An Overview of Bond Pricing Models and Duration of Bonds

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ABSTRACT
This paper presents and compares different bond pricing models, points out whether these models work well and if there are any limitations of these models, this paper concludes models for corporate bonds, convertible bonds, and zero-coupon bonds. Besides, this paper introduces Macaulay duration and Modified duration, evaluates how duration could affect the bond's sensitivity to interest rates, and the usefulness of duration in the bond pricing process. Considering bonds are such an important element of the capital markets, investors and analysts would like to know how the many characteristics of a bond combine to determine its intrinsic value. The value of a bond, like the value of a stock, decides whether it is a good investment for a portfolio and is thus an important stage in bond investing. Calculating the present value of a bond's estimated future coupon payments is what bond valuation is all about. The premise of using this article is under the International Financial Reporting Standards (IFRS).

Keywords: *Bond pricing model, duration, corporate bond, convertible bond, zero-coupon bond, yield to maturity (YTM).*

1. INTRODUCTION

Bond is defined as a type of financial contract or says a fixed-income instrument, which happens when government, financial institutions, and corporations raise loans and funds to finance projects and operations directly from the society, bonds are issued to investors and also are committed to paying an agreed level of interest rate while paying back the initial investment, so it's also referred to as fixed-income security [1-2]. The nature of the bond is proof of debt which has the force of law. Owners of bonds are debtholders, or creditors, of the issuer.

In the bond market, there are four primary categories of bonds. On some platforms, however, you may find foreign bonds issued by corporations and governments. These are corporate bonds, municipal bonds, government bonds, and agency bonds. There are many different types of bonds accessible to investors [3]. They can be characterized by the rate or type of interest or coupon payment, by being recalled by the issuer, or because they have other attributes. If we divide the categories by the methods of payment, there are zero-coupon bonds, level-coupon bonds, and floating-rate bonds [4-5].

The market prices bonds based on their particular characteristics. A bond's price changes daily, just like that of any other publicly traded security, where supply and demand in any given moment determine that observed price. Bond prices are inversely correlated with interest rates: when rates go up, bond prices fall and vice-versa. Bond prices are also greatly influenced by the creditworthiness of the issuer, from the federal government down to a junk bond issuer on the verge of default.

Bond valuation is the process of determining the fair price of a bond. Typically, this will involve calculating the bond's cash flow or the present value of a bond's future interest payments as well as its face value, which refers to the bond's value once it matures. A bond's interest payments and face value are fixed which allows investors to determine what rate of return a bond needs to provide to be considered a worthwhile investment.

This article reviews the exiting bond pricing model and duration which would benefit the researchers to find the research field. In Section 2, the article presents the bond pricing model. In Section 3, the paper introduces the different types of duration. In Section 4, the article illustrates the benefits and limitations of a zero-coupon

bond. Finally, the paper summarizes the conclusion in the last section.

2. BOND PRICING MODEL

There are lots of academic researches and papers that identify and analyze different models used in pricing bonds. This paper will discuss pricing models of corporate bonds, convertible bonds, and zero-coupon bonds.

2.1. pricing models of corporate bonds

Using a sample of noncallable bonds issued by enterprises with simple capital structures between 1986 and 1997, Eom et al., [1] directly examines five corporate bond pricing models: Merton, Geske, Longstaff, and Schwartz, Leland, and Toft, and Collin-Dufresne and Goldstein [6-8]. Researchers examine each of these models to compare their abilities to predict corporate bond spreads under similar consumptions and to see if the expected spreads are susceptible to systematic mistakes.

The results show that all of the models have significant spread prediction errors, but the magnitude and sign of the mistakes varied dramatically. Because the dispersion of predicted spreads is relatively large, as seen in the high standard deviations and significant average absolute prediction errors, the average error is a rather poor measure of a model's predictive power. All these five models provide extremely low spreads on bonds they perceive to be safe (typically low leverage and low asset volatility) and extremely high spreads on bonds they perceive to be very risky (usually high leverage and high asset volatility). The results always point to a substantially lower expected spread for lower-rated bonds, ceteris paribus, although most models generate their greatest spreads on junk bonds. While rating agencies place a strong emphasis on leverage, they also evaluate the risk of default from other sources, which may improve the models.

