The Valuation of European Option on Underlying Stock Price Based on AAPL index

Wanquan Wu[1,*,†], Shuya Zhang[2,†]

[1]*College of Business Missouris State University MO65897 Springfield, USA*
[2] *School of Finance, Dongbei University Of Finance And Economics, 116012, Dalian, China*
[*]*Corresponding author. Email: wu15615@live.missouristate.edu*
[†]*These authors contributed equally.*

ABSTRACT
Simulating option prices is one of the most commonly used arbitrage investment methods by financial investors at present. In this paper, the Black-Scholes model is used to simulate option price, which is widely used in option valuation. At the same time, taking the European option as an example, this paper selects the daily underlying stock price data of Apple Inc. from November 2018 to October 2021 and obtains a total of 745 observed values. Secondly, by searching and calculating the value of relevant parameters, the simulated option value and the price of call and put options are obtained. The results were in line with expected price movements. Finally, considering that the single target data cannot explain the robust results, sensitivity tests of different strike prices, different risk-free interest rates, and different maturity dates are carried out. The results show that the option prices under different conditions are consistent with the expected prices. To sum up, in this paper it is of theoretical and practical significance to simulate European option price based on the underlying stock price.

Keywords: *Black-Scholes model, European option, option price.*

1. INTRODUCTION

It is common for people to buy options as some sort of insurance when buying stocks. In 1990, Lauterbach, B., & Schultz, P examined more than 25,000 observations of daily prices of warrants. They concluded that “Perhaps the most common technique for pricing warrants is to use a version of the Black-Scholes model that includes an adjustment for dilution” [1]. In 2003, Amilon, H. studied option and hedging performance. He concluded that "In practice, daily hedging is not very realistic, but I see no reason why the same methods cannot be used for data with higher frequency, or ultimately, tick data" [2].

In addition, in 2005, Tsang & Markose compared the price of options priced by Black Scholes formula with the market price. They concluded that "the prices of the option and futures of a stock both reflect the market expectation of futures changes of the stock price" [3]. In the same year, Lee, C. F., Tzeng, G. H., & Wang, S. Y study the fuzzy decision theory. Then they concluded that “‘the impact of implicit ‘Fuzziness’ is inevitable due to the subjective assessment made by investors in a B-S OPM” and that “the fuzzy B–S model would result in a more realistic methodology for a B-S model” [4].

In 2010, Ivancevic, V. G. studies a non-linear option-price model based on an NLS equation with adaptive parameters. Finally, they proposed that the "adaptive–wave model, both the single NLS-equation and the coupled NLS-system which represents a bidirectional associative memory, is a Spatio-temporal dynamical system of great nonlinear complexity, much more complex than the Black-Scholes model” [5].

Apple stock is what investors often choose to buy and because of the high volatility, people may also choose to buy an option. Because of the popularity of apple stock, we are interested in options for apple stock.

In this paper, the B-S option pricing model is selected to simulate European option prices based on the underlying stock prices of AAPL from November 2018 to October 2021, and a total of 745 observed values are obtained. By using different strike prices, different risk-free rates, and different maturity dates, sensitivity analysis is conducted to check whether there is a large error in the option pricing model under different

parameter conditions. According to the above analysis, the trend of option value and stock price is consistent. When stock prices rise steadily, the value of options also rises steadily, which has a certain positive effect. The simulated option price is consistent with the expected price, and the sensitivity test further improves the accuracy of the option price.

The rest of this paper is divided into the following five parts. The second part is data selection. The third part is the model introduction. The fourth part is empirical analysis. The fifth part is sensitivity analysis. The sixth part is the result analysis.

2. COMPANY DESCRIPTION

Apple company is a large international company that specializes in electronic devices such as iMac, iPad, iPhone, air pods, iWatch, and online services including iCloud. It was founded by Steve Jobs, Steve Wozniak, and Ronald Wayne in 1976 in order to develop personal computers. They started to make their computer in a small bedroom and then operate in a small garage, did IPO in 1980, developed the famous apple Macintosh in 1984 iPod and Apple retail stores in 2001, and MacBook Air and iPhone 3 G in 2008. Step by Step, they become a multinational large company. As for their business philosophy, which contributes to their success, Johnson concluded that "Strategy of the company also includes expanding its distribution network to effectively reach more customers and provide them with a high-quality sales and post-sales support experience" [6]. L Besides, they only get involved in markets where they can make significant contributions, focus on truly important and meaningful projects, be honest to admit when they are wrong and believe in deep collaboration.

