Advances in Economics, Business and Management Research, volume 211
Proceedings of the 2022 7th International Conference on Financial Innovation and Economic Development (ICFIED 2022)

The Process of Test the Single-factor Capital Asset Pricing Model

Zhen Wang

The Australian National University
u7237023@anu.edu.au

ABSTRACT

The primary purpose is to test the Single-factor Capital Asset Pricing Model (CAPM) based on the Australian stock exchange market. At the same time, previous research explained the limitations and shortcomings of the Single-factor CAPM model and briefly introduced the optimization and change of the CAPM. This paper also points out whether investors and researchers can continue to use Treasury bonds as risk-free assets based on the high inflation rate in the United States.

Keywords: *Capital Asset Pricing Model; Fama-French three-factor CAPM; Australia Stock Exchange Market; Security Market Line; Cheney, Roll, and Ross five-factor CAPM.*

1. INTRODUCTION

The main purpose of the stock exchange is to provide capital and equity to companies. A company can raise additional funds for new capital by issuing new shares on the primary market. When investors buy stocks at a price higher than the stock's actual price, and stock sellers sell the stock at a price lower than that price, the return on the stock will decrease. The problem is how to price the stock more accurately [1].

A model for valuing financial assets that have been in use since 1960 is known as the asset pricing model (CAPM). The model assumes that only a single factor, market risk, can explain the expected return on an asset, and it also assumes that the market compensates investors based on the level of market risk given by their investment. This model was developed by Sharpe [2] and Lintner [3] following the work of Markowitz [4]. However, the researchers quickly identified problems with the model. Due to the variance-covariance sample matrix of the expected return, the market combination of short-selling positions and the beta risk market cannot explain the expected return.

Despite the severe problems with CPAM, the model is still assumed to be the most widely used by firms to estimate the cost of capital and portfolio valuation. According to previous research, approximately 45% of European companies use this model [5].

This article's central purpose is to show how to test the single-factor CAPM model by selecting ten stocks from different industries traded on the Australian Stock Exchange, rather than empirical evidence based on the effectiveness of the single-factor CAPM model the Australian stock market. A brief introduction to the evolution of the CAPM, such as the discovery and development of the multi-factor model.

2. SINGLE-FACTOR CAPM MODEL

The capital asset pricing model (CAPM) was introduced by Sharpe [2] and Lintner [3], following the work of Markowitz [4]. The CAPM model derives equilibrium expected returns on risky assets while providing a tool for comparing benchmark returns on possible investments. Consequently, the model is used to estimate expected returns on investments that are not yet traded in the market.

The CAPM model is based on the assumptions that:

i. taxes and transportation costs do not exist in the market,
ii. investors are price takers, and their wealth is minimal relative to the overall market,
iii. investors focus on single-period investments,
iv. investors can only buy publicly traded assets and can borrow at risk-free rates, and
v. investors are rational mean-variance optimizers, and therefore will use the Markowitz portfolio selection model.

Based on the above assumptions, the CAPM model assumes that only a single factor, market risk, can explain the expected reporting of the asset [6].

In other words, according to the definition in the model, the expected return $E(r_i)$ can be expressed as the Equation (1).

$$E(r_i) = r_f + COV[r_M, r_i] \times \frac{(E[r_m] - r_f)}{\sigma_M^2} \quad (1)$$

By defining β_i as the covariance of assets i's returns with those of the market scaled by market variance yield the Sharpe-Lintner CAPM:

$$E(r_i) = r_f + \beta_i \times (E[r_m] - r_f) \quad (2)$$

Where $E(r_i)$ is the except the return of stock *i*, r_f is the rate of return of Risk-free asset, and $E[r_m]$ is the expect a return on the market, the graphical show as following is the depiction of the relationship described by CAPM is called Security Market Line (SML):

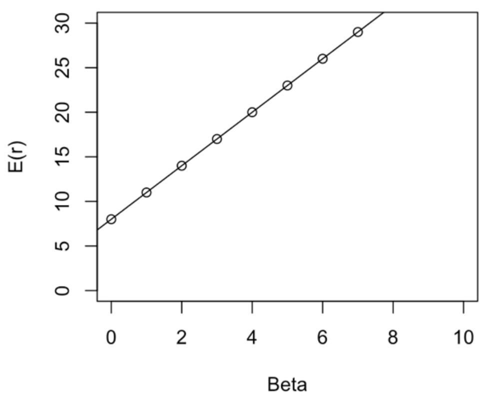

Figure 1: Security Market Line

3. TEST PROCESS

3.1. Selection Data

Because it needs to show how to test, therefore, instead of selecting 100 stocks according to the requirements of the traditional test, ten stocks of different industries were selected from the Australian Securities Exchange [7]. The annual 13-week Treasury bond yield during that period was also selected as the risk-free rate. The unit of analysis in this article is the monthly closing price from January 2009 to the end of 2013. Simple linear regression, multiple linear regression, and t-test will be used to analyze the collected data.

