

How to Find Optimal Portfolios among Nine Mutual Funds from US

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ABSTRACT

Portfolio selection is a common field in the financial world. Investors are always willing to find an optimal portfolio among several assets. In this paper, this report wants to find out a method that can help investors decide their optimal portfolios quickly through some simple comparison. This report use nine mutual funds in the US as the sample. This report set a certain range for the weight of single funds and built a portfolios enumeration model to enumerate all the possible portfolios which contain three single funds and use the Sharpe ratio to determine a portfolio's good or bad. The result is that a portfolio with a large Sharpe ratio has certain characteristics: the funds it contains always include a fund with a negative Sharpe ratio, a fund with the largest Sharpe ratio among nine funds and a risk-free asset and their weight floats only within a certain range. This method can facilitate investors' selection process because they could easily find an optimal portfolio by comparing the Sharpe ratio of each single fund.

Keywords: *Matual fund, Portfolio selection, Optimal Portfolio, Sharpe Ratio*

1. INTRODUCTION

When having capital, ordinary people will put that money into banks to earn interest and it is called a risk-free asset because people do not have to take any risks from this asset. Contrarily, when investors decide to invest their capital, what they can choose is not only the risk-free asset but also stocks, funds and financial derivatives existing in the real financial world. And these assets always have a higher return and higher risks than the risk-free asset. Therefore, in order to balance the expected return and risks, portfolios were something can be used by investors. A portfolio is a collection of stocks, bonds and financial derivatives held by investors or financial institutions. The purpose is to diversify risks and maximize benefits at the same time. Each portfolio will contain multiple assets, and each asset will have a corresponding weight to determine the proportion of investment. When one type of asset experiences some degree of recession, other types of assets provide stability for the portfolio, and the gains of other sectors offset any losses in that area. And this paper takes research on how to find an optimal portfolio among several single funds. The most efficient sections of the risk-return premium

spectrum are occupied by an optimal portfolio. It meets the criteria that no other collection with a higher expected return at the same standard deviation of the return exists. The concept of the optimal portfolio represents the best portfolio among these portfolios, that is, the portfolio that provides the maximum possible expected return under a given acceptable risk level. And people always use return per unit of risk to measure a portfolio, and it is also called the Sharpe ratio. So, in this paper, a method was created to help investors select optimal portfolios which means the portfolios with a large Sharpe ratio.

Research about portfolio selection problems is very popular, and many scholars have studied this topic from different perspectives. Rao et al. pointed out that the portfolio choice goal is to generate the highest returns in the context of the given level of risk of the portfolio [1]. Markowitz pioneered portfolio selection theory and stated the E-V model, which is widely used in portfolio selection [2]. In E-V model, it is important that investor desire to get a higher expected return and lower variance, and investor can use it when they build their portfolio. Markowitz provided a theoretical basis for investors and scholars. In the 1960s, capital asset pricing model

(CAPM) was developed by Sharpe, Lintner and Mossin [3-5]. Zadeh proposed fuzzy sets [6], and Watada takes fuzzy into consideration when he uses the E-V model [7]. Therefore, the mean-variance portfolio theory is not only the first work of modern portfolio selection theory but also one of the stones of modern finance. Recently, the research on the portfolio selection problem has achieved many results, and these theories have been widely used in practice for building optimal portfolios in investors and scholars (Elton et al.) [8]. Elton (1981) illustrated that modern portfolio theory examines the characteristics and analysis of individual securities as well as the theory and practice of optimally combining securities into portfolios [9]. Sankaran and Patil pointed out that portfolio has various diversification [10]. Abuallail pointed out that the copula function was widely used in Egyptian market interest rate and portfolio modeling [11]. Sharpe ratio is a very widely used method to measure portfolios. Sortino and Price replace the standard deviation with the downside deviation [12]. Many scholars had proposed ways to improve the Sharpe ratio, for example, Stutzer [13] and Shadwick and Keating [14]. Also, Feng and Palomar indicated that in the decade since the global financial crisis of 2008, managers have regarded risk management as more important than performance management [15].

However, although there are many types of research on portfolios selection and researchers have built many financial models to analyze the process of portfolios selection, there is not any methods that can help investors quickly find optimal portfolios among known single funds. So, different from previous studies, this paper uses simple statistical theory to compare the average, variance and Sharpe ratio of single funds and came out with a method that could find optimal portfolios rapidly through some simple comparison but not cumbersome research. Based on this idea, the weight of each asset is changed in the investment portfolio to determine the optimal asset portfolios, which have lower variance and higher Sharpe ratio and expected return. Then optimal portfolios can be found out. This has several advantages: First, this method can obtain any number of investment portfolios, which can give investors many choices and is very practical; second, the portfolio with high Sharpe ratio has an excellent risk-return effect, which is deeply loved by investors.