However, accuracy is an issue, as newer models tend to exaggerate the credit risk of companies with high leverage or volatility while underestimating the spread risk of safer bonds. The Leland and Toft model is an exception in that it consistently overestimates spreads on most bonds, especially those with large coupons. More precise structural models must avoid elements that add credit risk on riskier bonds while having little impact on spreads on the safest bonds.

2.2. pricing models of convertible bonds

The structural approach (or firm-value approach) and the reduced-form approach are the two main approaches used to value convertible bonds (stock-value approach). The type of input variables employed is the main distinction between these two procedures. The structural approach makes use of company-specific data, whereas the reduced-form approach makes use of market data.

Convertible bond models are typically expansions of Ingersoll's contingent claim framework and Black and Scholes' and Merton's option pricing models. Given the various hybrid elements that are common in convertible bonds, the valuation procedure is not as clear as it is for straight bonds and options. After reviewing the existing empirical literature, it is clear there remains scope for further examination of the appropriateness of methods used for convertible bond valuation. This review highlights several limitations to these studies with differences in the specifications of the pricing models generating inconsistent findings. For example, there is the critical question of using stochastic instead of constant interest rates in the pricing model. Chan et al. find that one of the most important features of the term structure is its dependence on interest rate volatility, thus term structure models that assume constant volatility are misspecified [9-10].

2.3. pricing models of zero-coupon bonds

The zero-coupon bond prices in the Vasicek and Cox–Ingersoll–Ross (CIR) interest rate models were computed using symmetry analysis as group-invariant solutions of the corresponding valuation partial differential equations. When a partial differential equation and its auxiliary conditions (i.e. initial/terminal and/or boundary conditions) admit a sufficiently rich' Lie point symmetry group, the technique utilizes in the computation is utilized [11].

The derivation of a general functional form of the terminal condition to complement the Vasicek (respectively, the CIR) equation such that the solution to the relevant problem is another possible application of the methods used to compute bond prices.

When a Markov-modulated model with jumps for the short rate is considered [12]. Researchers calculate the expected short rate using the main features of a telegraphic process with jumps and they use the Unbiased Expectation Hypothesis for forwarding rates and construct closed formulas for the zero-coupon bond price.

3. DURATION

Duration is a measure of the sensitivity of the price of a bond or other debt instrument to a change in interest rates. Duration can also measure how long it takes, in years, for an investor to be repaid the bond’s price by the bond’s total cash flows.

3.1. types of duration

There are mainly two types of duration: Macaulay duration and Modified duration. Macaulay duration

estimates how many years it will take for an investor to be repaid the bond's price by its total cash flows. Modified duration measures the price change in a bond given a 1% change in interest rates. A fixed income portfolio's duration is computed as the weighted average of individual bond durations held in the portfolio.

3.1.1. Macaulay duration

Macaulay duration finds the present value of a bond's future coupon payments and maturity value. For investors, most bond search and analysis software packages include this metric as a standard data point. Because Macaulay duration is a partial function of time to maturity, the longer the duration, the greater the risk or benefit for bond prices from interest rates.

The formula refers to equation (1) below, it is divided into two sections. The first part is used to find the present value of all future bond cash flows. The second part finds the weighted average time until those cash flows are paid. When these sections are put together, they tell an investor the weighted average amount of time to receive the bond's cash flows.

