

# Best Portfolio Choice by Using Markowitz Model: A Case Study of Ten Stocks from S&P500

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## ABSTRACT

Our paper practically implements the idea that people can have the best portfolio choice by Markowitz Model. Using a recent 20 years of historical daily total return data for ten stocks from S&P 500, one equity index (S&P 500) and a proxy for risk-free rate (1-month Fed Funds rate), we use Excel to arrange those data and categorize them with Solver Table. Then, to reduce the non-Gaussian effects, we aggregate the daily data into the monthly observations based on those monthly observations. Finally, we calculate all proper optimization inputs for the full Markowitz Model ("MM").

**Keywords:** portfolio optimization, Markowitz model, S&P 500, case study

## 1. INTRODUCTION

In our daily lives, there are many problems we have to face. One is related to every success in investing. It is very imperative to investors to know how to invest well and how to invest with higher profits and lower risks.

Markowitz put forward a method for portfolio optimization in 1952 [1]. It assists in selecting the most efficient portfolio by analyzing various possible portfolios of the given securities. Here, by choosing securities that do not 'move' exactly together, the Markowitz model shows investors how to reduce their risk. The Markowitz model is also called the mean-variance model because it is based on expected returns (mean) and the various portfolios' standard deviation (variance). It is foundational to modern portfolio theory.

In the research, I am studying to have the best portfolio choice by using Markowitz Model. I used the

raw data of 10 companies from "Yahoo" [2]. As a result, we found the two points and got the final answer for investors to invest.

Through the portfolio choice study, the consequence is that people usually choose the way which costs less and profits more. Hence, they need to use some models to help calculate the profits difference like Markowitz Model.

## 2. METHODS

In this section, the Markowitz method will show how to select an optimal portfolio. Figure 1 shows the efficient frontier. We can see from the figure that different stock has different change. As we can see here, the y-axis means the expected return, and the x-axis means the expected risks. The line which we call CML (Capital Market Line) intersects with the efficient frontier.

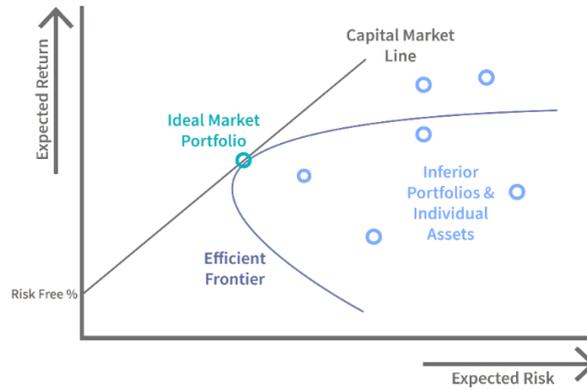


Figure 1. Efficient Frontier

According to [1,3-5], the followings are the formulas to calculate the Markowitz model.

$$R_t = (P_t - P_{t-1}) / (P_{t-1}) \quad (1)$$

Where,  $R_t$  is return of stocks  $i$ ,  $P_t$  is price on period  $t$ , and  $P_{t-1}$  is price in previous period.

Second calculate expected return of each stock with the following formula:

$$E(R_i) = \frac{\sum_{j=1}^n R_{ij}}{N} \quad (2)$$

Where,  $E(R_i)$  is the expected value of stock return  $i$ ,  $R_{ij}$  is actual return of stock  $i$  in period  $j$ , and  $N$  is number of observation periods.

Third calculate risk of each stock with the following formula:

$$\sigma_i^2 = \frac{\sum_{i=1}^n [X_i - E(X_i)]^2}{n-1} \quad (3)$$

Where,  $\sigma_i^2$  is stock variant  $i$ ,  $X_i$  is return of stock  $i$ ,  $E(X_i)$  is the value of the  $i$ -th expectations, and  $n$  is number of observations of historical data for large samples with  $n$  (most slightly 30 observations) and for small samples used  $n-1$ .

