

The Application of Markowitz Model Based Series of Companies' Stock

Ke Wei ^{1,*}

¹ Business School, University of International Business and Economics, Beijing, 100029, China

*Corresponding author. Email: 201806098@uibe.edu.cn

ABSTRACT

When building successful investment portfolios, investors need to have efficient and accurate analysis models. This paper conducts a comprehensive analysis of ten companies' stock prices to build permissible portfolios in three constraints to advise on different types of investors. This article first studies the stock market and select ten stocks according to their stock price, finance, and operational performance. Secondly, this paper introduces the method used to build permissible investment portfolios based on Markowitz Model and divide investors into three types according to their ability to have positions or arbitrary constraints. Then this article uses Excel solver to build our investment portfolio under three constraints and analyze the difference and similarities. Finally, through all the analysis, this paper conclude that Regulation T doesn't have a significant influence on the investment portfolio. And arbitrary constraints will lead to a 26% decrease in return. Investors can choose the best portfolio according to their situation and their preference for risk and return. This paper provides a better forecasting method to use Markowitz Model to build investment portfolios and give advice to different investors.

Keywords: Markowitz Model, Investment Portfolios, Capital Market Line

1. INTRODUCTION

An investment portfolio is a collection of assets like stocks, bonds, funds, financial derivatives, and other assets. Investors often use portfolios to diversify their investment choice and reduce the overall risk of stocks in the whole market. This is proven to be effective in practice, and it is very important for investors to build their own portfolios to achieve returns and control risks. There are many investment portfolios, and the composition of investments in a portfolio depends on a number of factors. The most important is the investor's tolerance for risk and investment horizon. Investors need to decide how much return they expect and how much risk they can tolerate. And then, they choose the right combination and allocate funds reasonably. A core view of investment portfolio theory is that when diversifying investment, investors should consider the risk and return and the correlation between independent investment. It is necessary to invest in independent and irrelevant fields rather than simply buying a package of securities to diversify investment. Buying products that are very similar cannot achieve the purpose of decentralization. Therefore, people must consider collocation, for example, funds, bonds, large and small-cap, value and

growth. They are all similar funds, which greatly reduces the effectiveness of the portfolio. In other words, the two-dimensional factors originally considered in decentralized investment have been raised to three-dimensional space.

This paper mainly analyzes ten companies and their stock price to build permissible portfolios according to Markowitz Model. And this research divides investors into three types and set constraints for each type. Then this article compares the results and give investment advice to investors to help them choose the right combination.

The building investment portfolio is very popular among investors because it can reduce the overall risk of assets and realize expected returns. Thomas and Michele put forward a Multivariate-Arch in Mean model to analyze how domestic fiscal and monetary policy and changes in the international economic environment might affect interest rate differentials across countries [1]. Takaaki and Tatsuhiro used portfolio theory to the prediction of train arrival times and found it can improve prediction accuracy [2]. Frank applied portfolio theory to biodiversity and researched the implications for the valuation and management of biodiversity [3]. Ian

asserted that managed futures could help capitalize on upside price volatility to manage downside risk [4]. Vesna thought selecting and using different databases were important when determining the importance of numerous relevant elements to create an optimal investment portfolio [5].

Modern Portfolio Theory normally contains two basic theories, the Markowitz theory of Portfolio and the asset pricing theory. In most countries, especially developed countries, Markowitz's portfolio theory has long been proven effective in practice and widely used in portfolio selection and asset allocation. Rasoulzadeh and Fallah argued that combining mathematical models of portfolio optimization created a wide range of applications in portfolio optimization [6]. Bakhtiar, Omid, and Ali proposed a novel approach for the determination of the optimal bidding patterns among GenCos in the deregulated power market using a hybrid of the Markowitz Model and Genetic Algorithm [7]. Vivek and Meenakshi studied diversification and highlighted how many securities should include in a well-diversified portfolio based on Markowitz Model [8]. Miyoung, Jihun, and Sekyung put forward a new method to determine optimal portfolio weights in a mean-variance framework when assets are more than observations [9]. Sahar researched the efficacy of algorithms in portfolio optimization and found that the efficiency of GA is high in portfolio optimization [10].

