The Valuation of Up and in Barrier Options on the Tesla Index

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ABSTRACT
This research researched to evaluate the valuation of in and up barrier methods in Tesla's index. This research used the pricing model of barrier methods and, specifically, the Black Scholes model. The model is favorable for some reasons such as being less costly than conventional models that attract investors who presume that there will not be a trigger on their barrier option. This research obtained the valuation of the up-and-in barrier option of Tesla's index and compared it to the standard European options price option. The comparison in the model involved a sensitivity analysis of the independent variables against the dependent variables where barrier, spot price, volatility, and strike price are the independent variables against the dependent variable of the option price. This research then adjusted the independent variables multiple times to observe option price behavior under the same conditions. The results were that the option price fell as barrier and strike price decreased. The spot price also increased with an increase in volatility and spot price**.** The evaluation is thus essential to forecast Tesla's performance in terms of profitability and growth based on the most appropriate barrier option. Investors will be more informed on the most appropriate barrier option to purchase with the most reasonable chances of return based on the option price and the sensitivity of the independent variables such as volatility and spot price. Tesla's investors will be in a position to make appropriate investment decisions.

Keywords: *Barrier Option, Tesla Index, Sensitivity analysis.*

1. INTRODUCTION

The pricing and hedging of vanilla options are now part of the common knowledge and the general interest has moved on to more complex products so practitioners in the market need to be able to price these financial products [1]. Barrier options are one of the typical examples of exotic options. A barrier option is a derivative whose payoff depends on whether the price reaches or exceeds the barrier. There are four types of barrier options include up-and-in, which means the option will be work if the price of the asset rises above the pre-specified barrier that is set above the strike price, and there is down-and-in, which will be active when the stock price goes below a pre-determined barrier that is set below the strike price. There is the up-and-out, which will lose efficiency when the stock's price moves above a barrier that is set above the strike price. The last one is down-and-out, which will lose efficiency when the stock price moves below a barrier that is set below the strike price. And when a barrier option is active, it works like a normal option. Since their price is much cheaper than the corresponding European options, barrier options are widely used in risk management as an appropriate hedge instrument by banks, corporations, and institutional investors [2]. Barrier options are one of the most popular and liquidly traded derivatives, especially in the foreign exchange (FX) markets and continuously monitored knock-in or knock-out calls and puts are among the most liquid products traded in the FX markets [3].

Rodrigo states that the Barrier option is an exotic option which payoff depends on whether the price reach or exceeds the barrier [4]. According to Scott's article, there are four types of barrier options which include up-and-in, which means the option will be work if the price of the asset rises above the pre-specified barrier, but it

will be worthless if the stock price cannot reach the barrier price [5]. Downey expresses that there is also a down-and-in barrier option, which will be active when the stock price goes below a pre-determined barrier that is set below the strike price [6]. This research also have the up-and-out, which will lose efficiency when the stock's price moves above a barrier that is set above the strike price as introduced in Mitchell's article [7]. The last type of barrier option is down-and-out, Scott's article shows that it will be invalid only after the stock price moves below a barrier that is set below the strike price [8].

To simulate the option price, this research used the Black-Scholes model. According to Hossan， Hossain and Islam's article, the Black-Scholes model has been developed in 1973 by Fischer Black and Myron Scholes [9]. Shinde and Takale point out that this model has been used to price European options on stocks that do not pay dividends, and the element of the formula include stock price, dividend the stock pays, strike price, time to expire, and the risk-free interest rate [10].

The research involved collecting Tesla's one-year stock price and evaluating the return. This research then used the return to calculate volatility and the standard deviation. The risk-free rate was also utilized as the yield rate for a one-year treasure rate. The option price is then calculated by simulating 1000 sets of future Tesla's stock. Ultimately, due to Tesla's appealing performance in the stock market, an up-and-in barrier option was the most appropriate for Tesla. In addition, under all circumstances, investors would spend less money on Tesla's barrier alternatives than on Standard European options. Based on the model, the general notion is that an investor could avoid risks by selling and buying options over time [11]. Thus, for investors who believe that stock prices will go high, it would be cheaper to purchase call options in Tesla's shares if they choose European options. The Black Scholes model used in equity-based products can lower the risk and enhance the yield [12].

Notably, the research is in a well-defined article comprising various sections that partition the content for easy understanding and flow. The article begins with a brief description of Tesla's business model and financial achievements for the past three years. The article further involves the method utilized in the research, which included the Black Scholes' barrier option pricing model. The article also incorporates a sensitivity analysis where this research analyzed the independent variable of price option against other variables (barrier, strike price, spot price, and volatility) and included a graphical representation of the results of each analysis under the same conditions. The final part of the article is the conclusion that provides closure on the most appropriate barrier method for Tesla. The conclusion also provides insights, on the sensitive periods of volatility, strike price, barrier, and spot price to establish whether the up-and-in option price is favorable.

