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Application and Commercial Extension of Game Theory in Strategy games

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ABSTRACT
Werewolves of Miller's Hollow is a popular game among small and medium-sized friends gatherings. It has a high degree of playability due to the mutual restraint of skill characteristics and the diversity of the game process. There is no restriction on the actual or false content of the speech during each round of voting, and werewolves can use incomplete information to establish an advantage to guide the situation. Since every player cannot be completely identified in the game, players need to speak according to the form of the scene to make other players believe them. This study will focus on the typical characters in the Werewolves of Miller's Hollow--Guardian. The characteristics are used to analyze the most suitable game strategy for werewolves. Therefore, it has a better understanding of how game theory is used in life, instead of just sticking to the numbers in the topic. At the same time, through the analysis of this game, it can be further extended to other games, and help people get the quintessence of the game theory.

Keywords: *Werewolves game, Game theory, Strategic issues*

1. INTRODUCTION

1.1. Background

Werewolf, the popular party game that originated from the Mafia game that Soviet psychologist Dmitry Davidoff (1986) invented, demonstrates the complex network of the application of game theory. Compared with other party games, Werewolf enables the player to logically think and thoroughly find the ingrained logic behind the speech or the incentive.

Regarding a dynamic game with incomplete information, Werewolf includes two opposing camps: the majority of civilians or gods and the informed minority of werewolves. Specifically, the majority group needs to chase down the clues of werewolves and vote them out during the daytime, while the minority group should try to disguise their identity and survive until the final stage. In particular, the majority group includes the gods with special abilities, which enables them to kill, save, and see the identity of others.

Game theory helps us understand a situation where decision-makers interact with each other. In the werewolf game, since it is difficult for players to grasp the accurate information of others' characteristics, strategy sets, and preference functions, most games are incomplete information games. The action that players do in the night is a static game of incomplete information, which means all players choose their actions at the same time but do not have perfect information. Complementary to it, the voting part is a dynamic game which means players take turns to decide their actions, and they do not have perfect information.

Sometimes the player's strategy is easy to grasp. For example, according to the game's postulate, considering that each player will take the best solution that he can take to achieve his side's victory. Individual games are also perfect information games, so Nash Equilibrium (s an action profile such that for any action of any player i) is also necessary.

1.2. Related Research

Bi and Tanaka concentrate on a werewolf-side strategy called the “stealth werewolf” strategy which means each of the werewolf-side players behaves like a villager, and the player does not pretend to have a special role. They calculated theε-Nash equilibrium of strategies for both sides under this limitation. The result suggests that the "stealth werewolf" strategy is not a good strategy for werewolf-side players. Bi and Tanaka limit the human-side strategies such that the seer reveals his/her role on the first day and the bodyguard never reveals his/her role which is worth referring to [1]. Goeree and Holt focus on the relevance of game theory for predicting human behavior in interactive situations. By relaxing the classical assumptions of perfect rationality and perfect foresight, Goeree and Holt obtain much improved explanations of initial decisions, dynamic patterns of learning and adjustment, and equilibrium steady-state distributions. Research describes new developments in game theory that relax the classical assumptions of perfect rationality and foresight. These approaches to introspection (before play), learning (from previous plays), and equilibrium (after a large number of plays) provide complementary perspectives for explaining actual behavior in a wide variety of games. The resulting models have the empirical content that makes them relevant for playing games, not just for doing theory [2]. Lin and Utsuro construct an agent that can analyze the participating players' utterances and play the werewolf gac the viewpoint of the given specific player. As a result, Lin and Utsuro develop a set of inference rules and apply them to infer the participating players' roles from a real werewolf game log. Furthermore, Katagami et al. also studied the effect of non-verbal information in the face-to-face werewolf game. These complex off-field factors create many different possibilities, and they have a huge impact on the game [3].

