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Application of Modern Portfolio Theory in Stock Market based on Empirical analysis

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ABSTRACT
With the rapid growth of the stock market, stocks have been viewed as one of the most popular investments. Investors then face a problem that is how to allocate resources among the variety of stocks with the aim to increase their wealth through investment activities. The paper aims to design an investment strategy for risk-averse investors. We study the modern portfolio theory and apply the mean-variance analysis to quantify expected portfolio returns and acceptable levels of portfolio risk. We would like to obtain an optimal portfolio and provide investors with investment strategies. We consider the daily adjusted closing stock prices between January 2010 and January 2021 of 5 companies: Facebook, Amazon, Apple, Netflix, and Google. Our results show that such mean-variance optimization is applicable. Furthermore, we can achieve a minimum variance portfolio with the lowest possible level of the risk of 23% (standard deviation), while the expected return is approximately 28%. Additionally, the best risk-adjusted portfolio can be achieved with a higher return of 33% at risk of 23%.

Keywords: *"component", "investment", "stock", "modern portfolio theory", "market".*

1. INTRODUCTION

Modern portfolio theory (MPT) is an investment strategy that is widely used to optimize the trade-off between risk and return for a diversified portfolio. We consider investors are risk-averse, that is, the investor would prefer a less risky portfolio rather than a risker one for a determined level of return.

The concept of a portfolio is introduced by Markowitz, which is a combination of different financial assets such as bonds, stocks, currencies, and so on [10]. Markowitz proposed the Modern portfolio theory, which is further developed by Sharpe [10][14]. This theory provides an investment framework for the construction and selection of portfolios, assuming that investors tend to maximize the expected return and simultaneously minimize the risk. The core idea of MPT is mean-variance analysis. Here, mean and variance are used to measure the return and risk of the investment portfolio.

MPT has been applied to a wide range of areas. For example, in the financial sector, it has been applied to improve laws in investment management and to corporate diversification for corporate risk management [5][7]; in the energy sector, it is used to determine the optimal power generation [3][2]. Besides these, it is also applied to solve conservation planning problems with the aim to identify the most efficient allocation of conservation resources [6][11][1][9][13].

However, there are some problems with the classical MPT. It is well-known that MPT is based on historical market data, and this might cause huge estimation errors. To reduce this error, Black and Litterman propose a model as known as the Black-Litterman model, where they incorporate investors' opinions on the future expected market-based on models lie capital asset pricing model (CAPM) [4]. Rom and Ferguson also point out there are two important limitations of the MPT. The firs one it that it assumes that the investment returns of all portfolios and assets can be represented by a normal distribution; and the second one is it uses the variance of portfolio returns as a measure of risk [12]. To overcome these problems, they introduce the theory of post-modern portfolio theory, where the risk is defined by the downside risk which is a variance of negative returns. Mahdavi-Damghani also shows that there are some criticisms for mismatching between theory and the real world [8]. The offset of accounting information may lead to a mistake in optimal portfolio selection. Even though the MPT has limitations, it is still one of the most widely used techniques when making investment decisions.

The remainder of the paper is structured as follows. In the next section, we give some assumptions of the

modern portfolio theory and a brief description of the mean-variance model. In Section 3, we show how the mean-variance model is applied in the stock market via a specific example. Finally, the last section draws a conclusion and discussion of our study.

2. METHODOLOGY

2.1. Assumptions

Modern portfolio theory is based on some assumptions. We give a list of assumptions and a short description. Assume

- Investors are risk-averse. That is, they prefer a higher return for a given level of risk or lower risk for a given level of return.
- The expected return of the portfolio is measured by the mean of returns, and risk is measured by the variance of expected return. The portfolio is only determined by these two factors.
- Investors only focus on the expected return and risk of the overall portfolio.
- Investors are price takers, and they can't change the market prices.
- The market is efficient; thus, investors have access to all information, which means mean and variance are known.
- There are no transaction costs and taxes.
- Stocks can be divided into infinitely many shares. The number of stocks is not necessary to be an integer.