2. FIRM DESCRIPTION

By Sara Gilson's article, Tesla is a vehicle company that aims to design and produce high-quality and performance electric vehicles. Its corn competitive advantage is their excellent teams that design unique electric powertrains. Tesla also hires team members who respond to computer aim design and crash simulation tests. They also try to develop light material which helps to maximize the mileage range of their cars. In their stores, there are usually limited finished inventory, because tesla's customer tends to customize their vehicle and spend 2500 dollar to place their order. Moreover, tesla invested to build a large supercharge network thought out the areas like North America, Europe, and Asia for the convenience of customers to charge their cars [13]. As Elon Musk, the CEO and chairman of Tesla, states, Tesla's mission is to bring the compelling electric car to the market as soon as possible to accelerate the arrival of sustainable transport [14]. By the report of form 10-Q of tesla, the total revenue of tesla at 2021 is 13757 million and the Gross profit is 3660 million [15]. By analyzing the closing price of tesla stock monthly, we can see how impressive the Tesla stock has performances in the market in recent 2 years. Here is a picture form by the data deprived of Yahoo finance. From November 2019 to July 2020, Tesla's stock price increase generally, but there is a relatively sharp increase after July 2020 and a sharp decrease after August 2020. After October 2020, the stock's price begins to increase rapidly. Then it decreases a little bit and remains relatively constant from December 2020 to September 2021. Then it increases rapidly after September 2021.

Figure 1 Tesla's stock price in recent 2 years

3. METHOD

3.1. PRICING MODEL OF BARRIER OPTIONS

For simulation, this research use the Black-Scholes model[17]. In 1973, Fischer Black and Myron Scholes published the BSM model with significant help from Robert Merton. They assumed future stock prices were

lognormally distributed, meaning the future stock price could be represented as a function of "z", a random variable that is normally distributed. The following formula is the Black-Scholes model.

$$S_T=S_0e^{(\alpha-\frac{1}{2}\sigma^2)T+z\sigma\sqrt{T}} \quad (1)$$

3.2. ADVANTAGES OF BARRIER OPTIONS

A barrier option is a derivative whose payoff depends on whether the price reaches or exceeds the barrier. There are four types of barrier options include up-and-in, which means the option will be work if the price of the asset rises above the pre-specified barrier that is set above the strike price, and there is down-and-in, which will be active when the stock price goes below a pre-determined barrier that is set below the strike price. There is the up-and-out, which will lose efficiency when the stock's price moves above a barrier that is set above the strike price. The last one is down-and-out, which will lose efficiency when the stock price moves below a barrier that is set below the strike price. And when a barrier option is active, it works like a normal option.

Out options are suitable for small price fluctuations and are easy to become invalid when the price fluctuates largely. However, options are suitable for large price fluctuations and are hard to become valid when the price doesn't fluctuate largely enough.

After a brief introduction of barrier options, This article compare barrier options with ordinary options.

Firstly, the main reason for the popularity of barrier options is that they are generally cheaper than conventional options, which is attractive to investors who believe that barrier price will not (or will) be triggered. The reason for the cheaper option price is that barriers limit the potential return to maturity of option buyers and reduce the risk exposure of option sellers. To be specific, for up-and-in options, the upward movement needs to exceed a certain limit to make a profit. If he is right, he can earn what he wants. Otherwise, even if the price of the underlying asset is higher than the strike price, that is to say, the ordinary option is profitable, but because the barrier price is not reached, the barrier option is still at a loss. On the contrary, a knock-out option will be purchased by someone who believes that the upward movement of the underlying asset price will have a certain limit before expiration. At this time, the option will also be cheaper than the corresponding ordinary option.

Secondly, buyers can use barrier options to hedge certain specific cash flows of similar nature. They can play a good risk avoidance effect in commodity and financial product investment. Besides, some customers need to make up for their risk exposure when the market price fluctuates sharply. At this time, options can meet this requirement.

Thirdly, barrier options can meet the individual needs of investors. Generally speaking, the buyer has a fairly precise view of the market direction. If he believes that the upward movement of the underlying asset price will exceed a certain limit before expiration, and he hopes to obtain the return of the call option and is willing to pay for the possibility of an increase. Then he can buy an upward knock-in option. As for Tesla, since most people on the market currently believe that its stock price will skyrocket, it is the best choice to enter an up-and-in call barrier option. Even if Tesla fell, his loss of option prices would be less than buying ordinary options.