The remainder of the paper is organized as follows. Section 2 explains some basic concepts in selecting portfolios and builds a portfolio enumeration model to find the optimal portfolios; Section 3 describes the variables and distributions of datasets; Section 4 presents the distribution of portfolios and portfolios with large Sharpe ratios using the model in section 2. Lastly, Section 5 concludes the paper.

2. MODELS AND METHODOLOGIES

In this study, this report consider the portfolio selection problems which can make investors achieve a better profit and less risk by choosing some different financial assets. This section was divided into three parts. The first one is some basic concepts and notations related to the analysis of portfolios, which include some statistical methods such as mean, variance, Sharpe ratio. Some effects of portfolio optimization were introduced in the second part. Finally, a method of enumerating possible portfolios is described in the last part.

2.1. Related concepts about analysis funds and portfolios

Generally, in this section, this report introduce some concepts in funds, portfolios and statistics, including the risk-free rate. In order to compute the profitability of funds and portfolios, people usually use statistics to analyze and interpret data. A fund is content that people can set money in and available for a certain purpose. And the portfolio is a combination of financial investments, including stocks and funds. When considering the profitability of one portfolio, people have to take the free-risk rate into account. The risk-free rate (RF) is a return of an investment that carries no risk. In this paper, the risk-free rate means the rate of treasury bills which is a short-term debt obligation backed by the U.S.

Considering two portfolios that have many funds, those formulations are always be used:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (1)$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \quad (2)$$

In equations (1) and (2), n is the number of the data. People can use those equations to calculate the sample mean and variance of the data. This paper use those two formulations to measure the profitability of each fund and portfolio.

Sharpe ratio, also known as the reward-to-risk ratio, which can comprehensively consider return and risks at one time is the rate that combines both standard deviation and expect. The Sharpe ratio is generally defined as:

$$\text{Sharpe Ratio} = \frac{E(R_p) - R_f}{\sigma_p} \quad (3)$$

where $E(R_p)$ is the mean portfolio return, and R_f is a risk-free return. Sharpe ratio is once the most popular way to use for portfolios. By comparing the Sharpe ratio values of different portfolios, because the higher Sharpe ratio means a superior plan, the portfolio with the largest value was selected.

2.2.Explanations of portfolio selection

This part briefly present some phenomena and their reasons. The first one is the diversification effect and the second one is leveraging. By knowing that strategy, people can find satisfying portfolios correctly.

The investors try to minimize portfolio risk and pursue higher expected returns. In general, people can find that the variance of the portfolio sometimes lower than the variance of any fund. This is named portfolio diversification effect which implies that funds that are not highly correlated in portfolio can decrease risks from individual assets. Because of this strategy, people can make our investment have lower volatility by choosing different assets.

The other number in order to measure profitability is expected value. In real life, many investors use debt to increase the return of a portfolio, which called leverage. This paper takes leveraging and diversification effect into consider.

2.3.Portfolio enumeration model

This paper constructs a model to select proper portfolios, which is better than individual assets, by enumeration.

Step 1: Initial selection. Generally, when building a new portfolio, some assets need to be chosen. Assuming that N is equal to the number of all assets, and n is equal to the number of selected assets in the portfolio. Then C_N^n is all possible situations

Step 2: Enumeration. Define W_i , and it presents the coefficient of each selected assets and i is an integer from 0 to n. As the formulation (4), the sum of W_i is one.

$$\sum_{i=1}^n W_i = 1 \tag{4}$$

Due to the step 1, there is a portfolio. Making the W_i between -1 and 2, there are many investment portfolios. In particular, expected value, standard deviation and Sharpe ratio of each portfolio can be calculated.

Similarity, there are C_N^n situations according to step one by enumerating all possible situations and save the data.

Step 3: Final choice. After enumeration those portfolios, this paper successfully calculate the Sharpe ratio of each portfolio. Using those data, people can exclude the bad ones which have higher variance, lower expected value, and lower Sharpe ratio than individual assets. It is also obviously that which portfolio pays the most per unit of risk.

The three steps are the method to find proper portfolio. The number of the possibly portfolios depend on parameter n. In this paper, this paper uses a quantile of n=3 to generate the portfolios.