For the research methods, this paper mainly uses Markowitz Model to build our portfolios. This article aggregates the daily data to the monthly observations and calculate all proper optimization inputs for the full Markowitz Model. Using these optimization inputs for MM, this paper finds the regions of permissible portfolios. This paper studies on investors' preference for return and risk and divide investors into three types according to their ability to have positions or arbitrary constraints. Then it compares the results under three constraints and try to explain the difference and similarity. Finally, after thorough and similarities research, it concludes that Regulation T by FINRA has little influence on investors' choice, and the influence of constraints is quite vital. Investors can choose one model and find the best investment portfolio.

For the overall arrangement, this article first introduced the data and companies in section 2. Then, section 3 focuses on the method used to set constraints and build permissible portfolios according to investors' preferences. In section 4, it uses Markowitz Model to build our investment portfolios and give advice to investors based on the similarity and differences of results. The last section presents our conclusions.

2. DATA AND COMPANIES

The paper collects a recent 20 years of historical daily total return data for ten stocks, which start from

5/11/2001 to 5/12/2021. These stocks are NVDA, CSCO, INTC, GS, USB, TD CN, ALL, PG, JNJ and CL. They perform better than most of the other stocks and they are in different industries. It also includes one (S&P 500) equity index and a proxy for risk-free rate (1-month Fed Funds rate) to build portfolios. And the introduction of these ten companies is as follows:

NVIDIA (NASDAQ: NVDA) is an artificial intelligence computing company, which was founded in 1993. The company is headquartered in Santa Clara, California, USA. NVIDIA is a global leader in programmable graphics processing technology. It focuses on building products that can enhance the human-computer interaction experience of personal and professional computing platforms. The company's graphics and communication processor has a wide market and has been adopted by a variety of computing platforms, including personal digital media PC, commercial PC, professional workstation, digital content creation system, notebook computer, military navigation system, and video game console. On May 21, 2020, NVIDIA released its financial report for the first quarter of 2021 as of April 26, 2020. Revenue in the first quarter reached US \$3.08 billion, up 39% from US \$2.22 billion in the same period. Datacentre revenue reached a record \$1.14 billion, an increase of 80% over the same period. GAAP's gross profit margin hit a record 65.1%.

Cisco Systems, Inc. (NASDAQ: CSCO) is a leading provider of Internet solutions, equipment and software products primarily used to connect computer networking systems. The company was founded in 1984 by Leonard Bosack, director of the computer center for the department of Computer Science, and Sandy Lerner, director of the computer center for the business School. According to a statement from Cisco Systems, the program is a global non-profit initiative, and the school does not pay any fees to the company. The Cisco Network Institute program uses e-learning as the teaching method, and students in any region learn exactly the same content in Cisco Network Institute.

Intel Corporation (NASDAQ: INTC) is an American company mainly developing CPU processors. It is the world's largest personal computer parts and CPU manufacturer and the first company to launch x86 processors. Its headquarters is located in Santa Clara, California, USA. Intel was founded on July 18, 1968, by Robert Noyce, Gordon Moore, and Andy Grove as Integrated Electronics, combining advanced chip design capabilities with industry-leading manufacturing capabilities. Intel also develops motherboard chipsets, network cards, flash memory, graphics chips, embedded processors, and other communications and computing-related products. On June 7, 2017, the 2017 Fortune 500 list was released, and Intel ranked 47th.

Goldman Sachs (NASDAQ: GS) is a leading international investment bank. Founded in 1869,

Goldman Sachs is one of the oldest and largest investment banks in the world, headquartered in New York. All of its operations are built on a tightly integrated global basis, with outstanding experts serving clients. Goldman Sachs Group also has the rich regional market knowledge and international operation capability. With the development of the global economy, the company continues to evolve and change to help clients identify and seize investment opportunities wherever they are in the world. On May 13, 2020, Goldman Sachs ranked 47th on the 2020 Forbes Global 2000 List.