4. RESULT

The variable symbols This research used and their meanings and values are listed as follows. The period, which is T, is one year. This research take the price of Tesla stocks on Oct 14, 2021, as the spot price, which is 818.32. The treasury yield rates of one year is 0.01%. Delta is dividends of one year for Tesla, which is zero. This research set the barrier price at 1000 and the strike price at 800. The strike price is slightly lower than the spot price so that this call option is more profitable and the risk of not exercising the right of buying stocks in the future and losing the option premium is lower.

Firstly, this research collected the Tesla stock price for one year and calculate the return. Then, this research use the return to calculate standard deviation and volatility.

The standard deviation is 0.033571. The correlation is 0.401754. The volatility is 0.53292.

Table 1. Summary of research results.

Standard Deviation	Correlation	Volatility
0.033571	0.401754	0.53292

Because the barrier option is a path-dependent option, this research need to simulate the monthly price to calculate the final price after one year. This research use the formula of the Black-Scholes model to calculate the monthly price. T is equal to 1/12. For the first month, S_0 is the spot price. However, for each month after the first month, S_0 is the price of the previous month this research have just calculated. This research simulate a thousand times for each month.

Finally, this research calculate the discounted average payoff of the barrier options, which is the option price. Our result is 186.658. This research also calculate the option price of the corresponding standard European options, which is 190.616. As this research discussed

above, barrier options are generally cheaper than conventional options.

From the formula and the process of our calculation, we can find that the price of a barrier option is affected by the barrier price, the strike price, the spot price, and the volatility of the underlying assets. Therefore, to further explore the influencing mechanism, this research make a sensitivity analysis.

5. SENSITIVITY ANALYSIS

Assumptions are made about how changes in independent variables influence their dependents under specified conditions. Four independent factors are taken into account: the barrier, strike price, spot price, and Volatility. Then, the option price is taken into account as a dependent variable. This research next adjust the independent variables in each group several times to see how the option price changes under the same conditions.

5.1. BARRIER

First, the Barrier is an independent variable; the strike price is fixed at 800 US dollars, the spot price is set at 818.32 US dollars, and Volatility is fixed at 0.5329. Barrier's base worth is 810 US dollars, and it rises to 1300 US dollars in 10 US dollar increments. This research ran the simulation 50 times and computed the option price under each of the 50 Barrier modifications. Finally, this research have a sensitivity analysis chart on Barriers.

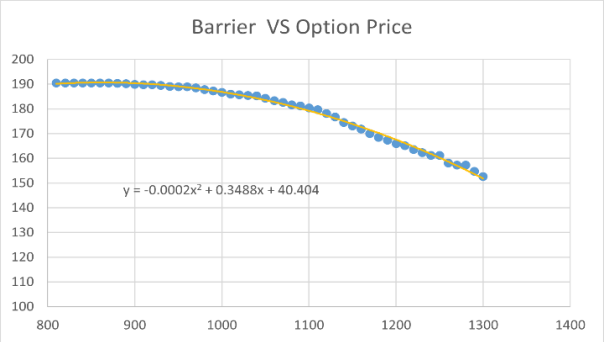

Figure 2 Barrier VS Option Price

As shown in Figure 2, as the Barrier increases, the option price will fall, and the slope of the decline will gradually increase. This is because the higher the barrier, the less likely the choice will be activated. This increases investors' risk, and investors are naturally hesitant to purchase options at high prices. The option's seller must reduce the option's price to entice investors to purchase it. And while the Barrier grows by unit, the risk taken by investors does not increase by department, and the risk they take increases significantly, causing the decrease in option prices to increase steadily.

5.2. STRIKE PRICE

The striking price is an independent variable in the second group, the Barrier is fixed at $1,000, and the spot price and volatility are the same as in the first group. The original strike price is $740, which is raised to $935 in $5 increments. This research simulated the strike price sensitivity analysis graphic 40 times in total.

Figure 3 Strike price VS Option Price

According to Figure 3, when the strike price rises, the option price falls, and it falls in line with a single slope. The higher the strike price, the less the investor may gain as long as the option is triggered and the investor decides to execute the option on the expiration date within the same barrier. Furthermore, the rise in strike prices raises the risk for investors. A too-high strike price reduces the likelihood that investors would choose to exercise the option, resulting in a loss of revenue and options premiums. As the strike price rises, investors will only be ready to purchase the opportunity at a reduced price.