Fourth calculate covariance with the following formula:

$$\sigma_{R_A, R_B} = \frac{\sum_{i=1}^n [R_{Ai} - E(R_A)] \cdot [R_{Bi} - E(R_B)]}{n} \quad (4)$$

Where,  $\sigma_{R_A, R_B}$  is covariance returns between stock A and B,  $R_{Ai}$  is return of stock A of condition  $i$ ,  $R_{Bi}$  is future return of stock B of condition  $i$ ,  $E(R_A)$  is the expected value of the stock return A,  $E(R_B)$  is the expected value of stock return B, and  $n$  is number of observations of historical data for large samples with  $n$  (most slightly 30 observations) and for small samples used  $n-1$ . Fifth calculate correlation coefficient with the following formula:

$$R_{A,B} = \frac{\sum_{i=1}^n (R_{Ai} - E(R_A)) \cdot (R_{Bi} - E(R_B))}{\sqrt{[\sum_{i=1}^n (R_{Ai} - E(R_A))^2] [\sum_{i=1}^n (R_{Bi} - E(R_B))^2]}} \quad (5)$$

Where,  $R_{A,B}$  is the correlation coefficient return of stock between A and B,  $R_{Ai}$  is future return of stock A condition  $i$ ,  $R_{Bi}$  is future Return of stock B condition  $i$ ,  $E(R_A)$  is the expected value of the stock return A,  $E(R_B)$  is the expected value of stock return B, and  $n$  is number of observation periods. Sixth calculate the proportion of funds from stocks candidate portfolio using the application program solver that is in Microsoft Excel. Seventh calculate expected return portfolio with the following formula:

$$E(R_p) = \sum_{i=1}^n (W_i \cdot E(R_i)) \quad (6)$$

Where,  $E(R_p)$  is expected return portfolio,  $W_i$  is the weight or portion of funds invested in stocks  $i$ ,  $E(R_i)$  is expected return stock  $i$ , and  $n$  is number of shares in the portfolio. The eight calculate portfolio risk with the following formula:

$$\sigma_p^2 = \sum_{i=1}^n W_i \cdot \sigma_i^2 + \sum_{i=j}^n \sum_{j=1}^n W_i \cdot W_j \cdot \sigma_{ij} \quad (7)$$

Where,  $\sigma_p^2$  is portfolio variant,  $\sigma_i^2$  is stock return variant  $i$ ,  $\sigma_{ij}$  is covariance between stocks  $i$  and  $j$ ,  $W_i$  is the portion of funds invested in stocks  $i$ ,  $W_j$  is the portion of funds invested in stocks  $j$ , and  $n$  is number of stocks in the portfolio.

### 3. RESULTS AND DISCUSSION

#### 3.1. Data source

I got the raw data of the ten stocks in the S&P 500 from Yahoo.com [2] to analyze this problem.

Table 1. The ten stocks selected from S&P 500

No	Company Name	Symbol
1	ADOBE INC	ADBE
2	INTERNATIONAL BUSINESS MACHINES CORP	IBM
3	SAP SPONSORED ADR	SE- SAP
4	SOUTHWEST	LUV

AIRLINES CO			
5	ALASKA GROUP INC	AIR	ALK
6	HAWAIIAN HOLDINGS INC		HA
7	BANK OF AMERICA CORP	BAC	
8	CITIGROUP INC		C
9	WELLS FARGO & CO	WFC	
10	TRAVELERS	COS	TRV

INC/ THE	
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They are financial services companies, technology companies and industrial companies. To do was delete weekend time in the table because the stock market did not work on weekends. In this case, I must fit the date to the date, respectively.

Table 2 shows the monthly data of the ten stocks. We can see from the table that the company named ADBE has increased almost 20 times in the end, but some, like company C, decreased their price in the end.

**Table 2.** Monthly data from 2001 to 2021

DATE	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK
2001/5/31	1256.94	19.89	111.80	35.11	29.91	476.68	23.54	50.60	20.00	7.18
2001/6/29	1226.34	23.51	113.00	35.09	30.30	491.47	23.22	50.97	18.49	7.23
2001/7/31	1214.25	18.75	105.21	36.08	32.11	467.01	23.03	44.09	20.01	7.91
2001/8/31	1138.24	16.81	100.08	34.49	31.04	427.33	23.14	42.26	17.90	8.16
2001/9/28	1046.31	12.00	91.84	25.92	29.75	377.88	22.35	41.74	14.85	4.99
2001/10/31	1066.25	13.21	108.22	25.70	30.05	424.72	19.86	46.48	15.91	6.10
...										
2020/7/31	4796.81	446.85	186.62	204.53	40.03	72.55	42.21	186.90	33.71	38.28
2020/8/31	5141.62	516.32	189.63	212.49	41.41	74.16	42.19	189.55	41.01	43.29
2020/9/30	4946.25	493.23	187.10	200.15	39.03	62.54	41.07	178.05	40.93	40.72
2020/10/30	4814.71	449.65	171.71	137.23	38.40	60.83	37.48	198.65	43.14	42.12
2020/11/30	5341.74	481.20	192.63	155.64	45.63	80.87	48.00	213.37	50.57	56.66
2020/12/31	5546.92	502.97	196.31	167.50	49.42	90.55	52.96	232.46	50.87	57.80
2021/1/29	5490.47	461.38	185.75	162.12	48.34	85.91	52.44	225.72	47.95	54.28
2021/2/26	5641.87	462.29	187.95	158.46	56.59	97.60	63.67	240.96	63.44	72.27
2021/3/31	5888.94	478.08	210.59	157.74	63.39	107.77	68.77	250.45	66.64	76.93
2021/4/30	6203.22	511.24	224.21	179.77	66.40	106.29	79.30	257.54	68.51	76.85
2021/5/12	6031.01	474.78	225.82	174.06	67.47	110.75	80.51	256.03	63.53	71.99