Now, based on Table 1, apple company has become a huge multinational company. By 2020, they have had more than 14,000 employees and according to the latest income statement obtained from Yahoo finance, they generate 365.82 billion dollars of revenue, 152.84 billion dollars of gross profit, and 94.68 billion dollars of net income.

Table 1. Income Statement

Relevant Indicator	Data
Revenue	365.82B
Revenue Per Share	21.9
Quarterly Revenue Growth	28.80%
Gross Profit	152.84B
EBITDA	120.23B
Net Income	94.68B
Diluted EPS	5.61
Quarterly Earnings Growth	62.20%

For the historical price of Apple stock, we obtained the Ami daily close price between October 30, 2018, and October 30, 2021, from Yahoo finance. According to the data, we make a historical daily stock price trend graph, which displays how the close price of Apple stock changed over the three years.

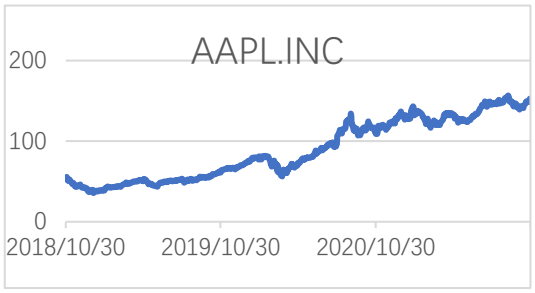

Figure 1 Historical Prices.

As can be seen in figure 1, in terms of the daily price of apple, although there are some fluctuations, the close prices saw a rising trend, growing from only approximately 60 dollars to almost 160 dollars over this period.

Table 2. Descriptive Statistical analysis

Variable Name	Max	Min	Average	St.dev
Price	156.69	35.55	89.43	37.50
Return	10.07	-10.52	0.13	1.94
Risk-free	2.51%	0	0.96%	0.010

Table 2 shows the descriptive statistical analysis of daily price, daily return, and daily risk-free rate during this period. By describing these data, during these years, the investor can earn a positive return on average. On top of that, the maximum return of apple stock is significantly higher than the risk-free rate, which proves that compared with the risk-free rate, the investors are able to earn more return for investing in apple. Li has studied the return of AAPL in the years of 2018 and 2019 and she reached a conclusion that "in terms of profitability, return on assets and return on equity, Apple is above the industry average" [7].

3. METHOD

A European option is an option contract that limits its execution to its expiration date. In other words, the call option and put option will only execute at the date of maturity. This means if an underlying asset, for example, a stock moves in price, the investor cannot exercise the option early.

For the European call option, the investors can only earn profits when the market price is above the strike price at the date of maturity. Therefore, the investors only purchase this type of option when they are confident about the market and expect the market price will be higher than the strike price. Otherwise, the investor would not choose to exercise this option and this option would expire worthlessly.

On the other hand, for the European put option, the

investor would sell it at the time of expiration, when the market price is lower than the strike price to earn profit and cover the cost of the premium. Otherwise, the investor would not exercise this option contract.

To price the European option, Zakamouline concluded that, "the break-through in option valuation theory starts with the publication of two seminal papers by Black and Scholes, and Merton" [8]. In other words, Black and Scholes is a good way to value an option. This is the B-S formula:

$$S_T = S_0 e^{(\alpha - \frac{1}{2}\sigma^2)T + z\sigma\sqrt{T}} \quad (1)$$

In this formula, according to Malliaris, "T is the number of days remaining until expiration of the option expressed as a fraction of a year" [9]. In other words, this formula is highly related to the maturity date. According to TSAI, "the price of a European option depends on its initial asset price, moneyness, time to maturity and volatility of the underlying asset". [10] Thus, the B-S formula that incorporates all of these factors is suitable for pricing European options. Therefore, in this paper, we choose the B-S model as the main method to price the option.

