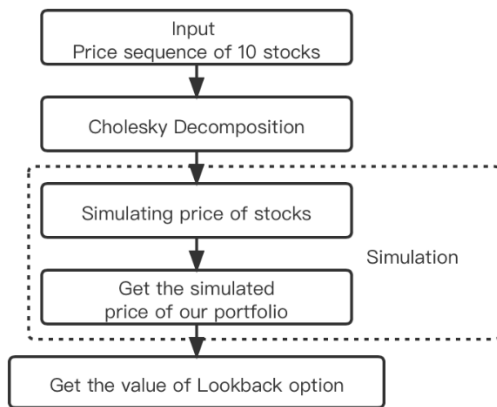


Figure 4 Flow chart of Python code

Three methods/equations were used to price option: Cholesky decomposition, Black-Scholes Model, and the floating strike lookback option calculation.

Since we combined the lookback option with the basket option, the calculation was different from the traditional floating strike lookback options. First, we adopted the method of simulating the price of a basket option, input the price sequence of 10 stocks, and used the cholesky decomposition to get the simulated distribution  $z$  of the stocks in the portfolio. Then we did the moment matching to get the target simulated  $z$  of each stock in each day. By doing this, we were able to consider the historical correlation between each stock in the portfolio. After that, we were able to simulate the price of each stock every day for ten days using the Black-Scholes model since the option is a ten-day option. Next, we calculated the price of the portfolio by getting the weighted sum of the chosen stocks. Finally, adopting the traditional way to calculate the floating strike lookback option, the price of the option was obtained.

#### 3.2. Cholesky Decomposition

Cholesky decomposition or cholesky factorization, discovered by André-Louis, is commonly used in the Monte Carlo method for simulating multiple correlated variables. It is a decomposition of a Hermitian, positive-definite matrix into the product of a lower triangular matrix and its conjugate transpose.

$$A = LL^* \tag{2.}$$

$L$  is the lower triangular matrix with real and positive diagonal entries, and  $L^*$  is the conjugate transpose of  $L$ .

Generating the random paths for each asset independently will not reflect how the assets or stocks in the portfolio have historically been correlated. When

dealing with option pricing of a basket of assets, cholesky decomposition helped us to generate sequences of correlated random numbers so that the historical correlation can be reflected.

#### 3.3. Black-Scholes Model

The Black-Scholes model or the Black-Scholes-Merton model is one of the essential mathematical equations to estimate the theoretical value of derivatives and other investment instruments. Taking into account the influence of time and risk, the equation posits that the price of stock shares and futures contracts follows a random walk with constant drift and volatility.

$$C = S_t N(d_1) - K e^{-rt} N(d_2) \tag{3.}$$

where:

$$d_1 = \frac{\ln \frac{S_t}{K} + \left( r + \frac{\sigma_v^2}{2} \right) t}{\sigma_s \sqrt{t}} \tag{4.}$$

and

$$d_2 = d_1 - \sigma_s \sqrt{t} \tag{5.}$$

In this equation,  $C$  means the call option price,  $S$  is the current stock,  $K$  indicates the strike price,  $r$  represents the risk-free interest rate,  $t$  is the time to maturity, and  $N$  is a normal distribution.

By using this mathematical model, one that can price and simulate the financial instrument such as stocks and European options. Here, we were using this equation to simulate the price of our chosen stocks in the portfolio.

#### 3.4. Lookback Option

The floating strike Lookback option is a type of exotic path-dependent option. And here, we focused on its call option: when it is a floating strike call option, the lowest price during the time to maturity is the strike price of the floating strike call option. And the payoff of this option can be calculated as follow:

$$f_c(s) = \max(S_T - S_{min}, 0) \tag{6}$$

But the option we calculated here is different from the traditional floating strike price lookback options. Instead of stimulating the price of single equity, we used the weighted simulated price of our stocks to get the payoff of our options.

### 4. RESULT

Each asset in the portfolio had a weight of 0.1 and we had ten stocks in our portfolio. For our 10-day floating strike basket-lookback option, the result of prices and payoffs of this option with 10000 times simulation are in Table 2.

