

# Various Methods Aim to Solve the Limitation of IRR

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## ABSTRACT

Both NPV and IRR are metrics that are used to calculate the profit and help investors to make decisions about their investments. While these methods help investors to identify the most profitable project, we cannot ignore some limitations of using them as well. Through analysing the strengths and limitations of these two methods, the modified internal rate of return and financial management rate of return is considered as the complement of IRR. This paper suggests that it is unrealistic to have a perfect metric without any limitation, but several methods can be combined and applied into practice.

**Keywords:** *internal rate of return, net present value, modified internal rate of return*

## 1. INTRODUCTION

NPV and IRR are helpful methods that are widely used by investors or risk managers, but several inevitable drawbacks appear to them when they apply these in the real life. Therefore, it's essential to find some alternatives to compensate for those drawbacks [1-3].

In this essay, we firstly define NPV and IRR and explain how they are beneficial to our decision-making [4]. By comparing the internal rate of return (IRR) with the cost of capital of the project ( $k$ ), we can distinguish which projects are worth investing in. During our research, we find some limitations of IRR, like the huge effects of investment scale cannot be indicated through IRR and it cannot calculate more than one cash flow which means that other methods are needed [5-6].

Hence, we think about other methods which have similar functions to these two to cover the limitations. We firstly analyse the reason two different IRR exist, and that is due to the huge negative cash flow in the final year which occurs frequently in reality. Modified internal rate of return (MIRR) is the first method we thought about since it turns complicated multiple cash flows into just two cash flows by compounding all the

positive cashflows and adding to the final year, all the negative cash flows will be discounted and added to the first year [7]. This seems a little out of reality, however, another method FMRR is introduced to solve the problem. In this metric, the reinvestment rate is not assumed to be equal to the discount rate. Instead, the discount rate is replaced by the safe rate which shows that there will be funds available to conquer the periodic negative cashflows. But this method is rarely used by people since it's too sophisticated.

Therefore, we conclude that no metric or model in the world is perfect, and they all have diverse limitations. So, when investors are making the decision, they are supposed to use more than one method to generate the best choice for them.

The remainder of the paper is organized as follows: Section 2 describes the sample and data; Section 3 explain the conditions of using IRR and compare it with the cost of capital; Section 4 introduces the advantages of both NPV and IRR, showing their limitations as well; Section 5 proposes two alternative methods which are MIRR and FMRR to compensate limitations we mentioned in Section 5. The last section presents our conclusions.

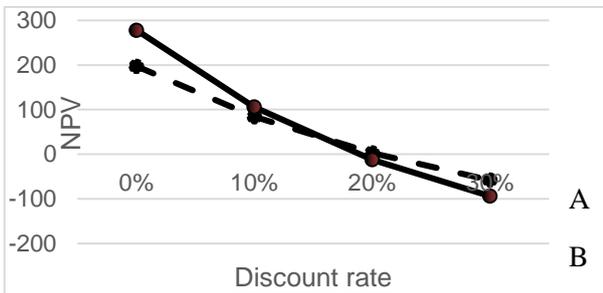
**2. DATA**

With an initial endowment of 430 in  $t = 0$ , an investor can invest in the following mutually exclusive and divisible investment projects:

**Table 1** Cash Flows (\$)

Project	CF <sub>0</sub>	CF <sub>1</sub>	CF <sub>2</sub>	CF <sub>3</sub>	CF <sub>4</sub>
A	-430	230	179	124	94
B	-430	70	138	240	260

Table 1 illustrated the cash flow of 2 available projects with different cashflows but the same initial endowment of 430.