This formula assumes that the future stock price can be represented as a function of "z", that during the lifetime of the option no dividend is paid, that risk-free rate and volatility are constant, and the movement of the market cannot be predicted.

4. EMPIRICAL ANALYSIS

4.1. Sample data analysis

The corresponding parameter estimates are given in Table 3. The return which is 0.16%, means the average return of the sample between 2018 and 2021. Std.DEV which is 60.96%, means the standard deviation of the sample. Variance which is 0.05%, means the mean dispersion of the sample.

Table 3. Parameter Estimates

Variable Name	Numerical Calculation	Introduction
Return	0.16%	Average return of sample
Std.DEV	60.96%	Standard deviation of sample
Variance	0.05%	Mean dispersion case of sample
Rate	0.04%	Treasury bill by 10/15/21
Beta	1.26	Systematic risk of sample
Delta	0.22	AAPL dividend
Sigma	0.22	Option pricing volatility

This paper obtains the relevant data for option pricing through calculation and data published by the underlying company, including Rate, Beta, Delta, Sigma. Based on these data, simulation analysis is carried out. According to the above analysis, the fluctuation changing trend of return and demeaned return can be obtained as shown in Figure 2 and Figure 3. The overall fluctuation range is -0.15 ~ 0.15, and the yield trend is more intensive between -0.05 ~ 0.05. As can be seen, price fluctuations are relatively flat.

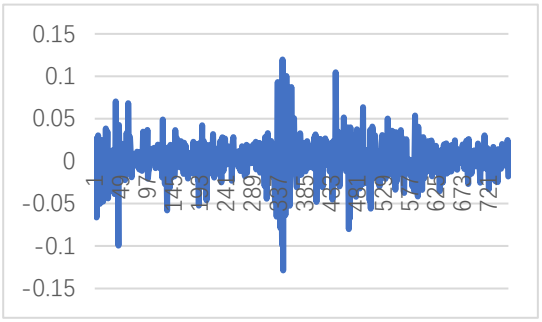

Figure 2 Return.

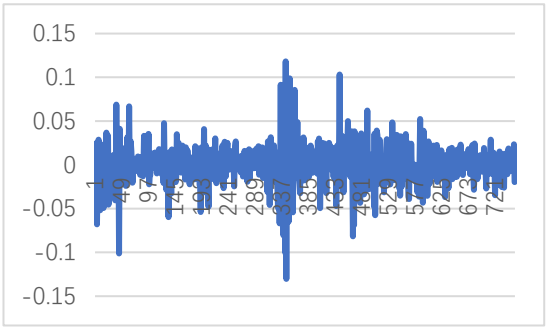

Figure 3 Demeaned return.

4.2. European option pricing

According to the option pricing formula and sample data selection, a total of 755 simulations were conducted. This paper chooses the European option pricing model with a maturity of 10 days and can only exercise the option on the maturity date. Figure 4 shows the movement of option values ST. Figure 5 shows the call and put price trends respectively. Table 4 shows the mean and discount of the call option and put option respectively.

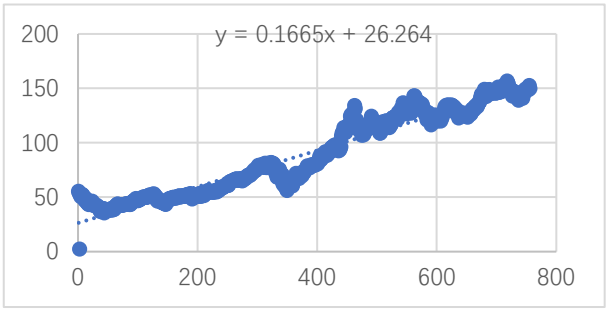

Figure 4 The movement of option values ST.