**Table 2.** The payoff and price of the ten-day option

	1	2	3	4	5	Average	S.D.
Payoff	51.866997	51.927291	51.945502	51.899091	51.898267	51.907430	0.02301282
Price	51.866174	51.926466	51.944677	51.898267	51.878996	51.902916	0.03256662



**Figure 5** Python output result

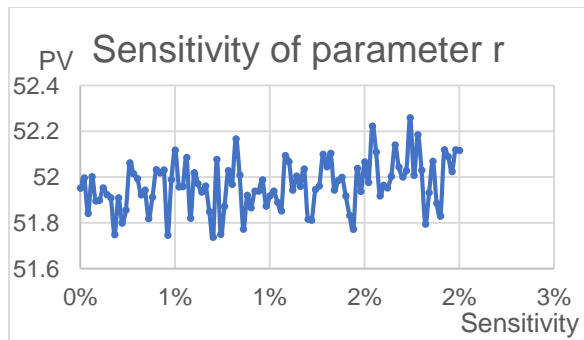
After calculating the price of the 10-day floating strike basket-lookback option for 5 times, the price should be 51.90 (see Figure x). The standard deviation here indicates that the price is stable. Because this is a ten-day option, the difference between the payoff and the price of the option is not obvious.

We speculated that many factors might influence the price of the basket-lookback option, such as the date to maturity and the risk-free rate. In order to further analyze the influencing mechanism, sensitivity analysis was performed.

**5. SENSITIVITY ANALYSIS**

**5.1. Risk-free rate**

We suspected that the risk-free rate might influence the price of our option. Thus we calculated the prices of the portfolio with  $r$  varying from 0% to 2%.

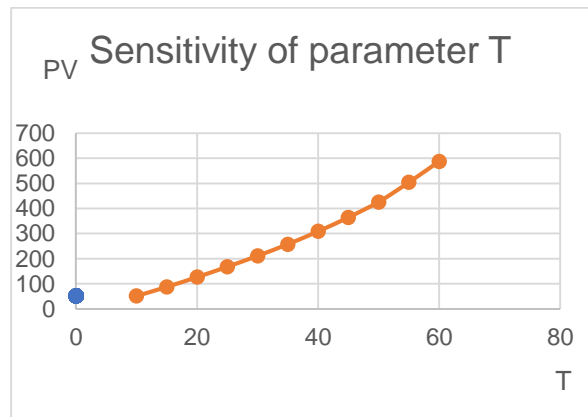


**Figure 6** Sensitivity visualization of  $r$

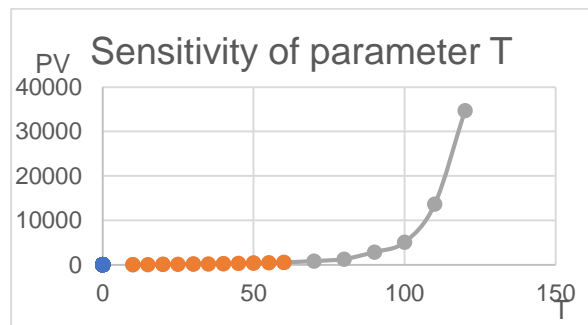
As the risk-free ratio varying, the price of the option fluctuated like stock prices(Fig 6). There was no significant upward or downward trend. Thus either adverse effects nor positive effects existed. The risk-free rate was not the potential influencing factor of the option price.

**5.2. Time to Maturity**

Then we simulated the price of our option with a changing date to maturity. When the maturity time went up from 10 days to 60 days, the value of the option increased stably (Fig 7). However, when the maturity time reached 120days, the price skyrocketed to nearly 35000 (Fig 8). A positive influence could be inferred from the incredible increase in the option price. The astonishing increase in the price of the 120-day options might be explained by the upward momentum of the driverless car and its related industry in the near future.



**Figure 7** Sensitivity visualization of  $T(60days)$



**Figure 8** Sensitivity visualization of  $T(120days)$

## 6. CONCLUSION

We have presented the logic of the portfolio construction related to driverless cars and the pricing simulation process of our basket-lookback option with floating strike price. The simulation results showed that the average price of a 10-day floating strike basket-lookback option was 51.90, and the standard deviation indicated that the price was stable and practical. According to our sensitivity analysis, on the one hand, the simulated price fluctuated with varying risk-free rates, which means the price of the option was sensitive to the parameter 'r' and the pricing model was robust to be applied in a real-life situation. On the other hand, when the length of maturity reached a certain level, the price got a sharp increase, which means this option had a great potential value and was worthy of investment with long maturity. Objectively speaking, the price of this basket-lookback option is relatively high compared with other options, so this option's application still needs to be optimized with a higher cost-effective implementation in the future. Our future research will aim at optimizing the pricing model and seeking the most cost-effective option price.

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