Founded in 1895, Us Bancorp (NASDAQ: USB) is headquartered in Minneapolis, MN. With \$219 billion in assets, U.S. Bancorp is a U.S. bank holding company headquartered in Minneapolis, Minnesota and incorporated in the State of Delaware. U.S. Bancorp is a nationally chartered bank regulated by the Office of the Comptroller of the Currency and the Office of the Treasury to provide a full range of financial brokerage services through its subsidiaries and ATMs, as well as banking, brokerage, insurance, investment, mortgage, trust, and payment services to consumers, businesses and institutions. It is the parent company of US. Bank N.A., the fifth-largest banking institution in the United States. It has 2,434 branches and 4,232 ATMs, mostly in the West and Midwest. It ranks 117th on the Fortune 500 list and is considered a systemically important bank by the Financial Stability Board. The company also owns Elavon, which processes credit card transactions for merchants, and Elan Financial Services, a credit card issuer that issues credit card products on behalf of small CREDIT unions and banks in the United States.

Td Bank is one of the world's leading online financial services companies headquartered in Toronto, Canada (NASDAQ: TD CN), with offices worldwide. The TD Banking consortium serves more than 14 million customers through four businesses in many of the world's leading financial centers: Canadian personal and commercial banking, including TD Canada Trust; Property management including TDWaterhouse's global operations; Wholesale banking, including TD Securities; And US personal and commercial banking through TD Banknorth. As of July 31, 2005, the bank had total assets of c \$368.4 billion and total shareholder equity of C \$15.775 million, making it the third-largest chartered bank in Canada in terms of total market capitalization.

Allstate (NASDAQ: ALL) founded in 1931, is the second-largest personal insurance and casualty insurance company in the United States and ranks among the 15 largest life insurance companies in the United States. Allstate has been a leader in promoting life-saving

improvements, such as the use of seat belts and airbags, safe driving education programs for teenagers, stricter traffic laws, and the production and construction of safer cars and roads. Allstate is also investing millions of dollars in research to develop safety programs, either directly or through its affiliate, Tech-Cor. Allstate ranked 205 on the 2020 List of the world's 500 most valuable brands.

Procter & Gamble (Procter & Gamble) (NASDAQ: PG) is headquartered in Cincinnati, Ohio. Founded in 1837, P&G is one of the world's largest consumer goods companies. P&G employs nearly 100,000 people worldwide, has factories and branches in more than 80 countries, and operates more than 300 brands of products sold in more than 160 countries and regions, including fabric and home care, hair and beauty, baby and home care health care, food and beverage, etc. In the fiscal year 2003-2004, the company had annual sales of \$51.4 billion. It ranks 86th in fortune magazine's latest list of the world's 500 largest industrial services companies.

Johnson & Johnson (Johnson & Johnson), founded in 1886, is the world's largest and diversified medical and health care products and consumer care company. According to the results published by Fortune and Business Week in 1997, Johnson & Johnson's market value index ranked 20th in the world and ranked among the ten most envied companies in the United States, with a global turnover of \$27.5 billion in 1999. Johnson & Johnson has established more than 250 branches in 60 countries around the world, with more than 115,000 employees and products sold in 175 countries and regions. Johnson & Johnson manufactures and markets products in the care products, pharmaceutical products, and medical devices, and diagnostics markets. It owns Johnson baby, Neutrogena, Klingkli, Jiao Shuang, Bundi Dakning, Tylenol, and many other well-known brands.

Colgate-Palmolive was founded in 1806. It is the world's leading consumer goods company that sells goods in more than 200 countries in oral care, personal care, home care, and pet food for the masses to provide high-quality goods. On June 8, 2016, the Brandz Global Top 100 Most Valuable Brands 2016 was released, and Colgate ranked 54th. Colgate ranked 140th in the world's top 500 Brands list 2018.

Figure 1 describes the monthly return of the stocks. The horizontal axis shows how many months it has been since 5/11/2001. And the vertical axis shows how many times the stock price is higher than the original price. It is clear that these ten stocks` price has risen over time. The stock price of NVDA has increased pretty much while others rise steadily.