3.1.2. Modified duration

The modified duration of a bond helps investors understand how much a bond's price will rise or fall if the YTM rises or falls by 1%. This is an important number if an investor is worried that interest rates will be changing in the short term. The modified duration of a bond with semi-annual coupon payments can be found with the below formula (2).

$$MacD = \sum_{f=1}^{n} \frac{CF_f}{\left(1+\frac{y}{k}\right)^f} \times \frac{t_f}{PV} \tag{1}$$

$$ModD = \frac{\text{Macaulay Duration}}{1+\left(\frac{YTM}{2}\right)} \tag{2}$$

3.2. factors affect duration

The longer the maturity, the higher the duration, and the greater the interest rate risk. Consider two bonds with a 5% yield and a £1,000 purchase price, but differing maturities. A bond with a shorter maturity period, such as one year, would repay its true cost faster than a bond with a ten-year maturity period. As a result, the shorter-maturity bond will have a shorter duration and lower risk.

The coupon rate of a bond is an important aspect in determining its duration. If two bonds are similar save for their coupon rates, the bond with the higher coupon rate will recoup its initial costs faster than the bond with the lower yield. The lower the length and the lesser the interest rate risk, the greater the coupon rate.

3.3. the usefulness of duration in bond pricing

Default risk and interest rate risk are the two most significant threats to a bond's investment value (interest rate fluctuations). Because both factors affect a bond's expected to yield to maturity (YTM), duration is used to assess the impact these factors may have on its pricing. Investors will demand a greater return or YTM to buy bonds if a company begins to suffer and its credit quality deteriorates, for example. To enhance the YTM of a current bond, its price must fall. The same criteria apply if interest rates rise and competitive bonds with a higher YTM are issued [13].

One study shows there are several interesting patterns across the credit rating portfolios when comparing the duration-adjusted CAPM to the Bai et al. model. Despite having higher average R^2 coefficients, the model of Bai et al. has the most explanatory power for the A and BBB portfolios, which account for the majority of the market. Incorporating the corporate bond market return into the Bai et al. model for these portfolios may improve its performance mechanically. Despite having two fewer components and lacking information from the corporate bond market, the duration-adjusted CAPM has better explanatory power for AAA and CCC rated bonds and equivalent performance for BB rated bonds. While the CAPM has difficulty pricing investment-grade corporate bond returns, results show that it matches duration-adjusted returns rather well.

A study result can also confirm that forecasting bond price change with duration and discounting bond cash flow directly with a 4% discount rate have a little difference. This is a minor variation, which indicates that discounting all cash flows immediately with 4% YTM yields nearly the same outcome (bond price 90, 55%). As a result, it can be concluded that length can be utilized to estimate bond price changes with a high degree of accuracy for bonds having a 10-year maturity.

For bonds with shorter maturities, deviations are smaller when anticipating price fluctuations. This suggests that duration is a good estimate and that MSE T-Bonds have lesser convexity. This also demonstrates that, when compared to the duration of the bonds, the combination of duration and convexity is a stronger measure for predicting bond price fluctuations on MSE. The duration of a bond influences its sensitivity to interest rate changes and indicates the approximate moment at which the risk of price changes outweighs the risk of reinvestment. Investors are protected from interest rate changes by holding bonds for the term.

4. BENEFITS AND LIMITATIONS OF ZERO-COUPON BOND

4.1. example of zero-coupon bond

The payment of interest, or coupons, distinguishes a zero-coupon bond from a regular bond. Regular bonds are commonly known as coupon bonds, pay interest all through the life of the bond and repay the principle at maturity. A zero-coupon bond does not pay interest but instead trades at a deep discount, resulting in a profit when the bond is retrieved for its full face-value at maturity.

The investor's return is the difference between the purchase price and the par value. The investor receives a payment equal to the principal invested plus interest earned, compounded semi-annually at a specified yield. For example, a bond with a face value of $20,000 will be matured in 20 years and yield at 5.5%, may be purchased for about £6,855. At the end of the 20 years, the investor will receive £20,000 and the difference between £20,000 and £6,855 represents the interest that compounds automatically until the year of the bond's maturity.