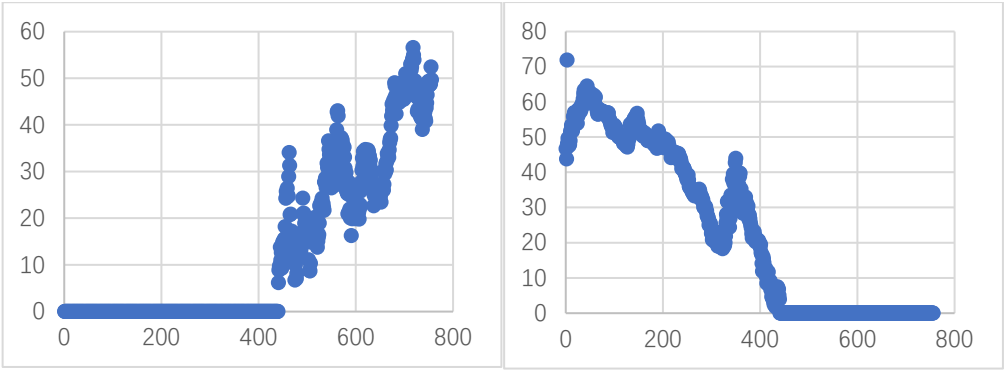

Figure 5 Option price of the call and put price trends.

Table 4. Average and Discounted

Option	Average	Discounted
Call	12.59	12.59
Put	23.31	23.31

5. EMPIRICAL ANALYSIS

Then, this paper carries out sensitivity analysis for different strike prices, different risk-free interest rates, and different maturity dates.

5.1. Strike price

Figure 6 shows option prices in different strike prices from 40 to 150.

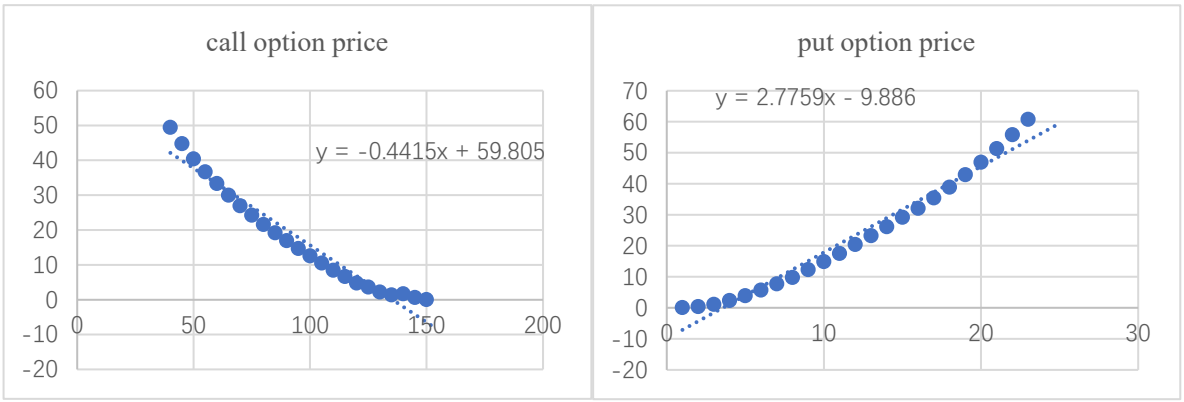

Figure 6 Option price with the strike price.

As can be seen from the figure, the call and put option basically conform to the price fluctuation rule. When the strike price is higher, call option price lower and put option price higher. The range of option price fluctuation is related to stock price fluctuation. Call option range from 50 to 0 and put option range from 0 to 60. The option price trend tends to be linear and there is no abnormal fluctuation.

5.2. Risk-free rate

Figure 7 shows option prices in different risk-free rates from 0.00% to 2.00%. As can be seen from the figure, the price fluctuation of the call option is basically between 10 and 15. That is not far off the price of the call option, which was originally set at a risk-free rate of 0.04%. The overall fluctuation is in a small range, and peak price occurred only in a small segment. In the same way, the price fluctuation of the put option is basically between 23 and 24. That is also not far off the price of the put option, which was originally set at a risk-free rate of 0.04%. When the risk-free rate is higher, call option price lower and put option price higher.