3. DATA ANALYSIS

The research group chose nine assets to form a portfolio with high return and low risk. These nine assets include Dreyfus Funds, Fidelity Funds, Keystone Retirement Annuity Fund, Putnam Ultra Short Duration Income Fund,,Vanguard Windsor Fund, Schwab US Dividend Equity ETF, equity market, Val market, and U.S. market. This paper choses these funds because they are representative mutual funds in US. The research utilized several methods to test the assets whether normally distributed and independent and identically distributed or not.

3.1.The test of normal distribution

Firstly, this paper used some graphs to verify the assets of the sample were normally distributed. This method was to utilize a graphical way to determine the yield of all assets were basically normal distribution. First of all, this paper extracted all the data of each asset, make them form nine histograms. After that, each group of data of mean and standard deviation were calculated. Then these parameters were used to form a standard normal distribution. Finally, the research compared the histograms with the line charts the parameters formed to see whether the data is close to normal distribution.

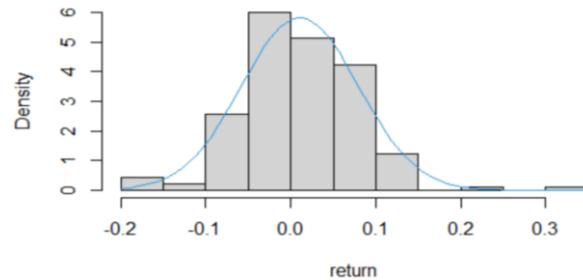


Figure 1 The histogram of eqmrkt (equity market) and corresponding normal distribution line

The graph above is an example of this method. Obviously, the histogram that the return of eqmrkt (equity market) constructed resembled to the normal distribution that built by the parameters of the mean and standard deviation of the return of eqmrkt (equity market).

The second method was the empirical rule of normal distribution. Empirical rule is that approximately 68% of the data is in the interval

$$(x - sd, x + sd) = x \pm sd \tag{5}$$

and approximately 95% of the data is in the interval

$$(x - 2sd, x + 2sd) = x \pm 2sd, \tag{6}$$

where sd indicates the standard deviation of the data. The rule can also be used to verify the data of the sample is normally distribution or not.

Table 1. Testify of empirical rule for each fund

Name of funds	$x \pm sd$	$x \pm 2sd$
Drefus	0.67	0.97
Fidel	0.71	0.96
Keystn	0.73	0.93
Putnminc	0.74	0.96
Scudinc	0.73	0.95
Windsor	0.74	0.94
Eqmrkt	0.71	0.97
Valmrkt	0.73	0.94
Mrkt	0.73	0.94

The table above was the result of the return of these nine assets. It could be seen that the data of the second column was close to 68% and the data of the third column was close to 95%. Therefore, the normal distribution holds in this sample.

3.2. The test of independent and identical distribution

In terms of the test of whether the data is independent and identically distributed, there are also two approaches. The first approach is to use a graph to illustrate that whether the pattern exists. If there is a pattern in the graph that the data formed, the i.i.d theory does not hold. If the line chart does not have a pattern, the i.i.d holds. The second method is to utilize the autocorrelation function to test it. For instance, if the correlation between the different periods is large, i.i.d is not workable. If the correlation between the different times is small, i.i.d is workable.

Firstly, graph is also an appropriate way to check the i.i.d. Observing whether there is a pattern in the distribution of the data is effective. This paper put every asset in a separate graph. The X axis represents time and the Y axis represents return on assets.

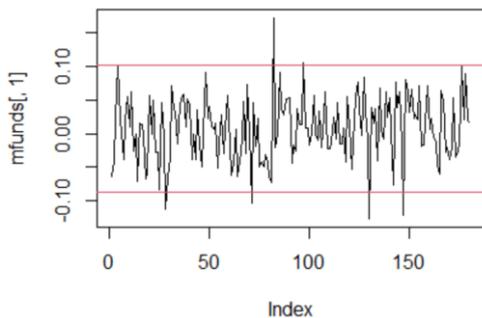


Figure 2 The line chart of Drefus (Dreyfus Funds)

The picture above illustrates the distribution of the Drefus (Dreyfus Funds). Pattern did not exist in this fund. Therefore, i.i.d held in this example. Similarly, other entries in the sample also had no pattern.

The second method was to use a function called ACF, which is the abbreviation of autocorrelation function. The function was utilized to check whether autocorrelation problem exists in the sequence.