Xu and Wang studied computer game theory and algorithms. The research gives the model of event game theory first, then analyzes problems and difficulties to the model solution. For the discrete game variables of dynamic games, the game is defined as an event game since the systems belong to a discrete event dynamic system (DEDS). The basic model frame is also given in the reference for the perfect information event game system. The fundamentals and methodologies of computer games are summarized and efficient tools to the event game could be detected. In addition, research also described how to develop the event theory and tools, some application problems are also introduced last [4]. Jotaro et al. investigated how werewolves' cooperation contributed to increasing the winning possibilities of the werewolves' team. The researchers used decision trees, payoff matrix, and computer model to develop the best strategies that the werewolves should use as the result of analyzing over four hundred game logs of Werewolf BBS focusing on the "stealth werewolf". The author confirmed that the winning percentage of werewolves increased by 65% at most when the number of werewolf utterances was very frequent. The result also testify that the winning percentage of werewolves dramatically increase when the number of whispers for werewolves was very frequent [5]. Issei and Yoshinobu applied computer models and databases to explore different camps' behavior in the role-playing game. In a preliminary experiment, the researchers discovered classification rules capturing characteristic behaviors among multiple agents were successfully obtained. They found out that wolves can recognize teammates, but others cannot, while seer players can perceive spurious seers easily because other players must not be a seer. By effectively exploiting this heterogeneity and preparing some mechanisms to estimate the intentions of other players, the researcher can develop complex and accurate classification rules for each character, which also set the foundation for the game setting of the werewolf game. Moreover, the classification of different role players' behavior could better develop the strategies that the players choose [6].

Eger and Martens researched how different roles in the Werewolves game would respond to the one-night game by using artificial intelligence. They discussed the different deliberative strategies our agents use to decide what they should say and when they should change their plans. To determine how these different deliberative strategies are perceived by human players, researchers conducted an experiment in which participants performed each of the three deliberative strategies uniformly. The results of this experiment show that commitment to a plan has a measurable impact on player perception and provides a trade-off between consistency and the potential for agency performance [7]. Xiong et al. applied computer simulations to model a simple version of the Mafia game, a similar game that follows the mechanism of a werewolf, which is conducted to collect the data while game refinement measure is employed for the assessment. In addition, the level of the player also affects the balance and complexity of the game. Furthermore, the researchers also apply this finding to a more complex game setting, which required natural conversation instead of model manipulation. However, as the number of players increases, then the refinement value of humans is almost closed to the pure AI simulation, which means while the total number of players was decided, the game's exciting level and progress are almost similar [8]. Katagami et al.'s study mainly focused on the analysis of nonverbal information during the werewolf game. The authors used the tagging to the contents utterances and nonverbal information expressed in the utterances in the game of werewolf. With the help of the. Game movie and the decision trees conclude that the nonverbal information in the werewolf game is essential in winning the game. But the importance of gestures and facial expressions remained unclear [9]. Xiong et al. studied the playing settings of the mafia game (which can be treated as the werewolf game). In the study, they mimicked the game with AI algorithms that follow the strategy of Nash Equilibriums. First, they built the game process model of the mafia game. Then they defined a game refinement value, GR, to indicate how well the game is designed. By analyzing the GR values calculated from different role settings and comparing them to different games, they found that the mafia game becomes less interesting when the game size gets too large. Also, they concluded several game settings that are most balanced [10]. Yao treated the mafia game as an experiment in human psychology and mass hysteria, or as a game between an informed minority and an uninformed majority. He first used the theorem given by Mossel et al. states that the mafia and the civilians are equally likely to win the game when the size of the mafia is the square root of the total number of players. After that, the author validated the correctness of the theorem. In the end, he managed to expand the phenomenon into more general scenarios[11]. Braverman et al. discussed the best strategies for the different roles in the mafia game corresponding to different scenarios. The primary question to be answered in their study is to determine how large and strong a subgroup should be to dominate the game. The team first found that a randomized strategy is optimal when the detective is absent. Then the mafia size is determined as the square root of the number of players. After that, they also conclude that the appearance of any single detective can make a great change to the game pattern [12].

Therefore, it is of great significance to study game theory. With the help of specific games, this paper will deeply study the application and skills of game theory.

2. METHOD

2.1. Problem Setting

The original game involves all surviving players making decisions during each stage of the game, and their decision-making processes vary from case to case, depending on every other player’s choice. For example, each player could give information that s/he is the seer by saying another player is a werewolf/good person. And other people need to choose to put credit on the statement or not. As the game progresses, the size of its subgames grows exponentially. Due to the tremendous complexity of the entire game tree and deficient programming computational skills, it is not likely to analyze each subgame and find equilibriums by hand. Therefore, the team decided to select one of the possible stages during the game to conduct research.