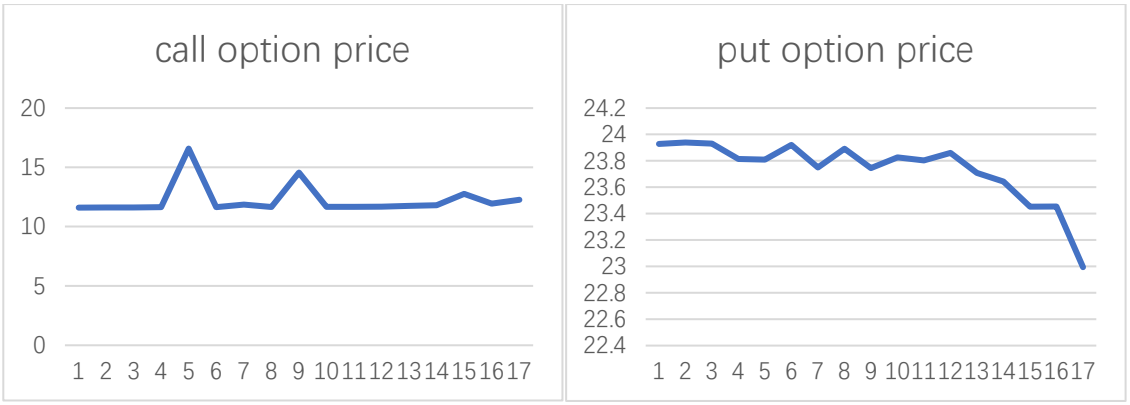

Figure 7 Option price with risk-free rate.

5.3. Maturity date

Figure 8 shows option prices in different maturity dates from 10 days to 200 days. As can be seen from the figure, the price fluctuation of the call option is basically between 11 and 14. That is not far off the price of the call option, which was originally set at a maturity date of 10 days. The overall fluctuation is in a small range, and peak price occurred only in a small segment. In the same way, the price fluctuation of the put option is basically between 23 and 24. That is also not far off the price of the put option, which was originally set at a maturity of 10 days. When the risk-free rate is higher, call option price lower and put option price higher.

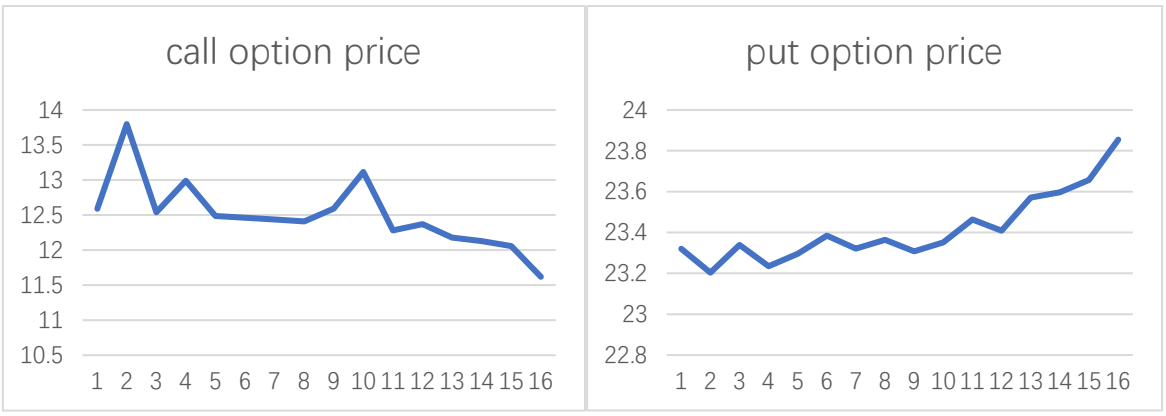

Figure 8 Option price with a maturity date.

According to the above analysis, whether it is the original data simulation or three different sensitivity tests, it can be concluded that the price range of the European call option is 10 ~ 15, and the price range of the European put option is 20 ~ 25. And the option price is related to the underlying stock price selected.

6. CONCLUSION

Simulating option prices is one of the most commonly used arbitrage investment methods by financial investors at present. In this paper, the B-S option pricing model is selected to simulate European option prices based on the underlying stock prices of AAPL from November 2018 to October 2021, and a total of 745 observed values are obtained. By using different strike prices, different risk-free rates, and different maturity dates, sensitivity analysis is conducted to check whether there is a large error in the option pricing model under different parameter conditions. The research results are as follows.