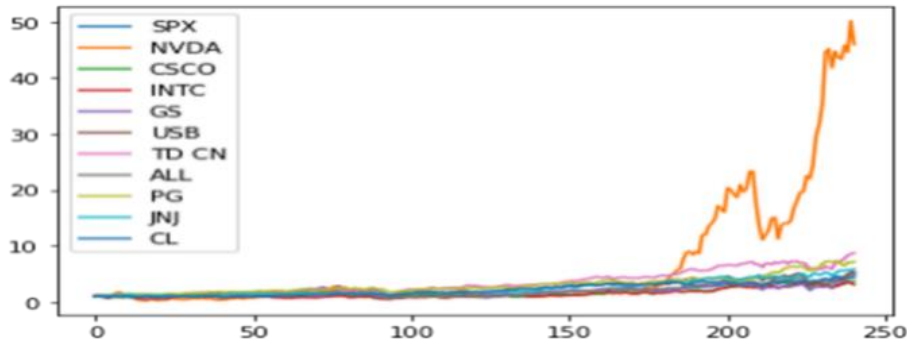


Figure 1 Monthly return of the stocks

3. METHOD

This research uses Markowitz Model to build the best investment portfolio under different constraints based on these eleven risky assets. Then it compares the results to conclude the similarity and differences between these portfolios and explain the possible reasons. Finally, it gives some advice for investors to make decisions based on our research.

In order to reduce the non-Gaussian effects, this article aggregates the daily data to the monthly observations and based on those monthly observations. It calculates all proper optimization inputs for the full Markowitz Model ("MM"). Using these optimization inputs for MM, it finds the regions of permissible portfolios, which include the Minimal Risk or Variance Frontier, Minimal Risk Portfolio, Maximal Sharpe Ratio or Efficient Risky Portfolio, Efficient Frontier, Capital Allocation Line, and Minimal Return Frontier.

Minimal Risk or Variance Frontier:

$$\begin{cases} \sigma(\vec{w}) \rightarrow \min_{\vec{w}} \\ \text{subject to: } r(\vec{w}) = \text{const} \end{cases} \quad (1)$$

Minimal Return Frontier:

$$\begin{cases} r(\vec{w}) \rightarrow \min_{\vec{w}} \\ \text{subject to: } \sigma(\vec{w}) = \text{const} \end{cases} \quad (2)$$

Efficient Frontier:

$$\begin{cases} r(\vec{w}) \rightarrow \max_{\vec{w}} \\ \text{subject to: } \sigma(\vec{w}) = \text{const} \end{cases} \quad (3)$$

Minimal Risk Portfolio:

$$\sigma(\vec{w}) \rightarrow \min_{\vec{w}} \quad (4)$$

And Efficient Risky Portfolio:

$$\begin{cases} r(\vec{w}) \\ \sigma(\vec{w}) \end{cases} \rightarrow \max_{\vec{w}} \quad (5)$$

This paper researches investor's preferences for return and risk. And it finds that investors can be divided into three types according to their ability to have positions or arbitrary constraints. Therefore, this article will see these permissible portfolios in three constraints, which are as follows:

Firstly, it will study a "free" problem, without any additional optimization constraints, to illustrate how the area of permissible portfolios in general and the efficient frontier, in particular, look like if no constraints.

Secondly, it simulates the Regulation T by FINRA, which allows broker-dealers to allow their customers to have positions, 50% or more of which are funded by the customer's account equity:

$$\sum_{i=1}^{11} |w_i| \leq 2 \quad (6)$$

Finally, it simulates some arbitrary constraints on weights, which may be provided by the client:

$$|w_i| \leq 1, \text{ for } \forall i \quad (7)$$

4. ANALYSIS

In order to reduce the non-Gaussian effects, it aggregates the daily data to the monthly observations. Table 1 shows the annualized data of SPX and ten stocks. NVDA has the highest return and alpha, which means investors can earn much more money compared with other stocks. On the other hand, it also has the highest StDev, beta and residual StDev, which means investors have to take more risks. CL has the lowest annualized average return and JNJ has the least residual StDev. PG has the least annualized StDev and beta, which means it is safer to hold PG's stocks. INTC and CSCO have a negative alpha value. Therefore, it is unwise to invest a lot of money in these two stocks.