**3.2. Main results**

This paper sets two constraints (Constraint 1: there are no constraints, which means free problems. Constraint 2:  $W_1=0$ , which means the risk-free market weights "0".) to run the graph with Solver Table in Excel.

Table 3 shows the results of the Markowitz Model based on two constraints. From Table 3, we can find that it is in the efficient position when the Ret is 6.5% or 9.5%.

**Table 3.** The results of the Markowitz Model based on two constraints

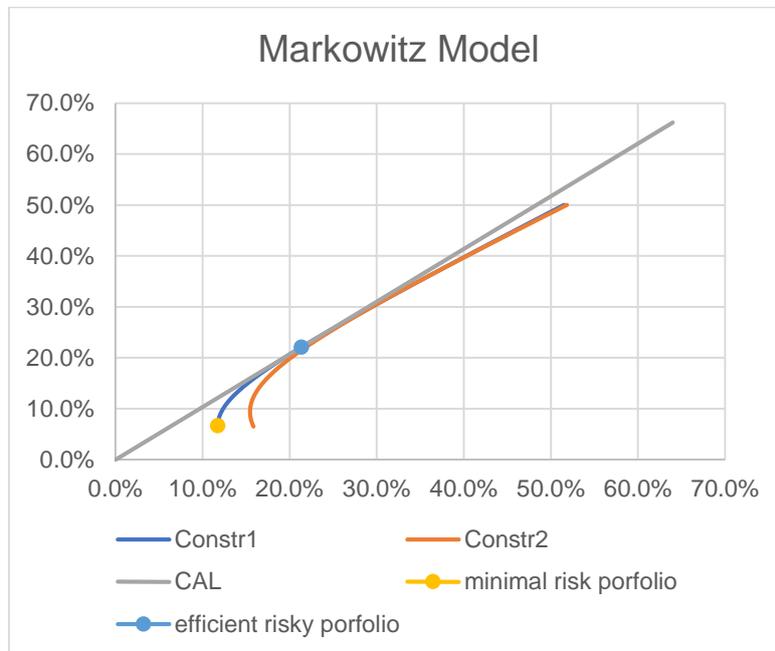
Ret	Constr1	Constr2	Ret	Constr1	Constr2
-10.0%	22.6%	28.22%	22.0%	21.3%	21.79%
-9.5%	22.1%	27.71%	22.5%	21.8%	22.23%
-9.0%	21.6%	27.21%	23.0%	22.2%	22.67%
-8.5%	21.2%	26.71%	23.5%	22.7%	23.12%
-8.0%	20.7%	26.21%	24.0%	23.2%	23.58%
-7.5%	20.2%	25.72%	24.5%	23.7%	24.04%