2.2. Model

We consider a portfolio, denoted by p. Suppose there are n stocks in this portfolio. We denote the return of individual stock i by R_i, and the expected return of it by $E[R_i]$, for $i = 1, \dots, n$. Let w_i be the weight of stock i in the portfolio, note that $\sum_{i=1}^{n} w_i = 1$, where $w_i > 0$. The expected return of the portfolio is measured by the weighted average of the return of individual stocks over a finite period, given by

$$E[R_P] = \sum_{i=1}^{n} w_i E[R_i]. \quad (1)$$

We use variance or standard deviation to measure the risk of the portfolio, which is denoted by σ_p^2 and σ_p, respectively. The variance of a portfolio consisting of two or more stocks depends not only on the variance of the stocks but also on the correlation between each other. The formula of variance is given by

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}, \quad (2)$$

$$\sigma_p = \sqrt{\sigma_p^2}. \quad (3)$$

where w_i and w_j are weights of stocks i and j in the portfolio and σ_{ij} is the covariance between stocks i and j. Note that the larger the variance or standard deviation, the higher the risk.

There are two rules for portfolio selection. The first one is to choose the maximum return for a given level of risk, that is

$$\max E[R_P] = \max \sum_{i=1}^{n} w_i E[R_i]. \quad (4)$$

subject to

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}, \quad (5)$$

and

$$\sum_{i=1}^{n} w_i = 1, w_i > 0. \quad (6)$$

The second rule is to choose a portfolio with minimum risk for a given level of return, that is

$$\min \sigma_p^2 = \min \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}, \quad (7)$$

subject to

$$E[R_P] = \sum_{i=1}^{n} w_i E[R_i], \quad (8)$$

and

$$\sum_{i=1}^{n} w_i = 1, w_i > 0. \quad (9)$$

2.3. Efficient Frontier

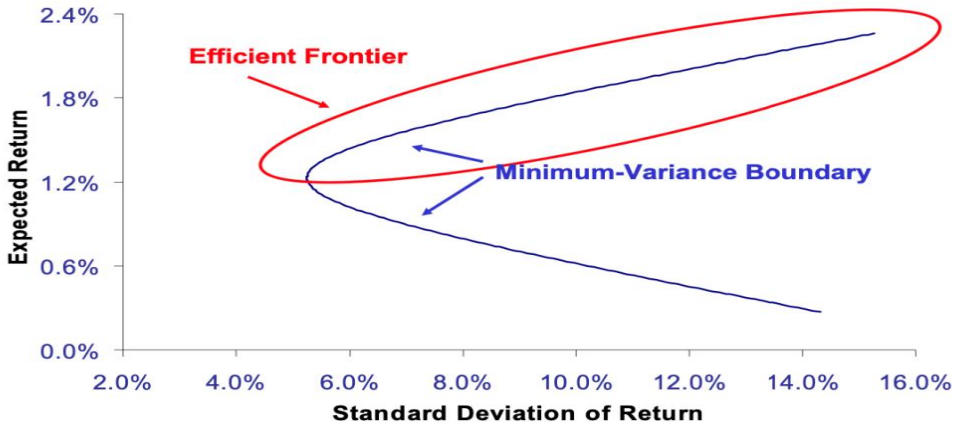

Figure 1 The feasible and efficient portfolios

Figure 1 shows the feasible portfolios, which is a bullet-shaped area. We can obtain the mean-variance boundary of the feasible region. The top half of the mean-variance boundary is called the efficient frontier.

The efficient frontier is a set of optimal portfolios that have the highest expected return for a given level of risk or the lowest risk for a given level of return. Portfolios that are below the frontier are considered sub-optimal since the expected return is lower compared with those along the frontier. Similarly, portfolios that are right to the frontier are also sub-optimal because the risk is higher for the determined expected return. Thus, rational investors would select portfolios along the efficient frontier.

The efficient frontier is the foundation of the MPT, which gives investors ideas on how to construct a portfolio with the aim to maximize expected return at a determinate level of risk. It helps investors to analyze the potential risk and returns and then choose the optimal investment strategies.

2.4. Sharpe ratio

The Sharpe ratio is a widely used technique for measuring risk-adjusted return, which helps understand the return relative to its risk. The formula of the Sharpe ratio uses the expected return of the portfolio in excess of the risk-free rate, $E[Rp] - r_f$, divided by the standard deviation of the portfolio, σ_p, here r_f is the risk-free rate. The Sharpe ratio is given by

$$Sharpe\ Ratio = \frac{E[Rp]-r_f}{\sigma_p}. \quad (10)$$

The Sharpe ratio shows the risk-adjusted performance of a portfolio. A higher Sharpe ratio is preferred because it helps investors determine the investment strategy that gives a higher return or a lower risk.

3. DATA ANALYSIS

The aim of this paper is to apply the mean-variance model in the U.S. stock market in order to create an investment portfolio with several stocks that obtain the maximum return for the lowest risk.