3.2. Fitting Regression

The risk-free rate is first converted to monthly units using the Equation (3), while the monthly return for each stock is calculated using the Equation (4).

$$r_p = \frac{r_n}{n} \quad (3)$$

$$r_i = \frac{P_{t+1} - P_t}{P_t} \quad (4)$$

Simultaneously, the excess return for each stock and the market excess return is calculated by the Equation (5):

$$Excess\ \ Return = E(r_i) - r_f$$
(5)

Using market excess return as the independent variable and each stock's excess return as the dependent variable fitted Sharpe-Lintner CAPM regression (Equation (2)). Table 1 shows each stock beta coefficient value obtained from the excess return over the risk-free rate.

Table 1. Beta for each stock

Stock	Beta
BHP	1.5017
CBA	0.9292
FXJ	1.8943
IIN	0.9262
JBH	0.9843
QAN	1.9866
RHC	-0.0065
STO	0.9401
TLS	0.1903
WES	0.7909

The results of the model will be extended to the stock market line - see Equation (6).

$$R_{it} - R_{ft} = \alpha_i + \beta_i(R_{mt} - R_{ft}) + \varepsilon_{it} \quad (6)$$

Where $R_{it} - R_{ft}$ is the excess return of asset *i* at time *t*, and $R_{mt} - R_{ft}$ is market risk premium of asset *i*.

3.3. Hypotheses Test

Based on the estimates of beta, a second-pass regression is performed using Equation (7) to estimate *γ0* and *γ1*, where the beta estimates are used as independent variables.

$$\overline{r_i - r_f} = \gamma_0 + \gamma_1 b_1 \quad (7)$$

To further argue that the critical element of the expected return-beta relationship described by SML is that the expected excess return of the security is determined only by systematic risk (measured by beta) and independently of the unsystematic risk measured by the residual variance $\sigma^2(e_i)$, these estimates are extended based on Equation (7) to obtain Equation (8), and performed hypothesis test.

$$\overline{r_i - r_f} = \gamma_0 + \gamma_1 b_1 + \gamma_2 \sigma^2(e_i) \quad (8)$$

$$H_0{:}\ \gamma_0 = 0, \gamma_1 = \overline{r_M - r_f}, \gamma_2 = 0$$

H_a: $\gamma_0 \neq 0, \gamma_1 \neq \overline{r_M - r_f}, \gamma_2 \neq 0$

The data required to test this hypothesis are risk-free assets, monthly closing prices of the S&P/ASX 200 index (market returns), and monthly closing prices of each stock from January 2009 to December 2013. The first-pass regression is used to find the parameters β for each stock, and then these parameters are used in the second-pass regression. Suppose the one-factor CAPM model holds, $\lambda 0$ should not be significantly different from zero, $\lambda 1$ should be equal to the average market excess return ($\overline{r_M - r_f}$), and $\lambda 2$ should not be significantly different from zero. The calculated average market excess return is 0.0073, representing the assumed value of $\lambda 1$.

Table 2. t Statistic and p values for hypothesis

	γ_0	γ_1	γ_2
Coefficient	0.0234	-0.0188	1.5770
Standard error	0.0081	0.0075	2.1342
t Statistic	2.8963	-3.4708	0.7389

3.4. Test Conclusion

The results of the second-pass regression and t-statistics are summarized in Table 2. Based on the summary information provided by Table 2 and use 5% significant level. The null hypothesis of $\lambda 0 = 0$ can be rejected which means that the value of $\lambda 0$ was significantly different from zero, $t(7) = 2.8963 > 2.365$ (cut-off value). The null hypothesis of $\lambda 1 = 0.0073$ can be rejected which means that the value of $\lambda 1$ was not equal to 0.0073, $t(7) = -3.4708 < -2.365$ (cut-off value). Since the *t* Statistic of $\lambda 2$ is 0.7389, which is bigger than the lower cut-off value -2.365 and smaller than the upper cut-off value 2.365. The null hypothesis of $\lambda 2 = 0$ failed to be rejected, which means that the value of $\lambda 2$ was not significantly different from zero.

