# Review for the Different Portfolio Methods Such as Mean-variance Analysis and Fama Factor Model

Tianrun Wang[1,*]

[1] *Jining Confucius International School, 272113, Jining, Shandong, China*
[*] *Corresponding author. Email: guanghua.ren@gecacademy.cn*

**ABSTRACT**
Portfolio Theory contains two important contents: the mean-variance analysis method and the portfolio effective boundary model. When people invest, they are essentially choosing between uncertain returns and risks. They maximize the expected return under a given level of expected risk or minimize the expected risk under a given level of expected return. Two important models will be used, Markowitz's portfolio theory and the Fama-french model. We would explain the difference in the returns of different stocks. And found that these models have certain shortcomings, it analyzes a practical example that gives an annotation for the proposed model and method. The proposed investment scheme will bring the corporation the greatest profits in terms of theory.

***Keywords:*** *portfolio theory, mean-variance analysis, Markowitz's portfolio theory, Fama-french model*

## 1. INTRODUCTION

Investment portfolio theories guide the way an individual investor or financial planner allocates money and other capital assets within an investment portfolio. An investing portfolio has long-term goals independent of a market's day-to-day fluctuations; because of these goals, Investment portfolio theories aim to aid investors or financial planners with tools to estimate the expected risk and return associated with investments [1]. Why should riskier companies have higher returns? Intuitively, an investor would require a higher expected return in exchange for accepting greater risk. Kent L. Womack in his paper [2] said that we do observe this relationship when we look back at historical long-run returns of stocks, bonds, and less risky securities, With complete certainty that the cash flows will all be paid when promised, an investor would discount the asset at the risk-free rate. As the degree of uncertainty increases, the return required to justify the risk will be much higher, resulting in a much lower price the investor would be willing to pay, simply because of the higher required discount rate.

This paper is investigated of portfolio theory and how to make an investment decision-making tool. In the investment world, there are different investment motives. One of the most prominent is the return on investment. However, choosing investments based solely on returns is not enough. There are two important models involved in this article, Markowitz's portfolio theory and the Fama-french model, and the advantages and disadvantages of these two models are also talked about in this article.

Firstly, this article talks about the internal mechanism of investors to establish an effective boundary, reduce investment risk and maximize expected utility. Systematically analyzes Markowitz's portfolio theory, as well as the portfolio problem with risk-free assets and short-selling constraints, and uses the mean-variance model to more accurately reflect the attitude of investors to income and loss. Further reveals the influence of preference structure on portfolio investment. Fama pointed out, based on previous studies, that the CAMP model cannot explain the differences in the returns of different stocks. In addition to Beta risk, firm size and book-to-market ratio are also important factors in capital asset pricing. Accordingly, a three-factor model is proposed. The model believes that the excess return of an investment portfolio ( including a single stock ) can be explained by its exposure to the three factors: market asset portfolio ( Rm − Rf ), market value factor ( SMB ), and book-to-market ratio factor ( HML ). Vassalou and Xing [3] is the first study that examines default risk in the context of the Fama-French model. They find that default risk is priced in the cross-section of equity returns and they conclude that default risk is systematic. They also find that SMB and HML contain default-related information, but they conclude that this is not the main

 

reason why SMB and HML are significant explanators of equity returns. To the best of our knowledge, Vassalou and Xing's analysis of default risk in the context of the Fama-French model has only been performed in the US. Therefore, in the spirit of Lo and MacKinlay [4], the main aim of this paper is to assess the external validity of the default risk hypothesis by testing whether default risk is priced in the cross-section of equity returns and whether SMB and HML are proxying for default risk in a market outside the US[5].

## 2. MARKOWITZ MODEL

The securities investment portfolio is an investment portfolio generated by investors choosing securities assets. However, since investment securities will be affected by various factors and will be uncertain, investors will often adopt risk diversification methods to reduce risk, in order to maximize the utility and rewards of the investment. In selecting investment securities, it is important to know the relationship between risk and return. Investors tend to prefer a combination of returns greater than risk. When the return is constant, the risk is low.

For Markowitz's "Portfolio Selection", this marked the emergence of modern asset portfolio theory. The "mean-variance model" he proposed is based on the hypothesis of prohibition of securities lending and risk-free borrowing. The mean and variance of the rate of return find the limit of validity of the portfolio - the combination with the smallest difference under a certain level of return. In order to reduce the risk of the investment portfolio, in addition to diversifying investments to diversify risk, stocks with low correlation coefficients should also be selected, and there should be a good asset allocation so as to bring you a higher return.The expected return on any asset is a weighted average return, weighted by the probability p that each return occurs. The expected rate of return is equal to the sum of the product of each rate of return and the corresponding probability.There's some expressions.

Expected return;

$$E(r) = \sum_{i=1}^{n} p_i r_i = p_1 r_1 + p_2 r_2 + \cdots + p_n r_n$$

$P_i$ is the probability of the rate of return,r means the r ate of return.