5.3. SPOT PRICE

The third group takes Spot price as the independent variable, the strike price is fixed at 800 dollars, the barrier is set at 1,000 dollars, and Volatility is the same as the first group. The initial spot price is 740, increasing to 870 US dollars in units of 5 US dollars. This research simulated a total of 25 times and got the sensitivity analysis graph of Spot Price.

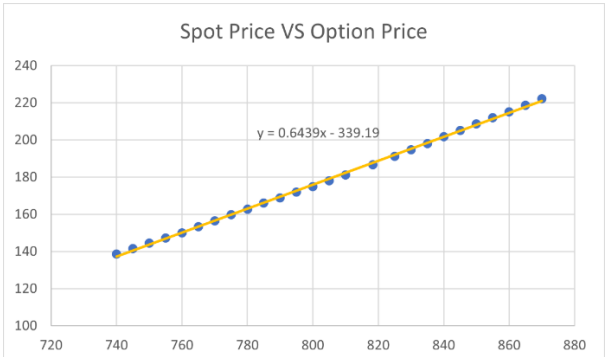

Figure 4 Spot Price VS Option Price

Figure 4 shows that the greater the spot price, the higher the option price. This is because, with the same

strike price, barrier, and volatility, investors are confident that Tesla's stock will climb and will purchase the option. When the stock rises, the higher the spot price, the more the option is activated. The higher the likelihood of execution, the lower the investor's risk, the larger the reward, and the higher the risk of the option seller. As a result, the higher the spot price, the more eager the investor is to pay to acquire the option, and the seller of the vote will raise the price to balance his revenue.

5.4. VOLATILITY

The last group takes Volatility as the independent variable. The strike and barrier prices are the same as in the third group, and the Spot price is 818.32, as in the first group. Volatility starts at 0.39 and rises to 0.63 in units of 0.01. One simulation was run 25 times, and the Volatility sensitivity analysis graph was generated.

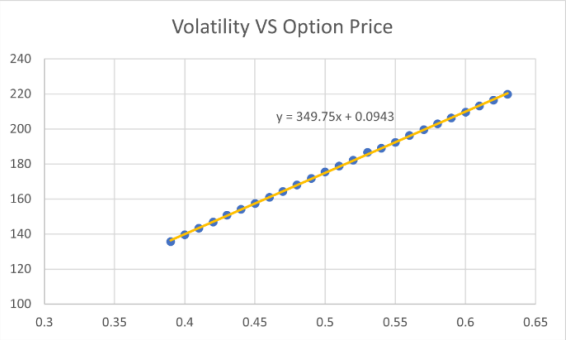

Figure 5 Volatility VS Option Price

As demonstrated in Figure 5, the higher the volatility, the higher the option price. This is because, given the same strike price, barrier, and spot price, the more the volatility of the stock, the greater the likelihood that the stock price would diverge from the execution price, and hence the more extensive the potential return. As a result, investors are prepared to pay more. To buy options at a price, for the seller of an alternative, the more the volatility of the stock, the greater the price risk it carries; hence the option's price must be increased.

6. CONCLUSION

In conclusion, this research choose tesla's stock to design an up-and-in barrier option, because tesla's stock performs very well in the stock market. It also has a lot of potential because of the characters of its business model, which represent it has a long-term goal. Therefore, the up-and-in barrier stock option is very suitable for it. This research collect tesla's stock price of one year to calculate the return, then this research use the return to calculate the standard deviation and volatility. This research choose the risk-free rate to be the yield rate of one year of treasure yield. This research simulate 1000 sets of future 12 mouth tesla stock to calculate the option price. This research pick a number near the spot price as the strick price of the barrier option

Under all other circumstances, Tesla's barrier alternatives will cost less than Standard European options. Bullish Tesla stockholders can use less money to purchase call options on the company's shares if they choose European options. And we can see that the rise in barrier and strike price will negatively influence the option price of Tesla's up and in barrier option, and the option price will fall as a result of this sensitivity study. This means that investors will have to deal with increasing levels of risk while also seeing lower levels of reward. The rising Spot and Volatility prices will have a beneficial effect on the option's price. As a result, the option's price will rise, lowering investors' risk and increasing profit. According to Tesla's stock movement over the previous two years, if the Barrier and Strike prices are set reasonably, option purchasers can avoid option costs and reap the benefits.

This research still have some deficiencies. This research only use a fundamental version of the Black-Scholes model based on several assumptions. In the future, Our research will use a more accurate model to simulate the price of barrier options. Besides, this research only collect the data of Tesla stocks for one year and this research only make 1000 simulations. In the future, our research will use a program to collect millions of thousands of statistics and simulate millions of times so that our results will be more accurate.

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