**Figure 1.** NPV graph

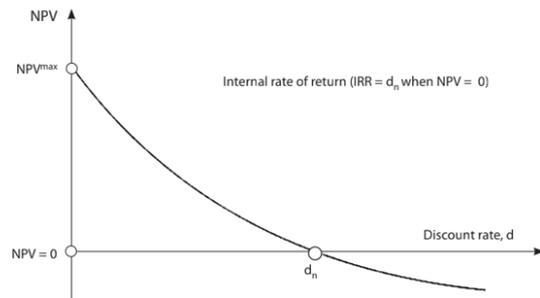
**Table 2** IRR results

	Project A		Project B
Initial investment	-430	Initial investment	-430
Cash flow 1	230	Cash flow 1	70
Cash flow 2	179	Cash flow 2	138
Cash flow 3	124	Cash flow 3	240
Cash flow 4	94	Cash flow 4	260
IRR	20%	IRR	19%

Based on Figure 1 and Table 2, it is clearly illustrated that at 15.3% discount rate, Project A and B are having the same NPV. Therefore, based on the comparison between NPV, it is suggested that between 0% to 15.3% discount rate, Project B should be the project to choose. But after 15.3% discount rate, Project A is better. Although the IRR results indicate that Project A is the better project to invest, it is unreliable in this case. As it is a percentage-based result that ignores the magnitude of investment and cannot determine the details.

**3. THE USE OF IRR**

To define the internal rate of return, it is closely related to the net present value. As the return on investment requires a period and the time value has to be accounted in money, the value of all future cash flows (positive and negative) over the entire life of an investment discounted to the present is the net present value (NPV). NPV indicates the return on the investment and provides investors with an indication of whether to engage in a project. Because the discount rate in NPV is uncertain, the net present value varies depending on the discount rate used in the calculations. Furthermore, the internal rate of return (IRR) is the discount rate applied to a project or investment where  $NPV=0$ .



**Figure 2** The point that intersects with the x-axis is called its internal rate of return (IRR).

Another way to evaluate the investment in a project is to analyse the internal rate of return. Companies frequently use it to determine which projects or investments are worthy of funding. For instance, when a company is attempting to compare the profitability of expanding existing projects versus establishing new projects, IRR can assist it in determining which option has the greatest potential to generate a quality rate of return and selecting the most effective decision. Even if the actual rate of return differs from the estimated IRR, projects with a much higher IRR than alternatives are likely to produce more value.

In addition, the IRR can be useful for companies considering a share buy-back program; if the IRR of the company's shares is lower than that of other potential projects, a stock repurchase may not be the best option.

To decide whether to invest, can also compare the internal rate of return (the maximum cost that can be borne) with the cost of capital of the project (k):

- When the internal rate of return > cost of capital (k), the actual cost of the project is within acceptable limits, the project is considered acceptable;
- When the internal rate of return < cost of capital (k), the actual cost of the project is over the acceptable limit, the project is rejected;

- And when the internal rate of return = cost of capital (k), the project is breakeven.

The formula for the Internal Rate of Return can be written as follows:

$$C_0 = \left[ \frac{C_1}{(1+k)} + \frac{C_2}{(1+k)^2} + \frac{C_3}{(1+k)^3} + \dots + \frac{C_n}{(1+k)^n} \right] \quad (1)$$

$$C_0 = \sum_{t=1}^n \frac{C_t}{(1+k)^t} \quad (2)$$

$$\sum_{t=1}^n \frac{C_t}{(1+k)^t} - C_0 = 0 \quad (3)$$

It should also be noted that the IRR of an investment is endogenous in the case that it is not obtained from the market or anywhere else; instead, it is strictly a function of the various cash flows of the investment itself, without being able to use the net benefit beyond the limits of proposed project. As a result, the method should focus solely on the specific distribution of income and investments. In general, if the investment and its return can be expressed in terms of cash flows, the IRR should be calculated using the equation below:

$$\sum_{t=1}^n \frac{CF_t}{(1+IRR)^t} - I = 0 \quad (4)$$

The IRR is the discount rate at which the NPV equals zero.

**4. STRENGTH AND LIMITATION**

IRR and NPV are basic and useful ways for people to make decisions when they invest. Whereas these two methods are like a double-edged sword.

There are several advantages to applying NPV in real cases. It is the net value of all present values of future cash flows plus the cash flow that invest money in during the period. From the formula of NPV, we can see that the NPV takes discounting rates (r) and time value of money (t) into consideration, which makes it can objectively measure the price of the project as a method of discounting cash flow. It also introduces risk factors into the decision-making of capital projects through the lowest rate of return, which can enlarge the risks. The answer of NPV has a uniqueness that there will not be another answer for one project, so when projects are mutually exclusive, it can provide an effective basis for decision-making; projects can be selected easily by comparing the size of the NPV.