Table 1. Annualized Data

	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL
annualized average return	7.542%	32.802%	9.714%	8.905%	10.825%	9.878%	11.010%	10.080%	9.437%	8.464%	7.105%
annualized StDev	14.850%	55.774%	30.809%	30.503%	29.572%	23.680%	18.134%	24.884%	14.587%	14.785%	15.350%
beta	1	1.979	1.321	1.188	1.410	0.971	0.787	1.056	0.405	0.540	0.454
annualized alpha	0	0.179	-0.002	-0.001	0.002	0.026	0.051	0.021	0.064	0.044	0.037
annualized residual StDev	0.000%	47.405%	23.762%	24.889%	20.881%	18.781%	13.865%	19.317%	13.288%	12.423%	13.787%

Then it can be calculated that the correlation coefficient of these eleven assets. The results are shown in Table 2. None of the stocks has a high correlation coefficient with others. The Goldman Sachs Group and

SPX have the highest correlation coefficient, which is 0.708092. In contrast, Intel Corporation and Colgate-Palmolive Company have the lowest correlation coefficient, which is 0.110064.

Table 2. Correlation Matrix

	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL
SPX	1	0.527	0.637	0.578	0.708	0.609	0.645	0.630	0.412	0.542	0.440
NVDA	0.527	1	0.487	0.524	0.343	0.160	0.338	0.157	0.060	0.165	0.069
CSCO	0.637	0.487	1	0.614	0.487	0.328	0.410	0.297	0.220	0.239	0.165
INTC	0.578	0.524	0.614	1	0.411	0.20	0.411	0.286	0.136	0.325	0.110
GS	0.708	0.343	0.487	0.411	1	0.472	0.494	0.417	0.173	0.296	0.203
USB	0.609	0.160	0.328	0.280	0.472	1	0.539	0.540	0.336	0.234	0.218
TD CN	0.645	0.338	0.410	0.412	0.494	0.539	1	0.417	0.231	0.273	0.212
ALL	0.630	0.157	0.297	0.286	0.417	0.540	0.417	1	0.346	0.452	0.407
PG	0.412	0.060	0.220	0.136	0.173	0.336	0.231	0.346	1	0.494	0.483
JNJ	0.542	0.165	0.239	0.325	0.296	0.234	0.273	0.452	0.494	1	0.527
CL	0.440	0.069	0.165	0.110	0.203	0.218	0.212	0.407	0.483	0.527	1

Then this article uses Excel solver to build our investment portfolio according to Markowitz Model.

And the results of three cases are shown in Table 3 and Table 4

Table 3. Efficient Frontier

Efficient Frontier					
constraint 2		constraint 3		constraint 1	
Return	StDev	Return	StDev	Return	StDev
0.113	0.12	0.113	0.12	0.113	0.12
0.141	0.14	0.142	0.14	0.142	0.14
0.158	0.16	0.165	0.16	0.165	0.16
0.171	0.18	0.183	0.18	0.185	0.18
0.184	0.2	0.199	0.2	0.204	0.2
0.192	0.22	0.214	0.22	0.222	0.22
0.203	0.24	0.227	0.24	0.240	0.24
0.213	0.26	0.240	0.26	0.257	0.26
0.223	0.28	0.253	0.28	0.274	0.28
0.232	0.3	0.266	0.3	0.290	0.3

0.243	0.32	0.278	0.32	0.307	0.32
0.254	0.34	0.290	0.34	0.323	0.34
0.264	0.36	0.302	0.36	0.339	0.36
0.273	0.38	0.314	0.38	0.355	0.38
0.412	0.768	0.325	0.4	0.371	0.4
		0.337	0.42	0.387	0.42
		0.349	0.44	0.403	0.44

Table 4. Capital Allocation Line

CAL					
constraint 1		constraint 2		constraint 3	
Return	StDev	Return	StDev	Return	StDev
0	0	0	0	0	0
0.140	0.140	0.166	0.161	0.170	0.165
0.280	0.279	0.331	0.321	0.340	0.329
0.560	0.558	0.662	0.643	0.680	0.659

There are some differences between the results of these three constraints. The efficient frontier of constraint 1 is similar to that of constraint 2. When the StDev is below 0.18, the results are totally the same. But as the risk rises, the return in constraint 1 becomes higher than that of constraint 2. And the gap between the two constraints becomes bigger when the risk increases. And the return of constraint is always lower than constraint 1 and constraint 2. And the slope of CAL in constraint 1 is about 1.031. This is bigger than constraint 2 and constraint 3, which are 1.030 and 1.004, respectively. Therefore, it can be concluded that constraint 2 doesn't have significance on the investment portfolio. And constraint 3 will lead to a 26% decrease in return.