4.2. benefits

The returns of zero-coupon bonds are predictable, as investors pre-know the returns of deeply discounted bonds after maturity in the form of par value, and are guaranteed a return of the full par value. For this reason, zero-coupon bonds are a suitable option for investors who have a specific financial goal in mind, and a long-term but specific time frame, their investments can target on children's education, marriage, and post-retirement goals.

reinvestment risk is the risk that an investor will be unable to reinvest a bond's cash flows (coupon payments) at a rate equal to the investment's required rate of return. Zero-coupon bonds are the only type of fixed-income investments that are not subject to investment risk – they do not involve periodic coupon payments. An example of zero-coupon u.s. treasury bonds can move up significantly when the fed cuts rates aggressively. These gains can more than offset stock-related losses, so treasury zeros are often an excellent hedge for stock investors. They also have solid long-run returns, similar to long-term treasuries. If an investor wants to bet on a bear market, treasury zeros frequently perform dramatically better than inverse etfs and short-selling stocks.

Besides, zero-coupon bonds usually have higher interest rates than other bonds. Since zero-coupon bonds do not provide regular interest payments, their issuers must find a way to make them more attractive to investors. As a result, these bonds often come with higher yields than traditional bonds. The amount varies, but a US Treasury zero often yields at least one percentage point more than its traditional Treasury counterpart, and sometimes a lot more. In 2018, see Figure 1, for example, a 10-year Treasury zero was yielding as much as an annualized 3.1%, while 10-year T-notes were at 2%.

4.3. limitations

Zero-coupon bonds are very sensitive to interest rates and very subject to interest rate fluctuations. This is because the calculation of its price is depended on the rate of interest, the formula is shown below (3).

$$\text{Bond Value} = \frac{maturity\ value}{(1+interest\ rate)^{years\ to\ maturity}} \quad (3)$$

Bond prices have an inverse relationship to interest rates, which means that their market value --- their price falls as interest rates rise. That's true of bonds in general, but zeros are especially sensitive as they do not make interest payments, the size of the payoff get from the bond depends entirely on its present value at the time that it is sold. This problem with zero-coupon bonds also leads to a higher default risk than traditional bonds. The reason behind this is that companies do not have to make regular interest payments to the investors.

What's more, investors will have to pay annual taxes on phantom income. Even though they not getting any interest payments, and won't realize the profit on their zero until the bond pays out at maturity, the IRS acts as if they are. It calls this "phantom income" and it wants a piece. So, investors likely have to pay taxes on the interest that "accrues" on the bond each year — not just federal, but state and local too. However, there are ways to avoid this. For instance, investors could buy zero-coupon municipal bonds which aren't subject to federal income taxes, nor to state according to Figure 1.

Figure 1 comparison of US zero-coupon bond and US treasuries in 10 years.

5. CONCLUSION

Bond valuation is the process of determining the fair price of a bond. The value of a bond is a key factor that decides whether it is a good investment for a portfolio so bond pricing is an important stage in bond investing. There are several models used in bond pricing, for

different types of bonds, different models are introduced. Researchers argue that the accuracy of these models is a problem, many studies' results prove that newer models tend to exaggerate the credit risk of companies with high leverage or volatility while underestimating the spread risk of safer bonds.

Duration is a factor that may influence bond pricing, it is a measure of the sensitivity of the price of a bond or other debt instrument to a change in interest rates. Default risk and interest rate risk are the two main hazards that might affect the investment value of a bond (interest rate fluctuations). Since both factors affect a bond's predicted YTM, duration is utilized to evaluate the possible impact these factors will have on its price.

This paper examines the advantages and limitations of a zero-coupon bond, these may be valuable factors to consider when investing in such bonds. Zero-coupon bonds are a suitable option for investors who have a specific financial goal in mind, and a long-term but specific time frame because the returns of zero-coupon bonds are predictable. Zero-coupon bonds are the only type of fixed-income investments that are not subject to investment risk and it usually has higher interest rates than other bonds since zero-coupon bonds do not provide regular interest payments. However, zero-coupon bonds are very sensitive to interest rates and very subject to interest rate fluctuations which may lead to higher default risks.

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