However, NPV cannot cover every aspect, and there are still limitations [8-9]. Firstly, it is hard to directly reflect the actual rate of return of the investment project from a dynamic perspective, and the calculation is relatively cumbersome. (The actual income level of the project is not shown). The NPV ignores the rate of return of the project, which makes it somehow

unpractical; the discount rate is not flexible, so it is difficult to determine; the NPV is unable to give the rate of return on investment of the project, which is not conducive to decision-making. What is more, in the case of different project investment amounts, the project with the highest investment efficiency cannot be determined.

Compared with the NPV method, The advantages of IRR are these: the internal rate of return (IRR) method is less difficult to operate and easy to make decisions. it can link the income during the life of the project with the total investment, point out the return rate of this project, and compare it with the benchmark investment return rate of the industry to determine whether the project is worth investing in. IRR also considers the time value of money(T), and the return of the investment project in all periods is considered; IRR also covers the inherent rate of return of the investment project.

However, the problems of IRR are these: the internal rate of return represents a ratio, not an absolute value. A scheme with a lower internal rate of return may have a larger net present value due to its larger scale and therefore is more worthy of construction. Therefore, IRR cannot reflect the impact of the investment scale. For that reason, when choosing a comparison between various options, the internal rate of return and the net present value must be considered together. Meanwhile, IRR is only applicable to projects with fixed cash flow. If the cash flow of each period is not equal, other methods need to be used to calculate IRR, which makes the calculation more complicated.

**5. ALTERNATIVE METHODS**

Although the IRR rule is very helpful and easy to manipulate, it has various limitations [10]. Multiple rates of return could be one of them. Multiple rates of return take place when various discount rates make zero NPV. A complicated cash flow could cause two or even more than two IRR.

To begin with, two internal rates of return could be caused when there is a huge negative cash flow in the final year. The NPV graph will have 2 interceptions with the horizontal axis as shown in Fig 3. This is a scenario that we frequently encounter in our life. For instance, when we invest in a nuclear station, there will be an enormous amount of expense in the final year to clean up the liquid waste produced by the station. Furthermore, if the cash flow is extremely irregular with some negative cash flows during the period, more than 2 interceptions could be formed as shown in Fig 4. This can also be a common phenomenon. For instance, an aviation corporation will have to repair or replace the elements, which might generate huge costs during that period and lead to a cash flow diagram that fluctuates.

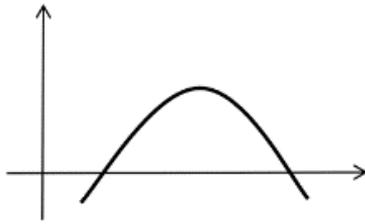


Figure 3. NPV graph

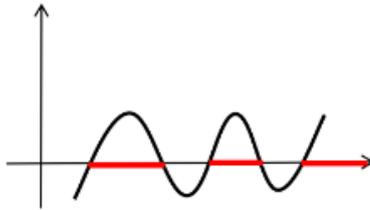


Figure 4. Interceptions

This is a dilemma since an investor may not be able to determine which IRR to use when there is more than one IRRs. To solve this problem, three methods could be utilized. NPV method, modified internal rate of return (MIRR), and financial management rate of return (FMRR) may be better alternatives when projects have multiple rates of return.

The first approach is simple, investors will only choose the projects with positive NPVs to invest in. For example, as shown in Fig 4, when the discount rate lies in the red segment, NPV is positive and the project should be invested. Although this method is simple, it may not enable us to compare the projects with multiple rates of return if we only know the range of discount rates for which the project is acceptable. A quantity may be required to illustrate which project provides the highest return when we have many projects with multiple rates of return.

Then, the MIRR model could be utilized, which can modify sophisticated cashflows into only 2 cashflows. In this model, future cash flow is assumed to be reinvested at the discount rate. For example, consider the project as shown in Table 3 where the reinvestment rate and discount rate are 10%.