Therefore, the Regulation T by FINRA has little influence on investors' choices. But if there are some constraints, the influence is quite vital. This article builds three permissible portfolios in three constraints, and these can be quite helpful for investors. They can choose one according to their ability to have positions or arbitrary constraints. Based on their preference for risk and return, they can find the best investment portfolio.

5. CONCLUSION

This paper researches the performance of ten stocks' prices and build effective investment portfolios. Based on Markowitz Model, it builds our portfolio based on recent 20 years of historical daily total return data and analyses the portfolio under three constraints. Finally, it gives some investment advice according to our results.

This paper first researches these ten companies' operation and finance performance. All of them perform pretty well, and some of them are top companies in their industries around the world. Then it analyses the annualized data and correlation coefficient based on raw data. It can be seen that NVIDIA's stock price has risen quite much, and the correlation coefficient is not very high. Finally, this article uses Markowitz Model to build our portfolios. It divides investors into three types according to their ability to have positions or arbitrary constraints. Based on research. Each type has its own constraints like Regulation T by FINRA and some arbitrary constraints. This research finds that Regulation T doesn't have a significant influence on the investment portfolio. And arbitrary constraints will lead to a 26% decrease in return. Investors can choose one constraint and find the best portfolio according to their situation and their preference for risk and return.

The shortcomings of this paper mainly include two aspects. First, it only studies on ten stocks, and there are much more stocks that can be included in our portfolio. Second, this paper only sets three constraints. The methods used to build portfolios may not be useful under other constraints. In the future, this research will include more stocks and build our portfolios under more constraints.

REFERENCES

[1] Thomas J. Flavin and Michele G. Limosani. (2004), Fiscal, monetary policy and the conditional risk premium in short-term interest rate differentials: an

- application of Tobin's portfolio theory. *International Review of Economics and Finance*, 16(1), pp. 101-112.
- [2] Takaaki Yamada and Tatsuhiro Sato. (2017), Portfolio theory application to prediction correction of train arrival times. *Int. J. of Computational Intelligence Studies*, 6(4), pp. 349-365.
- [3] Frank Figge. (2004). Bio-folio: applying portfolio theory to biodiversity. *Biodiversity and Conservation*, 13(4), pp. 827-849.
- [4] Ian Hudson. (2011), Managing Downside Risk With Upside Volatility: Enhancing The Performance Of An Investment Portfolio With Managed Futures. *Journal of Business & Economics Research (JBER)*, 6(4), pp. 2407-2407.
- [5] Vesna Trančar. (2017), The Influence of One's Own Database on the Accuracy of Forecasting Future Movements of Investment Portfolio Value. *Naše gospodarstvo/Our economy*, 63(2), pp. 42-48.
- [6] Rasoulzadeh Mehrdad and Fallah Mohammad. (2020), An overview of portfolio optimization using fuzzy data envelopment analysis models. *Journal of Fuzzy Extension and Applications*, 1(3), pp. 187-197.
- [7] Bakhtiar Ostadi and Omid Motamedi Sedeh and Ali Husseinzadeh Kashan. (2020), Risk-based optimal bidding patterns in the deregulated power market using extended Markowitz model. *Energy*, 191(C), pp. 116516-116516.
- [8] Vivek Chawla and Meenakshi. (2019), Diversified portfolio: Evidence from listed securities of bombay stock exchange (BSE) in India. *Asian Journal of Multidimensional Research (AJMR)*, 8(4), pp. 403-412.
- [9] Miyoung Lee and Jihun Kim and Sekyung Oh. (2018), A Portfolio Optimization Approach with a Large Number of Assets: Applications to the US and Korean Stock Markets. *Asia-Pacific Journal of Financial Studies*, 47(5), pp. 634-659.
- [10] Sahar Mojaver Rostami. (2018), Portfolio optimization based on minimum total risk acceptance level and its components using improved genetic algorithm. *Int. J. of Computational Systems Engineering*, 4(4), pp. 248-263.