In this scenario, there will be only four players remaining. One is a werewolf, one is a guardian, and two are the villagers. The next stage is night, where the werewolf selects one player to kill, then the guardian chooses one to guard. There are some further assumptions applied to implement the model.

Table 1. Further assumptions

Number#	Detail
1	The guardian and the werewolf know each other’s true identity from previous moves. As a result, the guardian will not choose to guard the werewolf during the night.
2	The guardian will always vote for the werewolf on the day.
3	Players do not communicate during the day; they make their votes by themselves.
4	Two villagers do not hold any belief; they do not know the identity of anyone but themselves. Thus, they decide their votes randomly. The chance of any other player receiving their votes is the same.
5	The payoff of victory is marked as 1, and 0 for losing.
6	Since the villagers are following stochastic moves, and they are on the same side as the guardian, their payoffs are identical to the guardian. Besides, their actions do not appear in the game tree.

As shown in Table 1, each player does not know exactly where s/he is in the game. The histories of the game are incomplete summaries of what has been played so far. It is reasonable to conclude the circumstance as a dynamic game of incomplete information. The structure of the game's extensive form of the first night is summarized in Figure 1 below. And the Nash Equilibrium can be found if all the simplified subgames can be calculated.

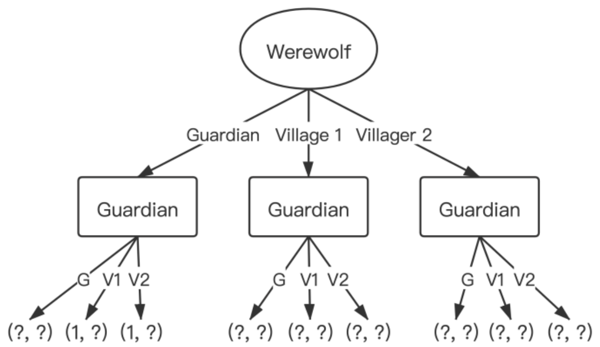

Figure 1 Structure of game extensive form at night 1

2.2. Simplification

Now, based on the result from the first night, there are two possible outcomes. Either the guardian can guard someone successfully or unsuccessfully. The following discussions will be conducted on the two circumstances.

2.2.1. Guard successfully

When the guardian guards someone successfully, all players can survive the night and these three situations are highlighted in Figure 2. Given the belief the guardian always votes the werewolf, Table 2 shows all possible outcomes the votes could be for the players except the werewolf in this case.

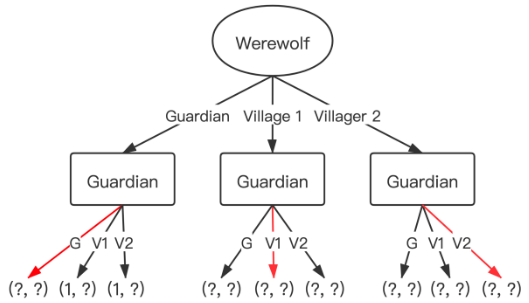

Figure 2 Structure of game extensive form when guarding successfully

Table 2. Structure of game extensive

Case#	W	G	V1	V2
Case1	3	0	0	0
Case2	2	1	0	0
Case3	2	0	1	0
Case4	2	1	0	0
Case5	1	2	0	0
Case6	1	1	1	0
Case7	2	0	0	1
Case8	1	1	0	1
Case9	1	0	1	1

However, due to the uncertainty of villagers, each case has a chance of 19 to arise. So, the werewolf could not determine the exact situation which case s/he is facing. Then it is necessary to calculate the chance for the werewolf to survive in each case. It is done by adding one to each column separately (the one vote from the werewolf).

Vote the guardian:

$$\left(0+\frac{1}{2}+0+\frac{1}{2}+1+1+0+1+\frac{1}{4}\right)*\frac{1}{9}=\frac{17}{36}=0.47 \qquad (1)$$

Vote the villager 1:

$$\left(0+0+\frac{1}{2}+0+1+1+0+\frac{3}{4}+1\right)*\frac{1}{9}=\frac{17}{36}=0.47 \qquad (2)$$

Vote the villager 2:

$$\left(0+0+0+0+1+\frac{3}{4}+\frac{1}{2}+1+1\right)*\frac{1}{9}=\frac{17}{36}=0.47 \qquad (3)$$

As calculated, the werewolf can survive the vote with a chance of 0.47 no matter which player s/he votes.