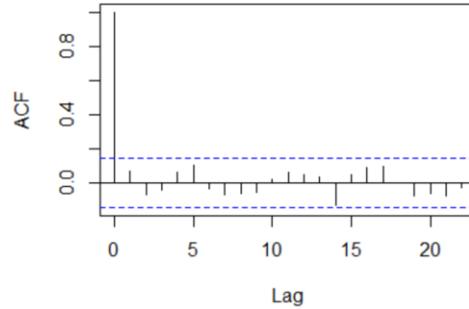


Figure 3 ACF analysis of Drefus (Dreyfus Funds)

The graph above demonstrated the coefficient of autocorrelation. If the coefficient value was greater than the value represented by the blue dotted line, it indicated a large degree of autocorrelation. According to the graph, it could be seen that almost all data located between the two blue dotted lines. Other funds had a similar efficient of autocorrelation to Drefus (Dreyfus Funds). As a result, autocorrelation was almost impossible in this sample.

4. RESULTS

Through the method mentioned above, this paper got a graph with all the possible portfolios and a table containing every portfolio's detailed parameters. This paper divide the results part into two aspects: results of Mean-Standard Deviation Method and results of portfolios with large Sharpe ratio. And this paper will have a discussion on the results which will have further analysis on the data and to talk about the future application.

4.1. Results of Mean-Standard Deviation Method

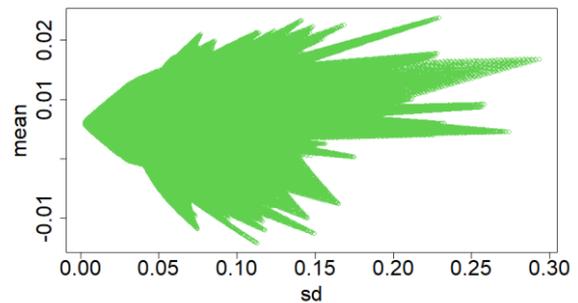


Figure 4 The dot graph of all the portfolios' corresponding stand deviation and mean

Fig. 4 shows all the possible portfolios which contain three funds choosing from total 9 funds. This graph is a dot graph and every dot represent a portfolio. Because the weight of each funds in the portfolios is confined, the shape of the dot graph is also confined. The head point of the fish is T-Bill which stands for the risk-free asset. If adding a line passing through point T-Bill and parallel to the x-axis, the points on this line have the same mean with T-Bill but have higher stand deviation and the points under this line have lower mean and higher stand deviation than T-Bill. This means that all these portfolios are infeasible because they are riskier than T-Bill but less profitable than T-Bill so that choosing the single funds T-Bill will be better instead of choosing those portfolios.

Then if wanting to find some optimal portfolios in this graph, people should focus on the Green fish's left part of the upper edge. At that part, the stand deviation of the portfolios is relatively low, and the mean is relatively high which will have high Sharpe ratio for each portfolio. And when comparing to the single funds, people could find that those portfolios may have same or lower risk than single funds but their expect returns are much larger than single funds. This can at least explain that those portfolios are optimal comparing to investing in single funds. People can conclude that the Sharpe ratio of portfolios can vary quite differently according to their weights and the optimal portfolios are exist.

4.2. Results of portfolios with large Sharpe ratio

This part will represent the result obtained from the table. The data is so huge that this paper sorted all the outcomes by their Sharpe ratio and intercepted some outcomes to facilitate our analyze. Table 2 shows the portfolios with top 20 Sharpe ratios. It is obvious that all these portfolios contain same funds, they are Fidel, Windsor, and T-Bill. Their Sharpe ratio are all around 0.21. And the weights of each funds also have some rules to follow. The weight of Fidel was always a negative number and close to -1, the weight of Windsor was the largest among three funds and close to 1, and the weight of T-Bill was always close to 0.7.

When turning to analyze the single funds in those portfolios, this paper found that Fidel has a negative Sharpe ratio and is the second smallest among 9 funds, and Windsor has the largest Sharpe ratio among 9 funds. And when keeping looking down the whole table, there is no change on the funds that the portfolios contain until 477th portfolio which change the Fidel to the Scudinc(Schwab US Dividend Equity ETF). Scudinc (Schwab US Dividend Equity ETF) has the lowest Sharpe ratio among 9 funds. This paper intercept part of the table from 477th to 489th (Table 3) and find that Scudinc (Schwab US Dividend Equity ETF) begin to appear instead of Fidel. So, this paper conclude that an optimal portfolio with high Sharpe ratio will have a fund with negative Sharpe ratio and its weight is negative, a fund with largest Sharpe ratio and its weight is higher than 1 and a fund which is a risk-free asset.