We use daily adjusted closing prices over ten years of data, from 01/01/2011 to 01/01/2021, obtained from Yahoo Finance 1 . Data has been collected for five companies: Facebook, Amazon, Apple, Netflix, and Google, and these companies are chosen from different industries. In our case, 25000 portfolios are randomly generated. We consider the rate of 0.1% as the risk-free rate, which is the 52-weeks treasury bill rate at the start of 2021 taken from the U.S. Department of The Treasury2.

Figure 2 shows the adjusted closing prices of five companies. We can see that Amazon and Google's stock prices are relatively higher than those of Netflix, Facebook, and Apple. Amazon's stock price grows fastest and has been the most expensive one since around 2017. The stock price of Google also experiences an increase but slightly less than that of Amazon. However, the other three companies are squashed at the bottom, the price of Netflix is higher than those of Facebook and Apple.

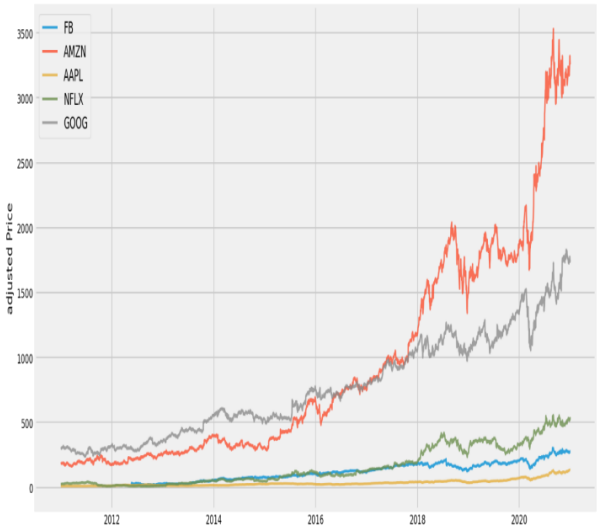

Figure 2 Adjusted closing prices of Facebook, Amazon, Apple, Netflix, and Google over 01/01/2011 to 01/01/2021.

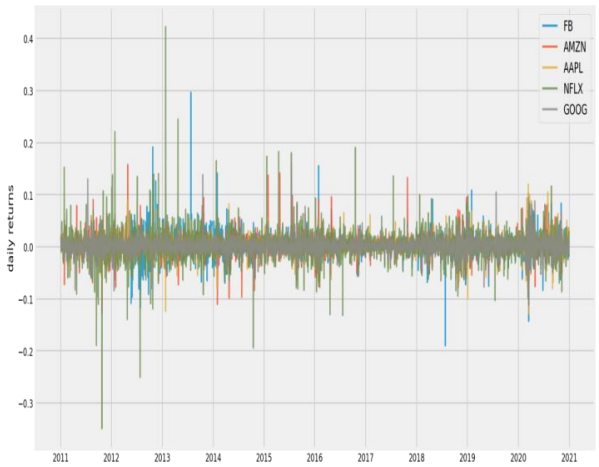

Figure 3 Daily returns of Facebook, Amazon, Apple, Netflix, and Google over 01/01/2011 to 01/01/2021.

Figure 3 shows the daily returns of five stocks. The daily returns are all fluctuated around zero. However, Netflix has the largest positive and largest negative spikes. Facebook and Amazon also have several large distinctive positive spikes and a couple of negative ones. Apple also shows some spikes that stand out from the plot. This means that the volatilities of Netflix and Amazon are quite large. This means that they are the riskiest stocks. Google is shown to be the most stable one among these five stocks.