And after the vote, three possible outcomes may happen:

Table 3. Three types of scenarios after the vote

Scenario#	Type
Scenario 1	The werewolf is voted out, the good people win.
Scenario 2	The guardian is voted out, the werewolf wins.
Scenario 3	One of the two villagers is voted out, the game continues.

To determine the chance of each outcome, the probabilities are calculated as follows. The i below indicates the case number from Table 2 and j represents the choice of the werewolf. i = 1, 2, 3,⋯ 9; j = W, G, V1, V2. According to Table 3,

Scenario 1, the werewolf is voted out:

$$p_{w,total} = \sum_i \sum_j p_{w,ij} = 55/108 = 0.51 \qquad (4)$$

Scenario 2, the guardian is voted out:

$$p_{g,total} = \sum_i \sum_j p_{g,ij} = 1/4 = 0.25 \qquad (5)$$

Scenario 3, one villager is voted out:

$$p_{v,total} = 1 - -p_{g,total} = 13/54 = 0.24 \qquad (6)$$

As a result, the probability of the game continuing is 13/54 = 0.24(6). Now consider the case when the werewolf survives.

When the game proceeds to the second night, the guardian does not have any other option but to guard the villager because s/he cannot guard the same person in two consecutive nights. If the werewolf keeps killing the guardian, the game ends with the payoff of (1, 0).

In the case of the werewolf choosing the villager to kill, given the belief the guardian always votes the werewolf, Table 4 shows all possible outcomes of the votes made by players except the werewolf.

Table 4. All Possible Vote Outcomes With 3 Players

Case#	W	G	V
Case1	1	1	0
Case2	2	0	0

Due to the uncertainty of villagers, each case has a chance of12 to arise. Now calculate the winning probability of the werewolf regarding two choices.

Vote the guardian:

$$(1 + 0) * \frac{1}{2} = \frac{1}{2} = 0.5 \tag{7}$$

Vote the villager 1:

$$\left(\frac{2}{3} + 0\right) * \frac{1}{2} = \frac{1}{3} = 0.33 \tag{8}$$

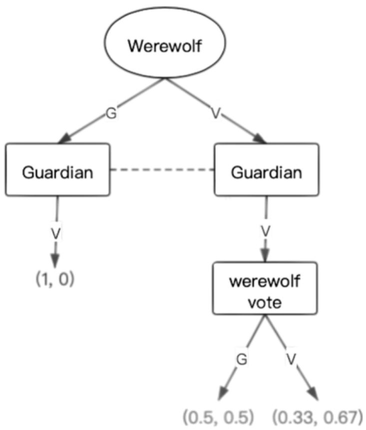

Figure 3 Subgame for the Night 2 when guardian guard himself the night 1

The werewolf knows the guardian guarded himself the first night if the guardian is not killed by daytime. Using the idea of backward induction, the best choice the werewolf has is to keep killing the guardian the second night and obtain victory. By this being clear, as long as the werewolf chooses to kill the guardian the first night and not being voted out the following daytime, he gets the victory. Then the payoff for the werewolf when the werewolf chooses to kill the guardian and the guardian also chooses to guard himself is:

$$1 * p_{g,total} + 1 * = 0.25 + 0.24 = 0.49 \tag{9}$$

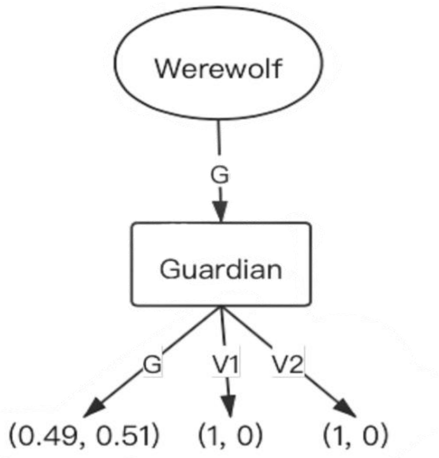

Figure 4 The simplified left branch of the game

In the scenario of a werewolf who chooses to kill a villager at night one, there is no difference between the two villagers and their behaviors, it is reasonable to treat the case where the werewolf managed to kill villager 1 and villager 2 the same.