Table 2. Portfolios having top 9 Sharpe ratios

Order	Fund1	Weight1	Fund2	Weight2	Fund3	Weight3	Sharpe ratio
1	Fidel	-1.000	Windsor	1.273	T-bill	0.727	0.21333
2	Fidel	-1.000	Windsor	1.242	T-bill	0.758	0.21322
3	Fidel	-0.970	Windsor	1.212	T-bill	0.758	0.21322
4	Fidel	-0.970	Windsor	1.242	T-bill	0.727	0.21321
5	Fidel	-0.939	Windsor	1.181	T-bill	0.758	0.21318
6	Fidel	-0.909	Windsor	1.151	T-bill	0.758	0.21311
7	Fidel	-0.939	Windsor	1.121	T-bill	0.727	0.21305
8	Fidel	-1.000	Windsor	1.303	T-bill	0.697	0.21300
9	Fidel	-0.879	Windsor	1.121	T-bill	0.758	0.21299

Table 3. Portfolios with Sharpe ratio ranked from 476 to 484

Order	Fund1	Weight1	Fund2	Weight2	Fund3	Weight3	Sharpe ratio
476	Fidel	-0.364	Windsor	0.394	T-bill	0.969	0.19652
477	Scudinc	-0.636	Windsor	0.636	T-bill	1.000	0.19648
478	Fidel	-0.849	Windsor	1.364	T-bill	0.485	0.19638
479	Fidel	-0.606	Windsor	0.969	T-bill	0.636	0.19636

480	Scudinc	-0.606	Windsor	0.606	T-bill	1.000	0.19633
481	Scudinc	-0.849	Windsor	0.546	T-bill	1.303	0.19627
482	Scudinc	-0.303	Windsor	0.273	T-bill	1.030	0.19624
483	Scudinc	-0.576	Windsor	0.576	T-bill	1.000	0.19615
484	Fidel	-0.212	Windsor	0.303	T-bill	0.909	0.19613

4.3. Discussion of results

According to the results obtained, selecting optimal portfolios among 9 single funds is something traceable. Initially, people should calculate those single funds' Sharpe ratio. Then, find the fund with the largest Sharpe ratio and funds with negative Sharpe ratio. The optimal portfolios will contain a fund with negative Sharpe ratio, a fund with the largest Sharpe ratio and a risky free asset. Their weights are not absolute values. The weight of the fund with negative Sharpe ratio should be negative. So, it does not mean to invest on this fund, it means short-sell the fund. Investors borrow the fund from somewhere and sell it at present and buy back the fund when the price go down.

Then, the weight of the fund with largest Sharpe ratio should bigger than risk-free asset which means investors invest most part of capital on the fund with largest Sharpe ratio and invest the rest on the risk-free asset. Through this method, investor can easily find optimal portfolios from a serious of fund. But Bodnar and Zabolotsky (2017) claimed that the large Sharpe ratio portfolios are exceedingly dangerous, according to both theoretical and empirical data [16], so they may not be optimal. However, there is a risk-free asset besides normal funds, and this paper adds the risk-free asset into the portfolios. The function of the risk-free asset is to neutralize large Sharpe ratio portfolio's existing risk, it can increase the expect return and reduce risks at the same time. In summary, this method will find the optimal portfolios only based on single funds' Sharpe ratio, but it has some restricts. For example, there should be a fund with negative Sharpe ratio and there is a risk-free asset available.

5. CONCLUSION

This paper has built a portfolio model to enumerate possible outcomes of portfolios and found those portfolios with large Sharpe ratios. So there is a method which can be summarized by the model this paper built to help investors quickly find the optimal portfolios they'd like to choose among several single funds. This paper concludes from Fig.4 that Sharpe ratio will change differently when the funds in the portfolios and the weights of those funds change. Then, people choose portfolios with top ranked Sharpe ratios as our main research objects. This paper found that a portfolio with large Sharpe ratio always contains three funds: a fund with negative Sharpe ratio, a risk-free asset, and a fund

with the largest Sharpe ratio. And their weights have incremental relationship: the weight of fund with negative Sharpe ratio should be negative, the weight of risk-free asset is in the middle and the weight of fund with the largest Sharpe ratio is the biggest. Investor should use their capital to short sell the fund with negative Sharpe ratio and invest in the funds with the largest Sharpe ratio. Also, the risk-free asset is indispensable because it insures the stable income of the portfolio. This paper builds the portfolios enumeration model and discuss a method which help people find the optimal portfolios in certain conditions, but this method has a certain limitation: it only supports the selection of three funds as a portfolio from multiple single funds because the model cannot bear the amount of data brought by portfolios with more than three funds. And it is uncertain that if portfolios with more than three funds would have larger Sharpe ratio than the portfolios enumerate in this paper. So, in the future, our research direction will be towards to the selection of portfolios with more than three assets. The model need to be improved so that it can bear a larger amount of data and analyze the data quickly in the future.

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