Covariance: A measure of how much two random ou tcomes "vary together"

$$\mathrm{Cov}[\tilde{x}, \tilde{y}] = \delta_{xy} = E\left[\left(\tilde{X} - \overline{X}\right)\left(\tilde{Y} - \overline{Y}\right)\right]$$

Correlation:

$$\mathrm{Corr}[\tilde{x}, \tilde{y}] = \rho_{xy} = \frac{\delta_{xy}}{\delta_x \delta_y}$$

There should be note that:

$\rho_{xy}$ must lies between -1 and 1

Two random outcomes are:

.perfectly positively correlated if $\rho_{xy} = +1$

.perfectly negatively correlated if $\rho_{xy} = -1$

.uncorrelated if $\rho_{xy} = 0$

Markowitz shows that under certain conditions an investor's portfolio selection can be simplified to balance two factors, namely the expected return of the portfolio and its variance. Risk can be measured by variance, and risk can be reduced by diversification. Portfolio risk depends not only on the variance of different assets but also on the covariance of the assets. Similarly, in trying to make variance small it is not enough to invest in any securities. It is necessary to avoid investing in securities with high covariances among themselves. We should diversify across industries because firms in different industries, especially industries with different economic characteristics, have lower covariances than firms within an industry [5].

Several assumptions of Markowitz's investment portfolio:

(1). Investors are rational individuals and will look for ways to avoid risk. This is another assumption of the efficient market hypothesis, but we now know from behavioral economics that market participants are not rational. It does not allow for "herd behavior" or investors who will accept lower returns for higher risk. Even gamblers clearly pay for risk, and it is possible that some stock traders will pay for risk as well.

(2). Investors will assess the risk of the portfolio based on the expected rate of return on the securities.

(3). At a certain level of risk, investors expect significant returns. If it's at a certain income level, investors prefer less risk.

(4). The market is complete and all market participants receive prices, market information is effective, and assets can be divided.

Their actions do not influence prices. In reality, sufficiently large sales or purchases of individual assets can shift market prices for that asset and others (via cross-elasticity of demand). An investor may not even be able to assemble the theoretically optimal portfolio if the market moves too much while they are buying the required securities. In reality, fractional shares usually cannot be bought or sold, and some assets have minimum order sizes. More complex versions of the MPT take into account a more sophisticated model of the world (such as one with non-normal distributions and taxes) but all mathematical models of finance still rely on many unrealistic premises as stated previously.[6]

(5). Short selling restrictions, i.e. short selling of stocks is not allowed

(6). Does not take into account the impact of the conversion on the rate of return

(7).the investors make decisions on the basis of the expected risk and profit only, so that the utility curve is expressed as a function of the expected profit and profit variant; [7]

An investment portfolio offered by Markowitz is determined by the securities that compose it and their weight. The expected return of the investment portfolio is the weighted average of the returns of the securities and the futures contracts. The risk of the investment portfolio is determined by the variance of the standard deviation By definition, this unified measure describes the extent to which the expected rate of return moves around its average value. If the change is drastic, it means that the rate of return is not stable i.e. the risk is great. The covariance between two or more stocks is expressed as the degree of correlation between those stocks. When their covariance is zero, a change in one has no effect on the others and is extremely irrelevant. When the covariance is positive, it shows that they are positively correlated. The greater the covariance, the stronger the positive correlation, indicating that several stocks are winning or losing at the same time. When the covariance is negative, it means that they are negatively correlated. The greater the covariance, the stronger the negative correlation. When one of the stocks is profitable, the other stocks lose money.

## 3. FAMA-FRENCH MODEL

4. Studied by Fama and French on the factors that determine the difference in the returns of different stocks in the US stock market found that the beta value of the stock market cannot explain the difference in the return of different stocks, while the market value, book-to-market ratio, and price-to-earnings ratio of listed companies can explain the difference in stocks. The difference in the rate of return. Fama and French believe that the above-mentioned excess returns are compensation for risk factors that are not reflected by β in CAPM. Fama and French pointed out in 1993 that a three-factor model can be established to explain stock returns. The model discovered that the excess return of an investment portfolio (including a single stock) can be explained by its exposure to three factors, which are: market asset portfolio ($R_m - R_f$), market value factor (SMB), book-to-market value ratio Factor (HML). Where $R_{ft}$represents the risk-free rate of return at time t; $R_{mt}$ represents the market rate of return at time t; $R_{it}$represents the rate of return of asset i at time t; E($R_{mt}$) $-R_{ft}$is the market risk premium, and SMBt is the market value at time t (Size ) The simulated portfolio rate of return of the factor, HMIT is the simulated portfolio rate of return of the book-to-market factor at time t.βi ( si and hi are the coefficients of the three factors respectively, and the regression model is expressed as follows:

$$E(R_i) - R_f = \beta_i[E(R_m) - R_f] + s_i SMB + h_i HML$$

The time sequence is written as follows :

$$R_{it} - R_{ft} = \alpha_i + \beta_i(R_{mt} - R_{ft}) + s_i SMBt + h_i HMLt + \varepsilon_{it}$$

The application of the Fama-French three-factor model is based on the theoretical assumption of a “bounded rationality”. And on this basis, draw a number of basic assumptions:

(1)There are a large number of investors;

(2)Any investor plans his own portfolio of investment assets during the same period of holding the securities;

(3)Investors' investment scope is limited to assets traded on the open financial market;

(4) There are no securities transaction costs (commissions and service charges, etc.) and taxes;

(5) Investors have the same expectations for the mean, variance and covariance of securities returns;

(6) All investors agree on the valuation of the securities and the economic situation.