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-7.0%	19.7%	25.24%	25.0%	24.2%	24.51%
-6.5%	19.3%	24.76%	25.5%	24.7%	24.98%
-6.0%	18.8%	24.29%	26.0%	25.3%	25.47%
-5.5%	18.4%	23.82%	26.5%	25.8%	25.95%
-5.0%	17.9%	23.36%	27.0%	26.3%	26.45%
-4.5%	17.5%	22.91%	27.5%	26.8%	26.94%
-4.0%	17.1%	22.46%	28.0%	27.3%	27.44%
-3.5%	16.7%	22.02%	28.5%	27.9%	27.95%
-3.0%	16.2%	21.59%	29.0%	28.4%	28.46%
-2.5%	15.9%	21.17%	29.5%	28.9%	28.97%
-2.0%	15.5%	20.76%	30.0%	29.4%	29.49%
-1.5%	15.1%	20.36%	30.5%	30.0%	30.01%
-1.0%	14.7%	19.97%	31.0%	30.5%	30.53%
-0.5%	14.4%	19.59%	31.5%	31.0%	31.06%
0.0%	14.1%	19.22%	32.0%	31.6%	31.59%
0.5%	13.8%	18.86%	32.5%	32.1%	32.12%
1.0%	13.5%	18.52%	33.0%	32.7%	32.66%
1.5%	13.2%	18.19%	33.5%	33.2%	33.20%
2.0%	12.9%	17.88%	34.0%	33.7%	33.74%
2.5%	12.7%	17.58%	34.5%	34.3%	34.28%
3.0%	12.5%	17.30%	35.0%	34.8%	34.82%
3.5%	12.3%	17.03%	35.5%	35.4%	35.37%
4.0%	12.2%	16.78%	36.0%	35.9%	35.92%
4.5%	12.0%	16.56%	36.5%	36.5%	36.47%
5.0%	11.9%	16.35%	37.0%	37.0%	37.02%
5.5%	11.8%	16.16%	37.5%	37.6%	37.58%
6.0%	11.8%	15.99%	38.0%	38.1%	38.13%
6.5%	11.7%	15.84%	38.5%	38.7%	38.69%
7.0%	11.8%	15.72%	39.0%	39.2%	39.25%
7.5%	11.8%	15.62%	39.5%	39.8%	39.81%
8.0%	11.8%	15.54%	40.0%	0.403214	40.37%
8.5%	11.9%	15.49%	40.5%	0.408756	40.94%
9.0%	12.0%	15.46%	41.0%	0.414306	41.50%
9.5%	12.2%	15.45%	41.5%	0.419861	42.07%
10.0%	12.4%	15.47%	42.0%	0.425423	42.63%
10.5%	12.5%	15.51%	42.5%	0.430991	43.20%
11.0%	12.8%	15.58%	43.0%	0.436565	43.77%
11.5%	13.0%	15.66%	43.5%	0.442145	44.34%
12.0%	13.3%	15.78%	44.0%	0.44773	44.91%
12.5%	13.5%	15.91%	44.5%	0.453319	45.48%
13.0%	13.8%	16.07%	45.0%	0.458914	46.06%
13.5%	14.1%	16.24%	45.5%	0.464514	46.63%
14.0%	14.5%	16.44%	46.0%	0.470118	47.21%
14.5%	14.8%	16.66%	46.5%	0.475727	47.78%

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15.0%	15.2%	16.90%	47.0%	0.48134	48.36%
15.5%	15.6%	17.15%	47.5%	0.486957	48.94%
16.0%	15.9%	17.43%	48.0%	0.492578	49.52%
16.5%	16.3%	17.72%	48.5%	0.498196	50.10%
17.0%	16.7%	18.02%	49.0%	0.503832	50.68%
17.5%	17.2%	18.34%	49.5%	0.509457	51.26%
18.0%	17.6%	18.68%	50.0%	0.515101	51.84%
18.5%	18.0%	19.03%			
19.0%	18.5%	19.39%			
19.5%	18.9%	19.76%			
20.0%	19.4%	20.15%			
20.5%	19.8%	20.55%			
21.0%	20.3%	20.95%			
21.5%	20.8%	21.37%			



**Figure 1.** Frontier line

After technologically fitting some indexes in Excel, I got this beautiful graph with Markowitz Model.

The blue point in figure 1 means the best portfolio choice, and in the situation of risk-equal, the profits investors gain in the blue point are the highest, which means investors will choose to invest in this point. The yellow point means the lowest risk situation, which means it is the safest place to invest, but the benefits may not be optimal.

#### 4. CONCLUSION

People or companies always invest among such curves because those are the place where they can get the highest profits and lowest risks with the same resource,

after researching the best portfolio choice with Markowitz Model, using Excel and Solver Table to calculate and graph.

In the research, we can know how to invest properly and a brand-new thinking system that tells us that all things have constraints. First, nothing is perfect, including Markowitz Model. What we can do is to put all resources we know together and analyze the plausible answer. Second, we have to do anything, including this research, carefully since if we miss one of the data, the final answer will be affected, and we have to do it again. Third, it is very important to memorize and comprehend key formulas because we cannot use them properly if we only memorize letters and signs on formulas but not the conception.

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