If the guardian fails the first night, the three players remaining to vote will be the werewolf, the guardian, and a villager. This situation is discussed and calculated in the previous section. When the werewolf votes the guardian, the payoff is (0.5, 0.5). When the werewolf votes the villager, the payoff is (0.33, 0.67). By noticing that, the werewolf will always choose to vote the guardian in this scenario.

If the Guardian fails, when it comes to the voting process, the idea and the procedures are the same as conducted above. First, analyzing we consider the situation where one villager is voted out. And there are two circumstances.

Table 5. Two cases of the voting process

Case#	Detail
Case 1	The villager werewolf tried to kill is still in the game
Case 2	The villager werewolf who tried to kill is not in the game.

As shown in Table 5, for case 1, the idea is similar to the circumstance where the guardian guards himself successfully the first night. With backward induction, it is known that the werewolf would keep killing the same player, and the werewolf will win.

For case 2, it is slightly more complicated. Here is the extensive form of this subgame.

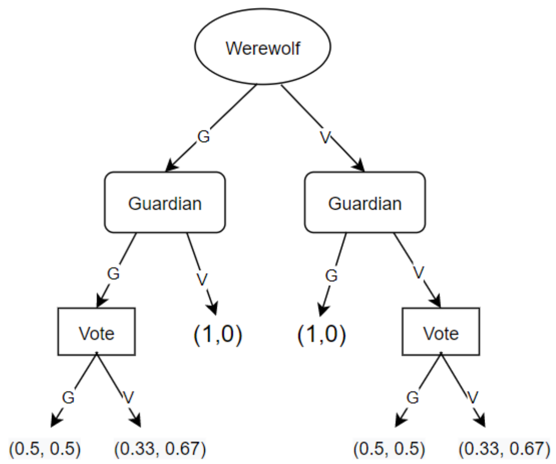

Figure 5 The night 2 subgame when the Werewolf's night 1 target is voted out

To find the payoffs, set the belief that the guardian will choose to guard himself that night with a probability p.

When the werewolf selects to kill the guardian, his payoff is:

$$0.5p + (1 - p) = 1 - 0.5p \tag{10}$$

When the werewolf selects to kill the villager, his payoff is:

$$p + 0.5(1 - p) = 0.5p + 0.5 \tag{11}$$

Set the two payoffs to be equal, the p is 0.5. This means if the p is less than 0.5, the werewolf should choose to kill the villager and vice versa.

The guardian does not receive the information from the werewolf. To simplify the game, we assume s/he will choose one player to guard at a 50 percent chance. Thus, the payoff for the werewolf of the whole process can be found as:

$$(0.5/8 + 0.33/8 + 1/4 + 1/4 + 0.5/8 + 0.33/8) = 0.71 \tag{12}$$

Then the payoff for the whole subgame can be treated as (0.71, 0.29).

3. ANALYSIS

Now the entire game is simplified. The complete game's extensive form is shown in Figure 6.

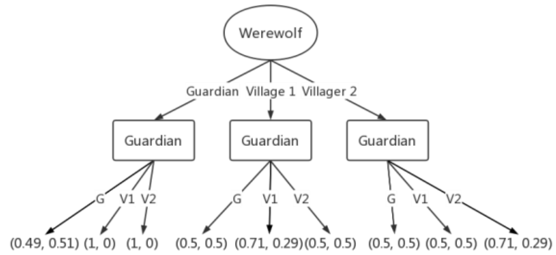

Figure 6 The complete game extensive form after simplification

To begin with, here is the payoff matrix that assists the author to compare each player's outcome, thus finding Nash equilibrium.

Table 6. The payoff matrix of the end-game of Werewolf game

Players		Guardian		
Werewolf	Actions	G	V1	V2
	G	(0.49, 0.51)	(1, 0)	(1, 0)
	V1	(0.5 0.5)	(0.71, 0.29)	(0.5 0.5)
	1	(0.5 0.5)	(0.5 0.5)	(0.71, 0.29)

Based on Table 6, the author can find the Nash equilibrium, which occurs when all the players have no incentives to deviate for a better payoff.

Step.1: In the condition that the werewolf chose to kill the guardian, the guardian will choose to protect herself since 0.51>0=0. In the condition that the werewolf chose to kill villager 1, the guardian will choose to protect either herself or villager 2, since 0.5=0.5>0.29. In the condition that the werewolf chose to kill villager 2, the guardian will choose to protect herself or villager 1 since 0.5=0.5>0.29.