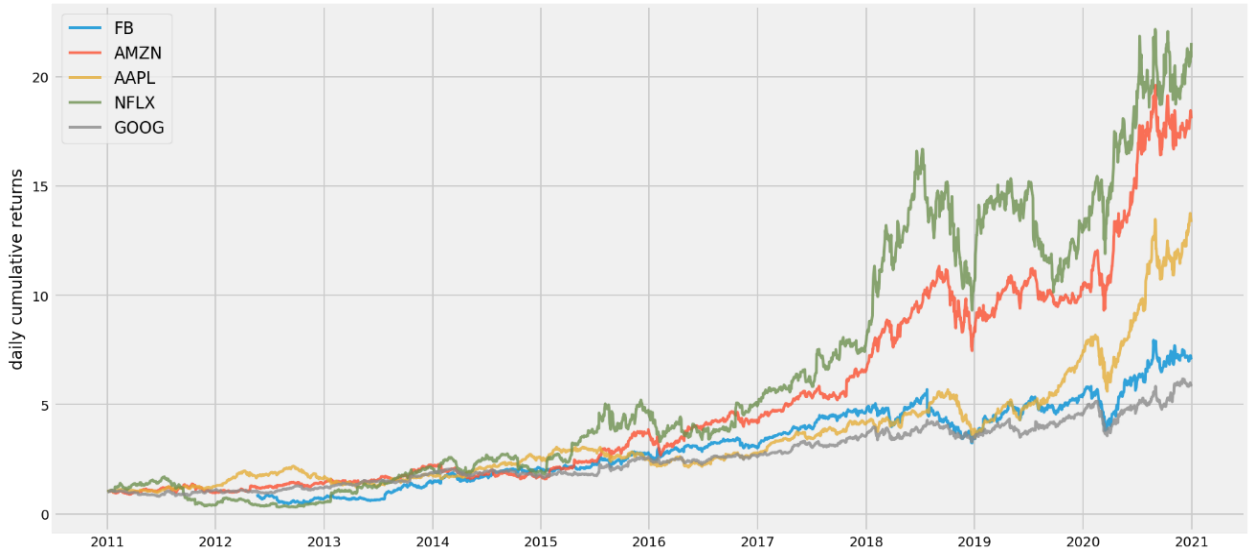

Figure 4 Daily cumulative returns of Facebook, Amazon, Apple, Netflix, and Google over 01/01/2011 to 01/01/2021.

Figure 4 shows the daily cumulative returns. We can see that these five companies increase at a similar speed from 2011 to 2015. However, after 2015, the daily cumulative returns of Netflix increase most rapidly, followed by Amazon and Apple. Facebook and Google experience gentle growth.

Table 1. Maximum Sharpe Ratio Portfolio Allocation and Minimum Volatility Portfolio Allocation

Maximum Sharpe Ratio Portfolio Allocation					
Annualised Return: 0.33					
Annualised Volatility: 0.25					
Companies	Facebook	Amazon	Apple	Netflix	Google
Allocation (weights%)	10.55	35.35	40.16	13.46	0.48
Minimum Volatility Portfolio Allocation					
Annualized Return: 0.28					
Annualized Volatility: 0.23					
Companies	Facebook	Amazon	Apple	Netflix	Google
Allocation (weights%)	7.83	15.09	35.09	3.04	38.95