Although this set of data suggests that the one-factor CAPM does not hold since this paper aims to demonstrate the process and methodology of the test only. Therefore, the conclusion does not prove that the one-factor CAPM model does not hold, and it also shows that in the actual empirical evidence process, researchers need to collect a large enough sample of observations to reduce the error to ensure the veracity of the statistical test.

4. LIMITATIONS OF SINGLE-FACTOR CAPM

As the estimation process shown in this paper, the data used are historical data, which means that all the test results and estimation results do not fully and perfectly explain the future situation. It is also worth noting that the CAPM model is based on many perfect assumptions, often incorrect in real life. For example, there is no ideal risk-free asset in the real market. Researchers and investors often use government bonds as risk-free assets to represent risk-free interest rates. However, given the current high inflation caused by the large amount of money printed by the U.S. government, this initiative has created questions about the accuracy of the risk-free rate. Estimating the risk-free rate more accurately seems to be a significant issue for investors and researchers in the U.S. market.

At the same time, previous studies have shown that there are many problems with the CAPM model. For example, in what came to be known as the Roll critique, Richard Roll demonstrated benchmark error. In other words, the CAPM cannot be tested unless the researchers know the actual market portfolio, that is, include all the individual assets in the market in the sample, and use these assets in the test [8]. Since then, many researchers have extended Roll's criticism by demonstrating that even if the CAPM is correct and the portfolio is highly diversified, it cannot produce a significant average return-beta relationship [9].

After making these criticisms, researchers realized the importance of changing the CAPM model, so Cheney, Roll, and Ross proposed a five-factor model of securities returns based on cycle t to describe the macroeconomy (see Equation (9)) [10]. Alternatively, the three-factor model proposed by Fama and French replaces the macro factors with company-specific factors related to the source of system risk (see Equation (10)) [11].

$$r_{it} = \alpha_i + \beta_{iIP}IP + \beta_{iEI}EI + \beta_{iUI}UI + \beta_{iCG}CG + \beta_{iGB}GB + e_{it} \quad (9)$$

$$r_{it} = \alpha_i + \beta_{iM}R_{Mt} + \beta_{iSMB}SMB_t + \beta_{iHML}HML_t + e_{it} \quad (10)$$

5. CONCLUSION

Despite many problems, the single-factor model is widely used in investment and corporate decision-making because the single-factor model is more concise and precise than other models. At the same time, based on the risk-free interest rate selection problem proposed in this article and the limitations of single-factor CAPM, investors should carefully consider the credibility and accuracy of the risk-free interest rate under the current situation and the answers given by the single-factor CAPM model.

REFERENCES

[1] Mossin, J. 1966, ‘Equilibrium in a Capital Asset Market’, *Econometrica*, vol. 34, no.4, pp 768-783.

[2] Sharpe, W.F. 1964, 'Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk', *The Journal of finance (New York),* vol. 19, no. 3, pp. 425.

[3] Lintner, J. 1965, 'The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets', *The review of economics and statistics,* vol. 47, no. 1, pp. 13-37.

[4] Markowitz, H. 1952, 'Portfolio selection'. *Journal of Finance*, vol.7, no. 1, pp. 77 - 91.

[5] Disatnik, D. & Benninga, S. 2012, 'The two-block covariance matrix and the CAPM', *International Journal of Portfolio Analysis and Management,* vol. 1, no. 1, pp. 32 - 42.

[6] Blitz, D., Falkenstein, E. & Vliet, P.v. 2014, 'Explanations for the Volatility Effect: An Overview Based on the CAPM Assumptions', *Journal of portfolio management*, vol. 40, no. 3, pp. 61-76.

[7] Bodie, Z, Kane, A, & Marcus, A 2017, *ISE EBook Online Access for Investments*, McGraw-Hill US Higher Ed ISE, New York.

[8] Roll, R. 1977, 'A critique of the asset pricing theory's tests Part I: On past and potential testability of the theory', *Journal of financial economics,* vol. 4, no. 2, pp. 129-176.

[9] Roll, R. & Ross, S.A. 1994, 'On the Cross-sectional Relation between Expected Returns and Betas', *The Journal of finance (New York),* vol. 49, no. 1, pp. 101-121.

[10] Chen, N., Roll, R. & Ross, S.A. 1986, 'Economic Forces and the Stock Market', *The Journal of business (Chicago, Ill.),* vol. 59, no. 3, pp. 383-403.

[11] Fama, E.F. & French, K.R. 1996, 'Multifactor Explanations of Asset Pricing Anomalies', *The Journal of finance (New York)*, vol. 51, no. 1, pp. 55-84.