Step.2: For the werewolf, when the guardian protects herself since the payoff will be 0.5=0.5>0.49, the werewolf could choose to kill the villager 1 or villager 2 randomly. When the guardian protects villager 1, the werewolf will kill the guardian because 1>0.71>0.5. Lastly, when the guardian protects villager 2, the werewolf will kill the guardian also because 1>0.71>0.5.

To clearly show the comparison's result, the payoffs of good camps are presented below.

Table 7. The payoff matrix of the end-game of Werewolf game with highlighted payoff

Players		Guardian		
Werewolf	Actions	G	V1	V2
	G	(0.49, 0.51)	(1, 0)	(1, 0)
	V1	(0.5 0.5)	(0.71, 0.29)	(0.5 0.5)
	1	(0.5 0.5)	(0.5 0.5)	(0.71, 0.29)

Referring to table 7, when the Nash equilibrium is (0.5, 0.5), which means that the werewolf will kill and the guardian will protect either villager, both players have no incentives to alter their actions.

4. DISCUSSION

The strategy discussed in this report assumes that all players are rational and their level of play is balanced. At the same time, in order not to fall into complicated logical disputes, the chain of suspicion is not considered too much. Still, the optimal strategy is determined intuitively by analyzing the benefits of the werewolves. The guard plays a traditional role in the werewolf killing, and its function is to protect a specific villager. We are studying a game segment where only one werewolf, one guard, and two villagers. In this case, since the guard and the werewolf already know each other's identities, this makes the guard's role more significant, and his survival can even determine the game's direction.

In the game, there are mainly dynamic games and asymmetric information games. The dynamic game refers to the sequence of actions of the participants, and the last step can observe the choice of the first actor and make corresponding choices accordingly. For example, voting-night alternation is a dynamic game. Since it is difficult for players to grasp accurate information about other people's strategies and preferences, most games contain incomplete information.

According to whether the guards can successfully guard, the situation is divided into two categories to discuss. The first type of werewolf choosing to kill the Guardian at night one, and safeguard successful protecting himself, the werewolf, is the indifference of whom to vote for, where s/he will have a probability of 0.47 to survive, the second night, the payoff of a werewolf is 0.49. The second type of choosing to kill the villager at night one and safeguard unsuccessful protecting him/her, the payoff is 0.33. And the payoff of protection is 0.71.

Ultimately, the best strategy for a werewolf facing two villagers and known guards is that the werewolf always chooses to kill the villagers.

5. CONCLUSION

The entire game process of Werewolves of Miller’s Hollow is a comprehensive application of game theory and psychology. When high-level players play together, taking into account the suspicion chain and Nash equilibrium will evolve into a fascinating game war. If analyzing the entire process, will be even more exciting. Game theory is a highly applied science. Theoretical models of game theory can generally find their prototypes in real life, and some rules of thumb in life can find similar elements in game theory.

Anything involving interaction between two or more people can be analyzed by game theory, combined with the common sense of psychology to gain advantages in daily life quickly. The crystallization analysis of human wisdom is the embodiment of game theory in specific life examples, and the particular life rules can best verify the frontiers of academic research, which is the meaning of the game theory.

Through the study of this topic, it also has a better understanding of how game theory is used in life, instead of just sticking to the numbers in the topic. At the same time, through the analysis of this game, it can be further extended to other games, and help us feel the happiness of the game theory when meeting with friends in life.

There are many shortcomings in this report. First of all, because the analysis process of multiple players needs to be solved with the help of computers, this paper chooses to study the game process, only one werewolf and one guarding and the two villagers. The second point is that assuming that the guard and the werewolf already know each other’s identities in asymmetric information. Only the villagers do not see any information. In real life, this is also a relatively unique situation. The third point is that many player characters have been cut, such as hunters, seers, etc., but these characters are indispensable in this game. Because of these more skills, the game between the good and the bad can also be more exciting. However, due to limited knowledge reserves, this part has not been studied. Finally, many psychological factors are also not taken into account. For example, during the game, werewolves or other villagers may take the initiative to propose that they are a particular priesthood to lead the situation to vote. However, this assumption will make the problem more complicated and impossible, so its not included in the report.

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