This model is very widely cited. Trimechet al., investigated the multiscale pricing model for the French stock market by combining wavelet analysis and the Fama-French three-factor model, showing that the explanatory power of the Fama-French three-factor model becomes stronger as the wavelet scale increases [8]. Besides, the relationship between the portfolio returns and the risk factors (i.e. the market, size, and value factors) depends significantly upon the considered time horizon. And in Wu reconstruct the Fama-French three-factor (F-F) model as a panel smooth transition regression (PSTR) framework to investigate the differentiated effects of investor sentiment proxies-the volatility index (VIX), credit default swap (CDS), and TED spread on the three risk premiums [9]. And the result showed that market premiums fall as investors in stock markets show extreme optimism or extreme pessimism. Except in rare situations, the size premium is significant and decreases with the increase in the VIX. Returns in holding growth stocks dominate holding value stocks when the investors show extreme pessimism or optimism. However, in the normal sentiment of investment, value stocks earn more returns than growth stocks.

C Gaun[10]talked in his paper, the present study extends the 1981 – 1991 period examined by Halliwell, Heaney, and Sawicki a further 10 years to 2000 and addresses several limitations and findings of that research. In contrast to Halliwell, Heaney, and Sawicki the current study finds the three-factor model provides significantly improved explanatory power over the CAPM, and evidence that the BM factor plays a role in asset pricing

However, the three-factor model does not represent the end of the capital pricing model. Recent studies have found that there are still many unexplained parts in the three-factor model, such as short-term reversal, medium-term momentum, volatility, and skewness. , Gambling and other factors.

## 4. DISCUSSION

Markowitz's model owes the huge application value in the market. Because the current securities market in my country is not enough to regulate investment risk, systematic risk will always occupy a relatively large proportion, but it can be seen that the risk of Markowitz's effective portfolio at the same level of expected return is much lower than that of other core titles. And at the same time by diversifying risks, while eliminating certain low-yield and high-risk certificates.

Markowitz's securities investment portfolio theory has problems in my country's application. First of all, the problem of the market effect. The market is only effective when the share price in the stock market can fully reflect timely market information. Due to the failure of the market itself, a fully efficient stock exchange is an ideal market, moreover, for the Chinese market, there are strengths and weaknesses.

Second, some people question the Markowitz assumption that investors are risk-averse and that variance is the most effective measure of risk in the industry. And when it comes to the measure of risk, the mean-variance model is just a special case of the utility function. Markowitz's portfolio theory is based on a series of strict premises, but increasingly empirical research findings are correct. Investment income obeys the doubt raised by the normal distribution hypothesis.

A predefined condition in the Markowitz model is that there are no transaction costs. Although a diversified investment can significantly reduce unsystematic risks, the associated transaction costs will also increase significantly, and as you diversify your investment, you need to spend more time managing it. And for the system risk, it is impossible to reduce the risk through wind dispersion. It may be caused by changes in some markets, such as interest rate changes, exchange rate changes, or economic recession. For system risk, it is all shareholders, which need to bear risk.

Finally, investors do not always invest according to the mean-variance model, and some investors are not completely rational. In fact, many investors do not completely follow this rational logic and may be swayed by their own emotions to influence their investment decisions.

## 5. CONCLUSION

Portfolio theory is abbreviated as portfolio theory primarily to study how people can diversify their investments to avoid unsystematic risks in investments and to maximize investment income when expected income is subject to a variety of uncertain factors. This article presents the widely used models of portfolio theory, the Markowitz mean-variance model and the Fama-French model.

For Markowitz's portfolio theory, it follows the goal of traditional portfolio theory, which is to balance risk and rate of return, but the difference is that Markowitz first proposed the quantification of differential risk and rate of return using the average value of the rate of return over a certain time. Indicates the security's rate of return, and the actual degree of variance is expressed by the variance. And through this theory, it is concluded that under normal circumstances there is a positive correlation between risk and return. The higher the return, the greater the risk; conversely, the lower the return, the lower the risk. Reasonable investors will choose a portfolio with high returns under certain risk conditions; under yield conditions, choose a portfolio of low-risk securities.

For the Fama-French model, the empirical results of the model show that the return on market risk, the book value/market value ratio, and the firm model can essentially explain the difference in the cross-section of expected return on stocks. , and can explain any abnormal performance except short and medium-term inertia effects If the factor is established, according to the Fama-French model, we can conclude that the stock market return of small companies is higher than that of large companies, and the return on value stocks is higher than that on growth stocks.

However, this model has shown a wide range of adaptability in relatively mature foreign stock markets, but its applicability in Chinese stock markets remains to be investigated. To sum up, we should appropriately introduce Western securities investment portfolio theory to research and guide the Chinese securities market, but we should not use mechanical methods. We should combine western investment portfolio theory with Chinese reality and build an investment portfolio theory suitable for China's national conditions system

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