First of all, from the trend chart of return rate and demeaned return rate, the overall fluctuation range is small, almost all within 0 ~ 10%, a few within 10% ~ 15%, both fluctuation trend is basically the same. Secondly, the fluctuation trend of option value simulated by the B-S option pricing model is consistent with the underlying stock price trend. When stock prices rise steadily, the value of options also rises, and the overall trend can be expressed by a linear equation. According to the option price trend chart, the price trend of the call option and put option is in line with expectations, basically fluctuating between 0 and 60, with an average of 12.59 and 23.31, respectively. Finally, a sensitivity test is added to verify the accuracy of simulated option price results. The results show that the option prices under different parameters are basically consistent with the expectation, and the prices of call option and put option fluctuate between 10 ~ 15 and 20 ~ 25 respectively. This shows that the simulated option price has high accuracy

and has a positive impact on stock price prediction of option value.

Amin, & Seyhun have studied the relationship between stock returns and options, and they concluded that “past stock returns exert a strong influence on the pricing of index options” [11]. This means the historical return of stocks can affect the price of the option. In this paper, however, we only take apple company as an index to value European options. It is not very reasonable because though Apple is a huge international corporation, it cannot represent the whole market. In the future, we would like to take more index to value options. On top of that, the Black Scholes pricing model also has certain restrictions. For instance, it assumes that during the lifetime of the option no dividend is paid, that risk-free rate and volatility are constant, and the movement of the market cannot be predicted but in the real world, usually, this cannot happen. Likewise, Janko has studied the drawbacks and limitations of the Black-Scholes model and concluded that "another strong prerequisite necessary for the derivation of the Black-Scholes model is the perfect derivative replication by the share and a risk-free instrument; however, this cannot be achieved without transaction costs" [12-13]. This means B-S formula does not consider the transaction cost. We will take these factors into consideration in the coming future.

REFERENCES

[1] Lauterbach, B., & Schultz, P. (1990). Pricing warrants: an empirical study of the Black-Scholes model and its alternatives. The Journal of Finance, 45(4), 1181-1209.

[2] Amilon, H. (2003). A neural network versus Black–Scholes: a comparison of pricing and hedging performances. Journal of Forecasting, 22(4), 317-335.

[3] Tsang, E., Markose, S., & Er, H. (2005). Chance discovery in stock index option and futures arbitrage. New Mathematics and Natural Computation, 1(03), 435-447.

[4] Lee, C. F., Tzeng, G. H., & Wang, S. Y. (2005). A new application of fuzzy set theory to the Black–Scholes option pricing model. Expert Systems with Applications, 29(2), 330-342.

[5] Ivancevic, V. G. (2010). Adaptive-wave alternative for the black-scholes option pricing model. Cognitive Computation, 2(1), 17-30.

[6] Johnson, K., Li, Y., Phan, H., Singer, J., & Trinh, H. (2012). The Innovative Success that is Apple, Inc.

[7] Li, B. (2020). Evaluation of Financial Risk in Apple Company.

[8] Zakamouline, V. I. (2006). European option pricing and hedging with both fixed and proportional transaction costs. Journal of Economic Dynamics and Control, 30(1), 1-25.

[9] Malliaris, M., & Salchenberger, L. (1993, March). Beating the best: A neural network challenges the Black-Scholes formula. In Proceedings of 9th IEEE Conference on Artificial Intelligence for Applications (pp. 445-449). IEEE.

[10] TSAI, P. C., & CHEN, H. J. (2020). LEVERAGE EFFECT IN VOLATILITY AND PRICE JUMPS: NEW EMPIRICAL EVIDENCE.

[11] Amin, K., Coval, J. D., & Seyhun, H. N. (2004). Index Option Prices and Stock Market Momentum. The Journal of Business, 77(4), 835–874. https://doi.org/10.1086/422440

[12] Janko, Z. (2018). Drawbacks and limitations of black-scholes model for options pricing. Journal of Financial Studies & Research, 2018, 1-7.

[13] Li, W., Cheng, Y., & Fang, Q. (2020). Forecast on silver futures linked with structural breaks and day-of-the-week effect. The North American Journal of Economics and Finance, 53, 101192.