Table 3. Cashflow Scheme

C	C <sub>1</sub>	C	...	C <sub>9</sub>	C <sub>10</sub>
0		2			
-	2	2	...	2	-13
6					

Due to the huge negative cash flow in the final year, two IRRs will be formed. However, we could modify the cash flow as shown in Fig 5. All the positive

cashflows should be compounded and added to the final year and all the negative cash flows should be discounted and added to the first year. After the manipulation, the cash flow has been transformed into a much simpler one with only 2 cashflows as shown in Fig 6. If we calculate the IRR for Fig 7, the result will be the MIRR.

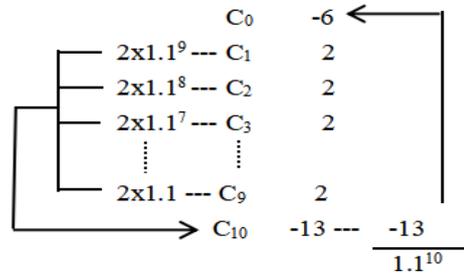


Figure 5. the process of the calculation

C <sub>0</sub>	-6 + (-13)/1.1 <sup>10</sup> = -11
C <sub>1</sub>	0
C <sub>2</sub>	0
C <sub>3</sub>	0
⋮	
C <sub>9</sub>	0
C <sub>10</sub>	2x1.1 <sup>9</sup> +2x1.1 <sup>8</sup> +...+2x1.1=30

Figure 6. the detailed results.

The derivation of MIRR may be summarized using the Identity shown below, where *Fv* represents compounded positive value and *Pv* represents the discounted negative value. In general, MIRR is the discount rate that discounts the sum of future value back to equal the initial endowment.

$$IRR = \sqrt[n]{\frac{-Fv}{Pv}} - 1 \tag{5}$$

Although MIRR is close to perfect, the assumption that discount rate is equal to reinvestment rate may not be realistic since they are not always equal in real life. In contrast, FMRR could be used because it is thought to be closer to real situations. In this model, the reinvestment rate is not assumed to be equal to the discount rate and the discount rate is replaced by a term called safe rate. A safe rate indicates that there will be funds available to conquer the periodic negative cashflows. The positive cash flow that occurs immediately before the negative cash flow will be used to cover that loss. The following considers the previous example using the FMRR model where the discount rate is 10% and the safe rate is 5%.

In the calculation, the negative cash flow will be discounted and added to the previous year. If the negative cash flow is eliminated by the previous cash

flow, the sum will be compounded and added to the final year. If the negative cash flow is too big to be eliminated, the sum will be discounted and added to the first year. In this case, the negative cash flow has a huge magnitude, so the sum of the two years will be added to the first year as shown in Fig 7. Then a modified cash flow is formed and one IRR could be calculated, which is FMRR as shown in Fig 8.

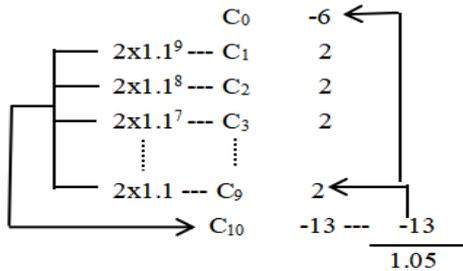


Figure 7. the process of the calculation

C <sub>0</sub>	$((-13)/1.05 + 2)/1.05^0 - 6 = -13$
C <sub>1</sub>	0
C <sub>2</sub>	0
C <sub>3</sub>	0
⋮	
C <sub>10</sub>	$2 \times 1.1^9 + 2 \times 1.1^8 + \dots + 2 \times 1.1^2 = 28$

Figure 8. the detailed results.

Although it could be an upgrade version of MIRR, few people have utilized it due to its complexity. In addition, this model is not feasible when two negative cashflows take place in 2 or more consecutive years. In that circumstance, MIRR could be better.

## 6. CONCLUSION

This article analyzes the different choices of investment evaluation methods. Compared with the NPV and IRR methods, we find that MIRR could be better as the evaluation way in the different investment projects. Consider the previous example using the FMRR model where the discount rate is 10% and the safe rate is 5%. However, the MIRR would be great for that result.

However, the paper also exists some limitations. We do not collect realistic data as an example. In the future, we would cooperate with the firms and utilize the firms' data to consider the best choices.

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