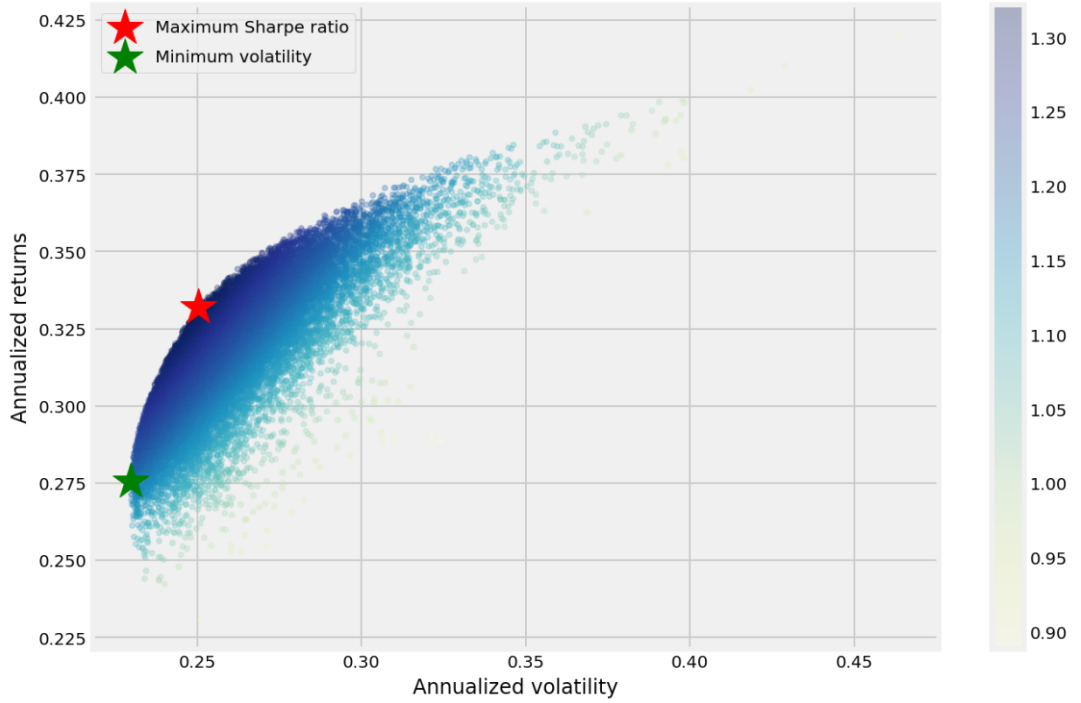

Figure 5 Portfolio Optimization and Efficient Frontier

Table 1 shows the composition of two special portfolios. The first portfolio shows the composition of the portfolio that achieves the maximum Sharpe ratio. We can see that, when 10.55%, 35.35%, 40.16%, 13.46%, 0.48% of the money invested in Facebook, Amazon, Apple, Netflix, and Google, respectively, we can obtain the highest Sharpe ratio with an annualized return is 0.25 and volatility is 0.33. The second portfolio shows the asset allocation of the minimum-variance portfolio, given 7.83%, 15.09%, 35.09%, 3.04%, 38.95% invested in Facebook, Amazon, Apple Netflix, and Google, respectively, with the annualized return of 0.28 and volatility of 0.23.

All feasible portfolios and the efficient frontier line are presented in Figure 5. Portfolios are randomly generated using different weightings. We plot these portfolios with a color map based on the Sharp ratio. The higher the Sharp ratio, the bluer. The top line of the area of blue dots is called the efficient frontier. All points along the efficient frontier give the highest return for a given risk. There are two best choices: one with minimum risk and another one with a maximum risk-adjusted return. The minimum volatility portfolio is represented by the green star, and the maximum Sharp ratio portfolio is represented by the red star. For the minimum risk portfolio, more than 60% of the money is allocated to Apple and Google, and around 3% is allocated to Netflix. According to Figure 3, Apple and Google are the least volatile stocks among the five stocks, and Netflix is the most volatile one. In the minimum risk portfolio, more than 2/3 of the money is invested in stable stocks; thus, the overall volatility is minimum. For the maximum Sharp ratio portfolio, the return is higher for the higher risk. Compared to the minimum volatility portfolio, a larger portion of the money is allocated to Amazon and Apple, where Amazon has quite high volatility and high returns (as shown in Figures 3 and 4). However, Google has less than 0.5% allocated to it in the maximum Sharp ratio portfolio, but around 39% in the case of minimum volatility portfolio.

Therefore, for a conservative investor who prefer lowest risk, the minimum-variance portfolio is a good choice. However, according to Markowitz, risk-averse investors seek different options for a portfolio with higher returns at an acceptable level of risk. Risk-averse investors can choose portfolios along the efficient frontier line, depending on the level of risk he or she wants to.

4. CONCLUSION

In the paper, we study the modern portfolio theory with its application in the stock market. Modern portfolio theory is a widely used for portfolio selection, particularly for risk-averse investors. The main idea of MPT is to use the mean-variance analysis to evaluate the performance of the portfolio. We use the weighted sum of the returns of individual stocks to measure the expected return of portfolio, and variance of individual stocks with correlation between stocks to measure the risk of the portfolio.

We consider the portfolio consisting of five companies: Facebook, Amazon, Apple, Netflix, and Google, over the period from 01/01/2011 to 01/01/2021.

We construct 25,000 portfolios by randomly generated weights, and then we can obtain an efficient frontier. We can give investors suggestions based on the efficient frontier. This is because an efficient frontier is a set of optimal portfolios with the highest expected return for a determined level of risk or the lowest risk for a given level of expected return. For example, if the investors are conservative, we could suggest them to invest heavily in Amazon and Google (more than 70%), and thus, they can achieve the portfolio with minimum risk. If the investors are risk averse, we could suggest them to invest 3/4 of their money in Amazon and Apple to obtain the maximum risk-adjusted return portfolio.

The MPT has several advantages. For example, it is easy to understand and implement for most investors. We can plot possible portfolios with the y-axis showing the risk of the portfolio and the x-axis showing the expected return, and optimal portfolios are lying on the efficient frontier. Moreover, by MPT, we can construct diversified portfolios, which have much lower volatility than each individual stock. However, it also has disadvantages. This model is unrealistic because it is based on some assumptions. For instance, it assumes that all investors are risk-averse and rational, and a well-diversified portfolio is achieved, which is not always true. For future work, we would consider more assets in this portfolio. With more assets, we might obtain relatively lower risks as well as higher returns. Adding risk-free bonds is also a way to lower the risk of portfolios.

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